

## Reply

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15 December 1991

The comments by Cane (1992, C hereafter) on Neelin (1991, N hereafter) are quite appropriate and raise very good points, especially in regard to the effects of a finite basin and the question of which regime of behavior for coupled modes most closely resembles the observed system. However, the manner in which coupled modes are constructed in a finite basin proves to be even more interesting than could be anticipated from the arguments given in C. We have made further progress in analyzing this and summarize here some aspects of forthcoming work that are directly relevant to the questions raised by C. We emphasize two points: 1) sea surface temperature (SST) modes in the fast-wave limit are well defined in a finite basin and have properties suggestive of some aspects of the observed coupled behavior; 2) the SST modes in the fast-wave limit are intimately connected to the modes found in, for instance, the Cane and Zebiak (1985, CZ hereafter) model. They provide a useful means of understanding many of the features of these modes.

In regard to point 1: while SST modes can exist independent of ocean boundaries, it is (as C points out) of great importance to consider the effects of boundary conditions (both oceanic and atmospheric) in a finite basin. Consider an ocean-model component with shallow-water dynamics for the simple case of a wind stress of the form

$$\tau'(x, y)/(\rho H) = \exp[\sigma t] \mathcal{A}_e(x) \exp[-\alpha y^2].$$

In a coupled model,  $\sigma$  may be complex and is internally determined, along with the  $x$  dependence of the equatorial stress, which depends on SST,  $\mathcal{A}_e(x) = \mathcal{A}_e(T; x)$ . The scales of atmospheric  $y$  variation are considerably larger than the oceanic radius of deformation, so the Gaussian  $y$  dependence simply provides a means of indicating the effects of meridional structure in the stress field, which prove to be secondary. We define  $i\phi = \sigma + r$ , where  $r$  is the oceanic Raleigh damping, and nondimensionalize using the Kelvin wave basin-crossing time, the basin zonal length scale in  $x$ , and the

oceanic radius of deformation,  $L_D$ , in  $y$  ( $L_D \sim 3$  degrees for a Kelvin wave speed of  $2.7 \text{ m s}^{-1}$ ). The oceanic damping time scale is fairly long, so  $r \ll 1$  is a useful approximation [ $r = (250 \text{ days})^{-1}$  yields nondimensional  $r = 0.25$ ] and atmospheric length scales are order of  $10^\circ$  so  $\alpha = L_D^2/(2L_a^2) \sim 0.05$ , which is even smaller than  $r$ . For a mode with small growth rate and frequency,  $\sigma \ll 1$ , the ocean comes into Sverdrup balance to  $O(\phi^2)$  in thermocline depth and  $O(\phi)$  in currents, as discussed in both N and C, yielding Eq. (50) of N [Eq. (1) of C], valid to  $O[\max(\phi^2, \phi\alpha)]$ , for  $h_e$ , the equatorial thermocline depth:

$$\partial_x h_e = \exp[\sigma t] \mathcal{A}_e(x). \quad (1)$$

The fast-wave limit is defined as the limit in which coupled evolution occurs on a time scale longer than other relevant time scales, especially that of adjustment by wave dynamics. The boundary condition required for (1) in the fast-wave limit is easily derived for winds of the aforementioned form by letting  $\sigma \rightarrow 0$ , where not multiplied by  $t$ , in an expression given by Cane et al. (1990, CMZ hereafter):

$$h_E = \exp[\sigma t] \int_0^1 \frac{\mathcal{A}_e(x_0) \sin 2\phi x_0}{(\alpha \cos 2\phi x_0 + i \sin 2\phi x_0)^{1/2}} \times dx_0 \left( \frac{i}{\sin 2\phi} \right)^{1/2} \quad (2)$$

where  $h_E$  is the eastern boundary value of the thermocline depth, constant in  $y$  in the long-wave approximation. The result can be sensitive to the order of limits (Cane and Sarachik 1981), but is well defined in a physically appropriate manner by keeping finite damping or by letting damping go to zero only after taking other limits. Equation (2) can be used as is, with  $\sigma = 0$ , but simplifies for  $r^2 \ll 1$  (without yet making assumptions about the relative size of  $r$  and  $\alpha$ ) to

$$h_E = \exp[\sigma t] \int_0^1 \mathcal{A}_e(x_0) \frac{x_0}{(x_0 + \alpha/2r)^{1/2}} dx_0. \quad (3)$$

The sensitivity of (3) to  $\alpha$  tends to be small, especially since typically  $\alpha/(2r)$  is fairly small. This boundary condition includes the effects of wave dynamics, especially the east-west asymmetry in the dynamics due

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to  $\beta$ , but the time scales of wave dynamics do not appear: in this limit, the waves have done their job in bringing the ocean into adjustment and coupled evolution proceeds through the SST equation. Since, by definition, the integral in (3) has no  $\sigma$  dependence, it extends trivially to general slow time dependence in a fast-wave limit coupled model through  $\mathcal{A}_e(T(t); x)$ .

