

1 **Temperature-Moisture Dependence of the Deep Convective**
2 **Transition as a Constraint on Entrainment in Climate Models**

3 SANDEEP SAHANY * J. DAVID NEELIN, KATRINA HALES

Department of Atmospheric and Oceanic Sciences, University of California, Los Angeles, Los Angeles, California

4 RICHARD B. NEALE

National Center for Atmospheric Research, Boulder, Colorado

* *Corresponding author address:* J. David Neelin, Dept. of Atmospheric and Oceanic Sciences, University of California, Los Angeles, 405 Hilgard Ave., Los Angeles, CA 90095-1565.

E-mail: neelin@atmos.ucla.edu

6 Properties of the transition to strong deep convection, as previously observed in satellite pre-
 7 cipitation statistics, are analyzed using parcel stability computations and a convective plume
 8 velocity equation. A set of alternative entrainment assumptions yield very different char-
 9 acteristics of the deep convection onset boundary (here measured by conditional instability
 10 and plume vertical velocity) in a bulk temperature-water vapor thermodynamic plane. In
 11 observations the threshold value of column water vapor above which there is a rapid increase
 12 in precipitation, referred to as the critical value, increases with temperature, but not as
 13 quickly as column saturation, and this can be matched only for cases with sufficiently strong
 14 entrainment. This corroborates the earlier hypothesis that entraining plumes can explain
 15 this feature seen in observations, and places bounds on the lower tropospheric entrainment.
 16 Examination of a simple interactive entrainment scheme in which a minimum turbulent en-
 17 trainment is enhanced by a dynamic entrainment (associated with buoyancy-induced vertical
 18 acceleration) shows that the deep convection onset curve is governed by the prescribed min-
 19 imum entrainment. Results from a 0.5 degree resolution version of the Community Climate
 20 System Model, whose convective parameterization includes substantial entrainment, yield
 21 a reasonable match to satellite observations in several respects. Temperature-water vapor
 22 dependence is seen to agree well with the plume calculations and with offline simulations per-
 23 formed using the convection scheme of the model. These findings suggest that the convective
 24 transition characteristics, including the onset curve in the temperature-water vapor plane,
 25 can provide a substantial constraint for entrainment assumptions used in climate model deep
 26 convective parameterizations.

1. Introduction

With the move toward higher resolution in global models, one might hope that additional aspects of the precipitation generating processes would improve. However, the interaction with traditional convective parameterizations can potentially become problematic, as the separation of scale between the grid size and convective plume size is reduced. There is a need for benchmarks that summarize statistics of fast time-scale motions in a manner that is relevant for the large-scale. The agenda here is to link a particular set of such benchmark statistics for deep convective onset from prior observational work (outlined below) with calculations of convective plume buoyancy similar to those used in convective parameterizations. This includes quantifying the dependence of deep convective onset on free tropospheric moisture as well as providing a constraint on the entrainment assumptions. We thus begin with a brief review of background on each aspect.

A number of observational studies have indicated that moist convection is sensitive to free-tropospheric humidity (Austin 1948; Malkus 1954; Brown and Zhang 1997; Sherwood 1999; Parsons et al. 2000; Bretherton et al. 2004; Sherwood et al. 2003). Some of these findings have also been confirmed from recent numerical modeling studies using convection-allowing models (Tompkins 2001; Grabowski 2003; Derbyshire et al. 2004). The transition of convection from shallow clouds to deep cumulonimbus involves many complex effects, and its realistic representation in climate models is essential for an accurate simulation of rainfall statistics.

Many climate models find it difficult to simulate this shallow to deep convective transition. Derbyshire et al. (2004) reported that commonly used convection schemes are likely too insensitive to free tropospheric humidity. While several factors may contribute, including separate shallow and deep convection schemes and coarse vertical resolution, one of the more significant of these is thought to be the entrainment profile used in the updraft cloud model. The entrainment assumptions used in a convection scheme can play a substantially important role in its ability to simulate the observed transition to deep convection. Entrainment

