

Have approx. analytic solution for convective regions:
convective constrains $T \Rightarrow$ vertical structure of baroclinic
pressure gradients

\Rightarrow vertical structure of $v \Rightarrow$ vertical structure of ω

Extend to full nonlinearity, non-convective regions,...

Use **analytic solutions** for leading **basis functions** in
Galerkin expansion in vertical

$$T = T_r(p) + \sum_{k=1}^K a_k(p) T_k(x, y, t) + T_R,$$

- Horizontal gradients of T matter; specify reference state $T_r(p)$ to improve accuracy.
- Simplest case: 1 basis function in T, q

Extra basis function for external mode \Rightarrow 2 in v

Model Summary – QTCM equations

$$\partial_t \mathbf{v}_1 + D_{V1}(\mathbf{v}_0, \mathbf{v}_1) + f\mathbf{k} \times \mathbf{v}_1 = -\kappa \nabla T_1 - \text{stress}$$

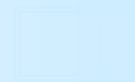
$$\partial_t \mathbf{v}_0 = \dots \text{ (barotropic component)}$$

$$\hat{a}_1(\partial_t + D_{T1})T_1 + M_{S1} \nabla \cdot \mathbf{v}_1 = \langle Q_c \rangle + \text{Rad} + H$$

$$\hat{b}_1(\partial_t + D_{q1})q_1 + M_{q1} \nabla \cdot \mathbf{v}_1 = \langle Q_q \rangle + E$$

Moisture sink and convective heating

$$-\langle Q_q \rangle = \langle Q_c \rangle = \varepsilon_c(q_1 - T_1)$$



Quasi-equilibrium schemes

Posit that bulk effects of convection tend to establish statistical equilibrium among buoyancy-related fields

Approach here depends on **convection** tending to **constrain vertical structure of temperature** field.

For now: **Smoothly posed convective adjustment**

Convective heating: (Betts 1986; Betts & Miller 1986)

$$Q_c = (T_c - T) / \tau_c$$

τ_c time scale of convective adjustment

T_c convective reference profile; **depends on h_b** for convection arising out of PBL

h_b planetary boundary layer (PBL) moist static energy after adjustment by downdrafts to satisfy energy constraint

T_c typically moist adiabat or closely related

Can be expanded about a reference state, $T(p)$

$$T_c = T_c + A(p)h_b' + \text{higher order}$$

$A(p)$ vertical dependence of the moist adiabat perturbation per h_b perturbation

Tends to reduce CAPE (convective available potential energy)

Analytical solution under quasi-equilibrium convective constraints

If T constrained to be close to QE temp T^c

$$T \approx T^c \approx \underline{T_r^c} + A(p)T_1^c$$

Primitive equations, momentum + hydrostatic:

$$\begin{aligned} (\partial_t + D_m)v + f\mathbf{k} \times v &= -\kappa \nabla \int_p^{p_o} T d \ln p + \nabla \phi_o \\ &\approx -\kappa \int_p^{p_o} \underline{A(p)} d \ln p \underline{\nabla T_1^c} + \nabla \phi_o \end{aligned}$$

baroclinic pressure gradients have strongly constrained vertical structure

Analytic solution in deep convective regions (cont.)

Vertical structure of baroclinic pressure gradients
⇒ structure of baroclinic wind V_1 . With barotropic component ⇒
$$v = v_o(x,y,p,t) + V_1(p)v_1(x,y,t)$$

Continuity eqn. ⇒
$$\omega = \Omega_1(p) \nabla \cdot v_1$$

The moist static energy eqn. becomes

$$(\partial_t + \mathbf{D})(\hat{T} + \hat{q}) + M \nabla \cdot v_1 = F_{net}$$

where M is the gross moist stability $M = \langle \Omega \partial_p h \rangle$

NB: Have not yet used convective closure on moisture.