On the Interpretation of the Gill Model

J. DAVID NEELIN*

Department of Earth, Atmospheric and Planetary Sciences, Massachusets Institute of Technology, Cambridge, Massachusetts

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1. Introduction

The simple model of tropical atmospheric circulation put forward by Gill (1980) has come into widespread use for studies of tropical air–sea interaction. Gill motivates the model in terms of internal modes of an atmosphere with a lid, forced by diabatic heating. For models of tropical air–sea interaction, it is necessary to parameterize this forcing in terms of the sea surface temperature (SST) in some fashion. In the simplest models, heating anomalies are assumed to be proportional to SST anomalies (for instance, Gill and Rasmussen 1983; Philander et al. 1984; Hirst 1986). Other models attempt to relate this heating to surface flux anomalies either directly (Zebiak 1982) or with some parameterized feedback of the low-level moisture convergence which acts to increase the effect of the surface flux anomaly (Gill 1985; Webster 1981; Zebiak 1986). Neelin and Held (1987) claim that, from the point of view of the moist static energy budget, the tropical convergence zones are associated, not with large surface flux, but with small values of the effective moist stability of the atmosphere. Their model attempts to parameterize this quantity in terms of SST.

All of these models are based on the assumption that latent heating forces significant low-level flow (which, in turn, can feed back on the latent heating). A very different perspective on this problem is provided by the model of Lindzen and Nigam (1987, LN hereafter). Boundary layer temperatures are strongly tied to SST by turbulent vertical mixing. These horizontal temperature variations have associated horizontal pressure gradients which, though seemingly small, can force boundary layer flow of about the observed magnitude in the tropics. Indeed, on the equator these gradients imply enormous convergence of the boundary layer mass transport, unless the pressure gradients are allowed some freedom to adjust dynamically. LN propose that this adjustment is due to a lag time in the uptake of boundary layer convergence by the cumulus mass flux which they refer to as the "back-pressure" effect. When this term is included, LN point out that their model resembles the Gill model but with the forcing in the momentum equations rather than in the height equation (which in Gill's interpretation would have been the thermodynamic equation), and with a very much larger damping coefficient in this equation than in Gill's version. It is the purpose of this note to indicate that the analogy to the Gill model can be extended to include the forcing term and to discuss the implications of this for interpretation of the atmospheric side of the tropical air–sea coupling problem.

2. Transforming the Lindzen–Nigam model into the Gill model

The LN model may be stated as follows, with some of the smaller terms neglected for the sake of clarity. Equatorial β-plane coordinates are used for the sake of continuity with Gill's presentation:

$$\epsilon u' - fv' + g\partial_x h' = (gH_0/(2T_0))\partial_x T'_s \quad (1a)$$

$$\epsilon v' + fu' + g\partial_y h' = (gH_0/(2T_0))\partial_y T'_s \quad (1b)$$

$$\epsilon_T h' + H_0(\partial_x u' + \partial_y v') = 0 \quad (1c)$$

where u and v are the mass weighted horizontal velocities averaged through the depth of the boundary layer, h is the perturbation height of the top of the boundary layer and is T_s the surface temperature. The forcing in (1a) and (1b) corresponds to the pressure gradients arising from thermally produced density gradients. As in LN, these equations are considered to apply only to the zonally asymmetric component of the flow, denoted by primed variables. Here H_0 is the mean depth of the boundary layer, T_0 a reference temperature for the linearization of the temperature dependence of density, and ϵ a mechanical damping due to vertical diffusion of momentum and surface drag. The ϵ_T is an inverse relaxation time for the adjustment of the boundary layer height as the cumulus mass flux responds to boundary layer convergence (τ_c^{-1} in the LN notation). The value of ϵ_T required to give a good simulation is

* Current affiliation: Dept. of Atmospheric Sciences, UCLA.

Corresponding author address: Dr. J. David Neelin, Department of Atmospheric Sciences, University of California, 405 Hilgard Avenue, Los Angeles, CA 90024.

about (30 minutes)$^{-1}$ while Gill uses the same value as the mechanical damping, (2 days)$^{-1}$. While the origin of Gill's choice for ϵ is unclear, the origin in LN is simply due to surface drag and the depth of the trade cumulus boundary layer.