dilution has a strong effect on the estimated convective available potential energy (CAPE) and closure assumptions in convection parameterization (e.g., Zhang 2009). Specifically, the relative roles of boundary layer humidity and the free tropospheric humidity can significantly change, based on entrainment assumptions. Consequently, the entrainment profile chosen for instability computations can strongly influence the estimated statistics of quasi-equilibrium. Lee et al. (2003) found that enforcing a minimum value of entrainment can yield a substantial increase in tropical intraseasonal variability. Given the importance of mixing assumptions in buoyancy computations, Neale et al. (2008) included the effect of entrainment dilution in the CAPE computation of the Zhang-McFarlane convection scheme used in the National Center for Atmospheric Research (NCAR) Community Atmosphere Model version 3.5 (CAM3.5), and noted substantial improvements in several aspects of tropical circulation. Bechtold et al. (2008) formulated the organized component of the total entrainment as a function of the environmental relative humidity and found improvements in rainfall simulation and tropical wave activity. Strong sensitivity to entrainment and other parameters affecting the onset of convection has been noted in Stainforth et al. (2005), Zhao et al. (2009), and Neelin et al. (2010). A number of studies have worked toward evaluating convective plume entrainment observationally (Raymond and Blyth 1986; Brown and Zhang 1997; Jensen and Del Genio 2006; Bacmeister and Stephens 2010; Luo et al. 2010) or via cloud-resolving modeling (Kuang and Bretherton 2006; Li et al. 2008; Romps and Kuang 2010) and one of the aims here will be to complement these approaches with independent criteria.

Examining the transition to strong convection at high time resolution, Peters and Neelin (2006, hereafter PN06) borrowed techniques from clustering transition systems in statistical physics and showed that they could be usefully employed to condense information about precipitation onset statistics, and for showing relations among properties. A key quantity is the value of the column water vapor (CWV) at which the transition to strong deep convection occurs, termed the critical CWV, w_c . Neelin et al. (2009, hereafter NPH09) examined the dependence on tropospheric temperature of this CWV. Taking into account bulk measures

of tropospheric temperature aided collapse of a number of leading precipitation statistics to relatively simple dependences on CWV and w_c . The critical value for the onset of convection was found to increase roughly linearly with tropospheric temperature, but this dependence differed substantially from that of the most obvious rule of thumb, column saturation. At higher temperatures convective onset occurs at lower saturation across a set of tropical regions.

This paper will focus on explaining this behavior and linking it to entraining plume calculations in a single plume framework such as can be used to constrain climate model convective parameterizations. It is thus worth reviewing an example of these results. Fig. 1a shows precipitation retrievals conditionally averaged on CWV and a bulk tropospheric temperature. These are displayed as a function of CWV for various values of this bulk temperature. Here a tropospheric average temperature $\hat{T}$ (200-1000 mb) is used (NPH09 obtained similar results for other measures). Such bulk measures are consistent with the use of CWV, and permit analysis of the leading order behavior of the moisture-temperature dependence of the onset of deep convection. Ideally one would like to carry out such estimates with additional vertical degrees of freedom but the approach makes use of the vast number of measurements available for CWV. Specifically, in Fig. 1a, microwave estimates of precipitation (for details see Section 3) are conditionally averaged in $\hat{T}$ bins of 1K and CWV bins of 0.3 mm, using temperature profiles from the National Center for Environmental Prediction-National Center for Atmospheric Research (NCEP-NCAR) Re-Analysis (Kalnay et al. 1996), and precipitation and CWV values from Tropical Rainfall Measuring Mission (TRMM; Kummerow et al. 2000) Microwave Imager (TMI; processed by Remote Sensing Systems with the Hilburn and Wentz 2008, algorithm), over the tropical western Pacific. A rapid pickup in precipitation can be seen to occur at a value of CWV that differs for each $\hat{T}$. This value is referred to as the critical CWV, w_c (following the PN06 and NPH09 terminology). A non-linear fitting of a power-law above w_c allows an objective estimate of where the onset occurs. Specifically, the power-law fit is of the form $a(w - w_c)^\beta$ (following

PN06 and NPH09), with $\beta = 0.185$ for all the curves.

