Have approx. analytic solution for convective regions:
convective contrains $T \Rightarrow$ vertical structure of baroclinic pressure gradients
$\Rightarrow$ vertical structure of $v \Rightarrow$ vertical structure of $\omega$
Extend to full nonlinearity, non-convective regions,…
Use analytic solutions for leading basis functions in
Galerkin expansion in vertical

\[ T = T_r(p) + \sum_{k=1}^{K} a_k(p)T_k(x, y, t) + T_R, \]

• Horizontal gradients of $T$ matter; specify reference state $T_r(p)$ to improve accuracy.
• Simplest case: 1 basis function in $T, q$
  Extra basis function for external mode $\Rightarrow$ 2 in $v$
Model Summary – QTCM equations

\[ \partial_t \mathbf{v}_1 + D_{VI}(\mathbf{v}_0, \mathbf{v}_1) + f k \times \mathbf{v}_1 = -\kappa \nabla T_1 - \text{stress} \]

\[ \partial_t \mathbf{v}_0 = \ldots \text{ (barotropic component)} \]

\[ \hat{a}_1 (\partial_t + D_{Tl}) T_1 + M_{S1} \nabla \cdot \mathbf{v}_1 = \langle Q_c \rangle + \text{Rad} + H \]

\[ \hat{b}_1 (\partial_t + D_{q1}) q_1 + M_{q1} \nabla \cdot \mathbf{v}_1 = \langle Q_q \rangle + E \]

Moisture sink and convective heating

\[ -\langle Q_q \rangle = \langle Q_c \rangle = \varepsilon_c (q_1 - T_1) \]
Quasi-equilibrium schemes

Posit that bulk effects of convection tend to establish statistical equilibrium among buoyancy-related fields.

Approach here depends on convection tending to constrain vertical structure of temperature field.

For now: Smoothly posed convective adjustment

Convective heating: (Betts 1986; Betts & Miller 1986)

\[ Q_c = \frac{(T_c - T)}{\tau_c} \]

- \( \tau_c \): time scale of convective adjustment
- \( T_c \): convective reference profile; depends on \( h_b \) for convection arising out of PBL
- \( h_b \): planetary boundary layer (PBL) moist static energy after adjustment by downdrafts to satisfy energy constraint
- \( T_c \): typically moist adiabat or closely related

Can be expanded about a reference state, \( T(p) \)

\[ T_c = T_c + A(p)h_b' + \text{higher order} \]

- \( A(p) \): vertical dependence of the moist adiabat perturbation per \( h_b \) perturbation

Tends to reduce CAPE (convective available potential energy)
Analytical solution under quasi-equilibrium convective constraints

If $T$ constrained to be close to QE temp $T^c$

$$T \approx T^c \approx T_r^c + A(p)T_1^c$$

Primitive equations, momentum + hydrostatic:

$$\left( \partial_t + D_m \right)v + f k \times v = -\kappa \nabla \int_{p_0}^{p} T d \ln p + \nabla \phi_o$$

$$\approx -\kappa \int_{p_0}^{p} A(p) d \ln p \nabla T_1^c + \nabla \phi_o$$

baroclinic pressure gradients have strongly constrained vertical structure
Vertical structure of baroclinic pressure gradients
⇒ structure of baroclinic wind $V_1$. With barotropic component
⇒

$v = v_o(x,y,p,t) + V_1(p)v_1(x,y,t)$

Continuity eqn. ⇒

$\omega = \Omega_1(p) \nabla \cdot v_1$

The moist static energy eqn. becomes

$$(\partial_t + D)(\hat{T} + \hat{q}) + M\nabla \cdot v_1 = F_{net}$$

where $M$ is the gross moist stability $M = \langle \Omega \partial_p h \rangle$

NB: Have not yet used convective closure on moisture.