The system (1) is forced by pressure gradients proportional to gradients of SST. By the transformation

$$h' = h' - (H_0/(2T_0))T'_s \tag{2}$$

$$\mathscr{H}_0 = \epsilon/\epsilon_T H_0 \tag{3}$$

this system is converted into the more familiar Gill model

$$\epsilon u' - fv' + g\partial_x h' = 0 \tag{4a}$$

$$\epsilon v' + fu' + g\partial_y h' = 0 \tag{4b}$$

$$\epsilon h' + \mathscr{H}_0(\partial_x u' + \partial_y v') = -\epsilon(H_0/(2T_0))T'_s \tag{4c}$$

with the forcing in the height equation. In the Gill model this forcing is interpreted as latent heating and the further, rather ad hoc, assumption is commonly made that latent heating is proportional to SST. The LN model gives this relation directly, without the assumption of arbitrary proportionality.

It should be noted that the transformation (2) does not apply perfectly to the full LN model but that the residual terms are very small.

3. Discussion

In LN, H_0 is simply the depth of the Trade wind boundary layer, about 3000 m. In the Gill model, the depth scale is interpreted as an equivalent depth, whose value is an order of magnitude smaller than this. The rescaling of the depth, (3), a device for obtaining the same "thermal" damping as in Gill, reduces H_0 by the ratio of the Gill damping to the LN damping. When ϵ_T is very large, one recovers the "no back-pressure" case with small $\mathscr{H}_0$, allowing a strong, narrow response along the equator. For $\epsilon_T = (30 \text{ min})^{-1}$, $\mathscr{H}_0 = 30$ m, which puts the model qualitatively in the same regime as in Gill. The equatorial radius of deformation, $L_D = (g\mathscr{H}_0)^{1/4}/\beta^{1/2}$, is about 600 km compared to 1000 km in Gill.

Interpreting the Gill model in terms of the LN model is not in contradiction with the finding that useful simulations of low-level flow can be obtained using latent heating as forcing. In the LN model, the motion forced by temperature gradients implies a vertical velocity at the top of the boundary layer, which is taken up by the cumulus mass flux. If one were to take the heating associated with this as a given forcing in the LN model, instead of using SST directly, the same low level flow would result, although the cause would be obscured. Neelin (1988), also interpreting the Gill model as a boundary layer model, uses forcing by latent heating or by a given vertical velocity at the top of the boundary layer. This approach verifies that the damped, linear momentum equations of the Gill model are a reasonable approximation in the boundary layer (although errors associated with turning of the wind with height do arise). However, it begs the question of how the heating and vertical velocity are related to SST, as addressed in the LN model.

Although the LN model is forced by gradients of SST, the analogy to the Gill results makes it clear that the convergence maximum will tend to occur near the SST maximum for forcing with y-scales on the order of L_D and x-scales shorter than or on the order of the decay scale, $(g\mathscr{H}_0/\epsilon)^{1/2}$. In the LN view, cumulus convection occurs as a response to the boundary layer convergence, and will thus appear over the regions of warmest SST.

The implication of this for coupled air–sea interaction studies is, first, insofar as the LN model represents the dominant process, the relation between the low level atmospheric response, including surface stress, and sea surface temperature is essentially linear. This would formally justify the Gill equations with a forcing proportional to the SST anomaly. Second, the interpretation of the atmospheric leg of the interaction dating back to Bjerknes (1969), wherein SST variations cause latent heating anomalies which in turn force low level flow, should be altered. If the LN mechanism is dominant in determining boundary layer flow, then it is forced by boundary layer temperature gradients which are directly linked to SST.

This interpretation in terms of the LN model is subject to some caveats. LN assume that latent heating begins too high to feed back significantly on boundary layer flow, contrary to the assumptions used by Gill (1985), Zebiak (1986) and Neelin and Held (1987). This results in decoupling of the surface flow from the upper level flow forced by latent heating—although the latent heating distribution is determined in large measure by the surface flow—whereas the Gill model views upper and lower level flows as a single response. Ignoring the specific role of latent heating in forcing surface wind would be correct if latent heating is restricted to the region above 3 km, as suggested by Houze (1982). While the formal equivalence of the LN and Gill models does not necessarily imply that motions induced by latent heat release are unimportant near the surface, the assumptions of the Gill model need to be reexamined and it is difficult to settle the issue definitively on the basis of current observations.

From the point of view of coupled air–sea modeling, the main difference between these approaches may be the type and degree of nonlinearity in the relationship between SST and vertical velocity at the top of the boundary layer. Examination of atmospheric anomalies related to El Niño, for instance, in Rasmussen and Carpenter (1982) or in GCM simulations (Lau 1985), suggests that there is some nonlinearity in the response of low level flow to SST anomalies. Whether some modification of the LN mechanism could account for this remains to be seen. (An obvious possibility involves allowing interaction between the surface drag coefficient and wind.) To a first approximation, however, the various models in current use for air–sea interaction studies seem to give similar results in terms of function, if not interpretation. In absence of a definitive answer to the question of how SST affects tropical flow, some comfort can be drawn from this resemblance.

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