Because w_c turns out to be an important measure, it is worth examining its dependence on temperature as shown in Fig. 1b. A natural thing to compare this to (Bretherton et al. 2004) is the column saturation value $\widehat{q_{sat}}$, which is also shown as a function of $\hat{T}$ in Fig. 1b. The w_c values (estimated here using NCEP temperatures) are very close to those reported in NPH09 (where temperatures from the European Center for Medium-Range Weather Forecasts (ECMWF) Re-Analysis (ERA-40; Uppala et al. 2005) were used). Figure 1b corroborates (with a different temperature data set) the findings of NPH09 that at higher temperatures the deep convective onset tends to occur at lower values of relative humidity, as can be seen from the increasing distance of the onset boundary and the column saturation line with increasing temperatures. As in NPH09, similar results are found when the w_c estimation is carried out for different tropical regions (the western Pacific chosen here). The empirical critical value thus does not have a simple relation to the column saturation value — it is this property that we aim to explore in this paper.

To illustrate the usefulness of this critical value, Fig. 1c shows an example of how various statistics related to deep convection can be collapsed to similar dependences when CWV is rescaled. The probability of occurrence of CWV values (for precipitating points), variance of precipitation, and precipitation (normalized by an amplitude factor determined from the power-law fits) are shown as a function of CWV rescaled by the corresponding estimated w_c values (as in Neelin et al. 2008). The curves for different $\hat{T}$ tend to collapse to a common dependence on the rescaled variable w/w_c . Thus, the critical value of CWV seems to be a robust indicator of the transition to deep convection over the tropical oceans, and can be useful in condensing properties of various statistical measures of precipitation.

To investigate physical mechanisms governing the CWV-precipitation relationship, Holroyd and Neelin (2009, hereafter HN09), examined entraining plume buoyancy using sonde data from Nauru. Using various entrainment schemes, they found that higher values of CWV are associated with higher plume buoyancies, especially in the upper troposphere.

Moisture content of the lower-tropospheric air was found to be particularly important for this buoyancy increase, by means of entrainment. Thus highly entraining plumes were found to produce relationships between CWV and onset of conditional instability matching aspects of the observed onset of deep convection and precipitation. This paper further pursues this agenda under the hypothesis that the bulk temperature-CWV relationship seen in the observed convective transition can be explained by such plume models. Quantifying the relationship of the onset of deep convection in observations through that in entraining plume models potentially provides additional constraints for convective parameterization schemes. This also helps to establish whether onset statistics such as are seen in Fig. 1a can provide useful metrics for climate models.

A first step is to verify whether the observed transition boundary as a function of temperature and moisture can be reasonably reproduced by simple plume calculations and offline versions of convection schemes used in climate models (section 2). In these calculations, conditional instability of a deep convective column and plume vertical velocity are used as representative measures of the onset of deep convection. For the computation of plume vertical velocity, an updraft velocity equation is coupled to the buoyancy calculation, with the caveat that the vertical pressure gradient term is ignored. Similar calculations were performed using an offline version of the Zhang-McFarlane convection scheme as modified by Neale and Richter (Neale et al. 2008, hereafter ZMNR). Analysis of these parameters as a function of bulk measures of water vapor and temperature, shows that the onset of deep convection is strongly influenced by the choice of entrainment formulation (section 3). In order to investigate if climate models can capture the temperature-moisture dependence of deep convective onset, we compare the model output from a high-resolution (0.5 degree) simulation of CAM3.5, with satellite observations (section 4). Agreement between the offline calculations using ZMNR and the high-resolution model output supports the use of the offline model to analyze the physics and sensitivity in a more controlled environment.

2. Buoyancy Computation and the Updraft Equation

a. Buoyancy Computation and Entrainment Cases

The buoyancy computations use the standard formulation:

$$r_k = (1 - \chi_{k-1} \Delta p) r_{k-1} + \chi_{k-1} \Delta p \tilde{r}_{k-1}, \quad (1)$$

where r is a conserved variable, $\tilde{r}$ is the corresponding environmental value, k is the pressure level, Δp is the pressure interval (5 hPa), and χ denotes the mixing coefficient. For the buoyancy computations, a theoretical parcel is made to rise from the 1000 hPa level, conserving total water specific humidity q_t and the ice-liquid water potential temperature θ_{il} , with no precipitation production, and thus all the condensate is retained by the parcel in the form of cloud water (below the freezing level) and cloud ice (freezing level and above). For clarity a simple single-step water-to-ice conversion at the freezing level is used (which permits this contribution to be assessed visually in buoyancy profiles).