Hao et al. (1992) use the boundary condition corresponding to (3) in a fast-wave limit version of the N model in a Pacific-like basin, with a simplified nonlinear SST equation including thermocline feedback and surface-layer effects. The zonal structure of the stress along the equator is given by the Gill (1980) model for the linear, integral operator  $\mathcal{A}_e(T; x)$ , with atmospheric boundary conditions corresponding to zero SST anomaly outside the basin. This coupled model exhibits nonlinear oscillations in the fast-wave limit, two examples of which are shown in Figs. 1 and 2. In both cases,  $\alpha/(2r) = 0.1$ , as in the estimate given previously. Using (2) instead of (3) gives no visible change and using  $\alpha/(2r) = 0$  gives extremely similar results. The two figures differ in that in Fig. 1 the effects of upwelling perturbations in the active surface layer are at the strong end of the realistic range, while in Fig. 2 the surface-layer currents are inactive. The coupling

coefficient has been increased in Fig. 2 such that the system is maintained slightly above the first bifurcation from the climate state.

In both cases, we find oscillations due to nonlinearly equilibrated SST modes in propagating regimes (westward and eastward, respectively), with interannual periods (about 2.5 and 2.75 years, respectively). The SST field is eastward trapped in the basin and exhibits a substantial stationary-oscillation component, although the propagating component is essential to the period of oscillation. Although there are by definition no effects of wave time scales in these oscillations, a feature of interest is the phase relation between thermocline depth in the west and that in the east. An increase in thermocline depth in the west precedes each warm event in the east, especially in Fig. 2, where the thermocline feedback dominates. Such phase relations in observations or in more complex models are thus not alone sufficient to ascertain whether wave adjustment times are crucial to an oscillation.

In regard to point 2): the period of the SST mode oscillations in the fast-wave limit is easily increased from the values shown in Figs. 1 and 2 to infinity by small changes in parameters; varying a parameter governing the strength of the surface-layer feedbacks be-

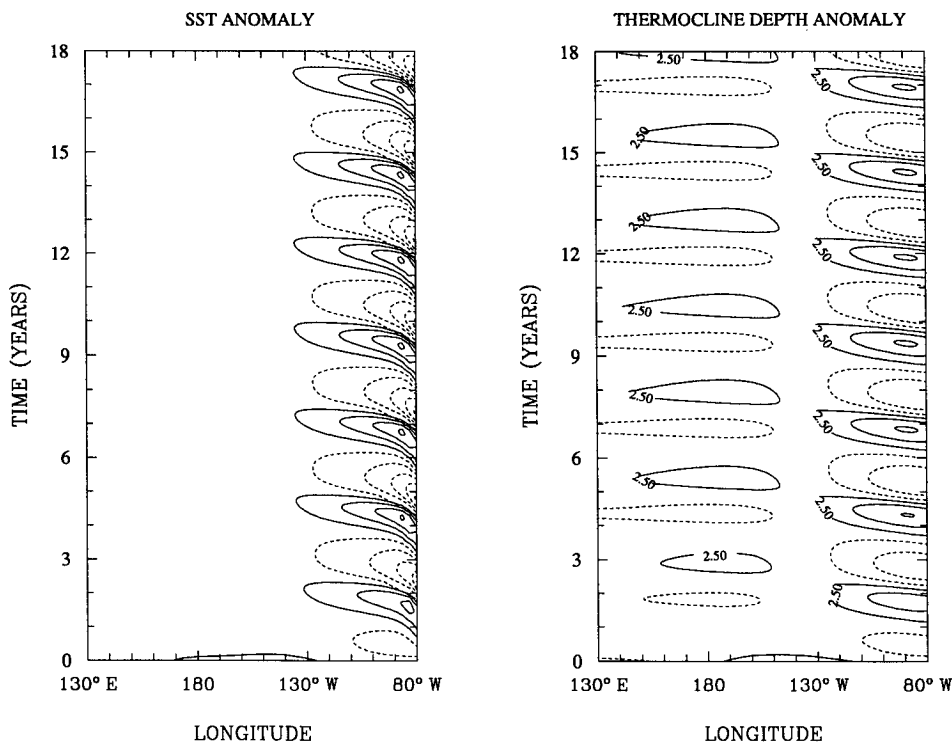


FIG. 1. Evolution of anomalies along the equator over 18 years from a nonlinear, fast-wave limit coupled model in the case of strong feedbacks due to active surface-layer upwelling: (a) SST (contour interval  $1^{\circ}\text{C}$ ; dashed lines  $-0.5^{\circ}\text{C}$  and below), (b) thermocline depth (contour interval 5 m; dashed lines  $-2.5$  m and below).