To examine the impact of entrainment formulations, a set of simple assumptions for the dependence of the mixing coefficients are examined. The following entrainment cases are considered (illustrated in the Appendix):

(i) No Entrainment. The mixing coefficient for entrainment of environmental air is set to zero for the entire atmospheric column. Properties of a rising parcel are determined by its temperature and moisture content at the level of initiation.

(ii) Constant Entrainment. In this formulation the mixing coefficient has one value through the vertical column (50-950 hPa) above the atmospheric boundary layer (ABL), with a larger fixed value in the ABL (950-1000 hPa). To study the sensitivity of the plume calculations to the prescribed values of the mixing coefficient, four different profiles are used. In the ABL, all of them have the same value of the mixing coefficient ($0.18 \text{ hPa}^{-1} \approx 19.83 \text{ km}^{-1}$), whereas for the rest of the vertical column, the prescribed values are 0, 1, 2, 4 in units of 10^{-3} hPa^{-1} specified in pressure coordinates and these are thus referred to as C0, C1, C2, and

185 C4, respectively. The corresponding values for C1, C2, and C4 are 0.1 km^{-1} , 0.2 km^{-1} , and
 186 0.4 km^{-1} in the lower troposphere (around 900 hPa), the conversion for p-coordinate values
 187 in the mid-troposphere (around 500 hPa) are approximately 0.06 km^{-1} , 0.12 km^{-1} and 0.24
 188 km^{-1} , respectively, and in the upper troposphere (around 200 hPa), they are approximately
 189 0.03 km^{-1} , 0.06 km^{-1} and 0.12 km^{-1} , respectively. The lower tropospheric values are similar
 190 to or moderately higher than those used by some convective parameterization schemes;
 191 for example, Tiedtke (1989) assumed an entrainment rate of 0.1 km^{-1} for deep convective
 192 clouds and Brown and Zhang (1997) use similar values. A constant, high value of the
 193 entrainment coefficient is used in the ABL in C0-C4 to make it clear that varying free
 194 tropospheric entrainment is the dominant factor in results here. Sensitivity tests have also
 195 been conducted, for instance, using C4 values from 950 to 600 or 700 hPa and C2 values in
 196 the upper troposphere.

197 (iii) Modified version of the Zhang-McFarlane convection scheme (ZMNR). Neale et al. (2008)
 198 made modifications to the Zhang-McFarlane convection scheme used in the NCAR CAM3.5,
 199 by using a prescribed entrainment rate (1 km^{-1}) for the CAPE computation, thus relaxing
 200 the assumption of a non-dilute plume (no mixing with environmental air) in the standard
 201 Zhang and McFarlane (1995) version. The motivation for this change was primarily to
 202 alleviate the bias in the model-simulated near-surface thermodynamic conditions and to
 203 increase the sensitivity of deep convection to free tropospheric humidity.

204 (iv) Deep Inflow A. This is similar to the LES-based estimate of the vertical dependence of
 205 the mixing coefficient reported in Siebesma et al. (2007). In this case, the plume mixes with
 206 the environmental air through a sufficiently deep layer in the lower troposphere, unlike the
 207 constant entrainment case, where most of the mixing happens in the first few layers. The
 208 mixing coefficient in this formulation has an inverse dependence on height, and is given by
 209 the following relation (same as HN09):

$$\chi_k \Delta p = c_\epsilon z_k^{-1} \Delta z_k, \quad (2)$$

210 where χ_k is the mixing coefficient for layer k expressed in pressure coordinates as hPa^{-1} , Δp
 211 is the pressure interval between two consecutive vertical levels (5 hPa), Δz_k is the depth of
 212 the layer, and $c_\epsilon = 0.4$. In height coordinates, the mixing coefficient for Deep A has a value
 213 of almost 1 km^{-1} at 970 hPa, 0.39 km^{-1} at 900 hPa and 0.06 km^{-1} at 500 hPa.
 214 (v) Deep Inflow B. Similar to HN09, an idealized updraft vertical velocity profile is chosen
 215 such that it increases almost linearly at lower levels (in height coordinates), with zero at 1000
 216 hPa ($\approx 84 \text{ m}$) and maximum at 430 hPa (7 km). This is similar to the updraft velocities
 217 reported in Robe and Emanuel (1996), and some of the relevant observational studies (e.g.,
 218 LeMone and Zipser 1980; Cifelli and Rutledge 1994; LeMone and Moncrieff 1994). The
 219 mixing coefficients are then computed from the vertical gradient of the specified updraft
 220 vertical velocity profile (noting that for this case, the specified profile is simply a means of
 221 roughly justifying the mixing profile, as opposed to the interactive mixing of case (vi) where
 222 the updraft profile changes). For simplicity, the mixing coefficient is set to zero above 430
 223 hPa, since there is negligible increase in mass flux beyond this level. In height coordinates,
 224 the mixing coefficient for Deep B has a value of almost 2.8 km^{-1} at 970 hPa, and 1 km^{-1} at
 225 900 hPa.
 226 (vi) Interactive (dynamic) Entrainment. Unlike the previous cases, where the mixing coeffi-
 227 cient is fixed *a priori* and is thus independent of the evolution of the plume, in this case it
 228 is a function of the updraft vertical velocity, and hence is, in part, dynamically determined,
 229 rather than being statically prescribed (e.g., HN09, de Rooy and Siebesma 2010). The total
 230 entrainment essentially consists of two components, namely, the minimum prescribed value
 231 (turbulent entrainment), and the dynamic entrainment associated with buoyancy-induced
 232 vertical acceleration, as follows:

$$\chi_k = \chi_{min} + \frac{(\omega_k - \omega_{k-1})}{\Delta p(\omega_k + \omega_{k-1})/2}, \quad (3)$$

233 where χ_k is the mixing coefficient for layer k , χ_{min} is the minimum prescribed mixing, Δp is
 234 the pressure interval between two consecutive vertical levels (5 hPa), and ω_k is the pressure

vertical velocity for layer k (computed from the updraft vertical velocity, see eqn. 5). In the ABL (1000-950 hPa), entrainment is specified to be the same as Deep B, i.e., corresponding to assuming linear plume growth.

In order to explore the sensitivity of the plume calculations to the minimum prescribed value, we perform two sets of computations, one with a prescribed minimum of 0.002 hPa^{-1} , and the other with 0.004 hPa^{-1} , denoted I2 and I4, respectively. Note that the minimum prescribed values for I2 and I4 are same as the free tropospheric values used in C2 and C4 above.

In discussion we loosely group the C0 and non-entraining scheme as low entrainment, C2, I2, Deep A, C4, I4, and Deep B as high entrainment, with the caveat that the vertical structure differences must be taken into account when making finer distinctions (e.g., comparing C2 and C4, or Deep A and Deep B).

b. Updraft Equation

The plume equation is used to diagnose updraft velocity both for analysis and also to test the interactive entrainment assumptions. Interactive or “dynamic” entrainment attempts to take into account effects of vertical acceleration of the plume which would enhance entrainment by a mass flux balance, assuming that turbulence prevents the width of the plume from decreasing (Houghton and Cramer 1951; Ferrier and Houze 1989; de Rooy and Siebesma 2010, HN09). To estimate the importance of such effects, we use a plume velocity equation similar to those used in prior studies (e.g., Simpson and Wiggert 1969; Sud and Walker 1999; Gregory 2001; Siebesma et al. 2007; Kim and Kang 2011) to diagnose updraft vertical velocity from buoyancy:

$$\frac{\partial w_u^2}{\partial z} = -\frac{c}{z}w_u^2 + aB, \quad (4)$$

where w_u is the updraft velocity, B is the buoyancy term, and c and a are constants. In

the Simpson and Wiggert (1969) formulation, $a = 2/(1 + \gamma)$, with $\gamma > 0$, while Siebesma et al. (2007) make assumptions regarding the pressure gradient term such that the effects of buoyancy are actually enhanced, i.e., $a > 2$. We have done sensitivity tests with a from 1 to 3, but in fact the sensitivity can be more easily seen from the analytic solution below.