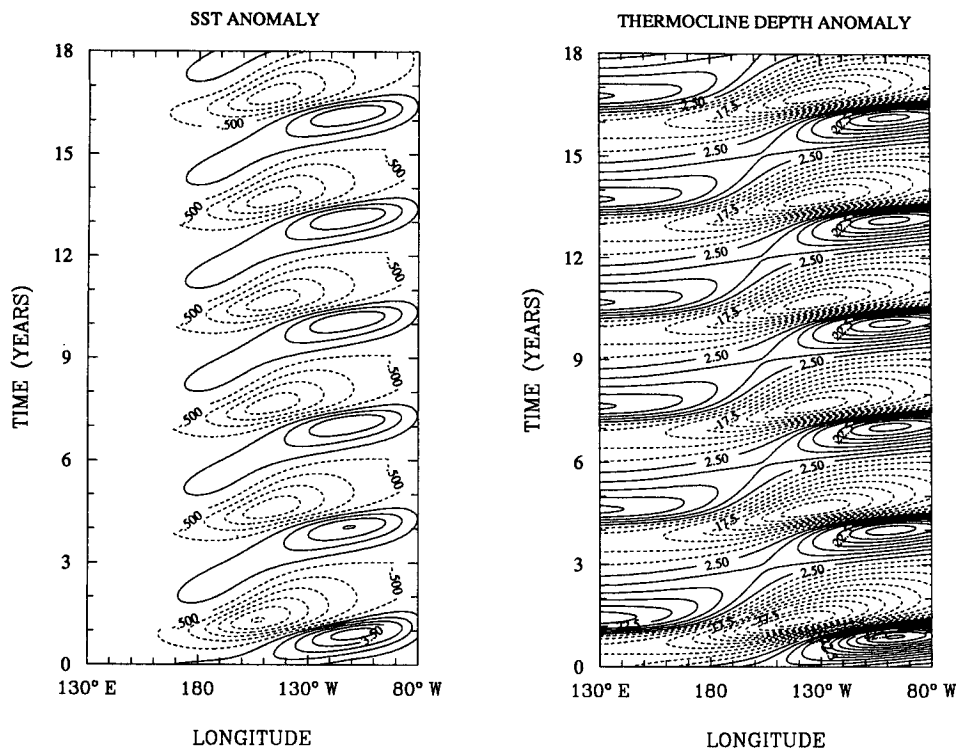


FIG. 2. As in Fig. 1 but in the case of inactive surface-layer upwelling.

tween the two cases shown yields a large range where there is no oscillation due to zonal propagation, but rather pure stationary growth of the SST modes. Jin and Neelin (1992a,b) and Neelin and Jin (1992) (collectively JN hereafter) explore the parameter space more completely, including what happens when dynamical time scales become more important, that is, as one moves away from the fast-wave limit in parameter space. It is found that these stationary SST modes can be perturbed to produce oscillatory modes whose spatial pattern and growth characteristics are those of the SST modes in the fast-wave limit but whose period depends on wave dynamics. For these modes, dynamically induced phase lags, for instance, in the thermocline depth, are indeed essential to the oscillation, basically for the reasons discussed in C, although zonal advection can also play a role. These are the modes found in the CZ model. It also turns out that these modes are continuously connected to the modes found in the limit explored by CMZ, which may be termed the “fast-SST limit” since SST is assumed to adjust fast compared to wave time scales. The continuity hinges on the fact that at strong coupling wave dynamics in the conventional sense is never important, resulting in a direct connection between unstable stationary SST modes in the fast-wave limit and the corresponding modes in the fast-SST limit.

To put it succinctly, the JN results suggest everyone

is looking at the same modes, but in slightly different regimes. The propagating SST modes found in some GCMs (e.g., Meehl 1990; Lau et al. 1992) are in fact connected to the mixed SST/subsurface-dynamics modes found in the CZ model. Modes such as those found in the Neelin (1990) model, which have some wave effects but resemble fast-wave limit counterparts, lie in between. The SST modes in the fast-wave limit provide a useful means of examining the coupled problem because they permit simple—in some cases analytical—solutions in a finite basin, which show how the spatial structure is determined and what the growth mechanisms are, even when wave effects are important to the period.

Thus, it is not surprising that the observed system lies somewhere between the extremes, exhibiting important features of the regime with standing SST oscillations and subsurface memory, but also showing features of the regimes with propagating SST anomalies. It is reasonable to hypothesize that year-to-year variations in these aspects are just the result of stochastic or secular perturbations of parameter regime creating slight variations in the behavior of the same mode.

*Acknowledgments.* Preparation of this reply was supported in part by NSF Grants ATM-8905164 and ATM-9215090. The authors acknowledge valuable discussions with M. Cane.

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