The plume velocity equation is treated as a separable add-on to the buoyancy calculations (note that the buoyancy calculation does not have the same form as that of the plume equation for the vertical velocity). This is done in order to have a buoyancy computation that closely parallels the standard calculations that do not use a vertical velocity, while at the same time being able to explore some of the consequences for models where a plume velocity is calculated. The mixing coefficient for buoyancy is influenced by the vertical velocity only for the interactive entrainment cases.

A solution for this formulation of the updraft velocity is:

$$w_u^2 = z^{-c} \int_0^z a \dot{z}^c B d\dot{z} + w_0^2 \left(\frac{z}{z_0} \right)^{-c}, \quad (5)$$

where w_0 is the initial perturbation velocity, and z_0 is the height where it is applied. This solution is similar to that of HN09, but with a second term on the rhs included. This term represents the initial perturbation to the updraft velocity of the plume, due to, for example propagating gust fronts initiated by convective downdrafts. Equivalently, one could define a perturbation pressure gradient through the layer of the gust front that would yield this vertical velocity at the top of the layer. For results shown, we have used a value of 10 m/s for w_0 and 350 m for z_0 . Most of the observational studies on gust fronts have been done for the mid-latitudes, for example, Wakimoto (1982), and they report similar or higher values for the speed and depth, depending on the stage of development of the front.

From this solution it may be seen that increasing a by some factor ‘ f ’ and increasing w_0 by square root of ‘ f ’ will leave the shape of the solution unchanged, although its amplitude will be proportional to ‘ f ’. Thus for instance whether or not a plume reaches the upper troposphere will be independent of a (with this corresponding change of w_0) because regions

of negative and positive buoyancy rescale by the same factor. The amplitude also cancels in the calculation of the dynamic entrainment contribution, so for purposes here sensitivity to a is equivalent to sensitivity to independent changes in w_0 (aside from the amplitude of w_u , which is not presented). Results are shown for the case of $a = 1$. For the entrainment used here (a strong momentum entrainment case $c = 1$ is shown, though smaller values have been tested) the influence of w_0 drops off fairly rapidly with height [compared to, e.g., Gregory (2001) where the effects reach the upper troposphere], although w_0 is important to the plume punching through layers of negative buoyancy in the lower troposphere.

3. Transition to Deep Convection as a Function of Tropospheric Temperature

In this section we explore the dependence of critical column water vapor w_c , a measure of the transition to deep convection (PN06), on tropospheric temperature. As summarized in the introduction, NPH09 computed w_c over various tropical ocean basins, using the ERA-40 temperatures and the TMI column water vapor, exploring its dependence on bulk measures of tropospheric temperature. We compute w_c values over the western Pacific using the same method as that used in NPH09, but using temperature profiles from the NCEP-NCAR Reanalysis. The w_c values thus computed are plotted as a function of CWV and $\hat{T}$, and shown in Fig. 1b. These are repeated in Fig. 2 in order to compare them to plume calculations below. Also shown in Fig. 2 are w_c values reported in NPH09 (which used ERA-40 temperatures). The slopes of the deep convective onset boundaries for the two temperature sources agree quite well, although the w_c values estimated using the NCEP temperature profiles are marginally higher than those estimated from the ERA-40 profiles. The critical value for the onset of convection has an approximately linear dependence on $\hat{T}$, increasing at a rate of about 2.2 mm K^{-1} in the range of temperatures analyzed. This change is approximately $3.6\% \text{ K}^{-1}$, while the column saturation value increases at a rate of around

308 6% K⁻¹, emphasizing that the ratio of w_c to the column saturated value has a temperature
 309 dependence. In order to test if simple convective plume representations such as those used in
 310 current convective schemes can capture this behavior, and to improve our understanding of
 311 the physical mechanisms underlying the deep convective transition, we perform plume com-
 312 putations with different entrainment formulations and various assumptions for the vertical
 313 structure of temperature perturbations.

314 *a. Plume calculations using typical temperature profiles from ERA-40*

315 We select a set of temperature profiles typical of those for conditions that have a reason-
 316 able chance of deep convection from the ERA-40 data set. For this, temperature profiles over
 317 the tropical western Pacific are first binned at 1 K intervals of corresponding $\hat{T}$ values. Then,
 318 for each bin, we choose only those temperature profiles that have a corresponding CWV value
 319 greater than $0.9w_c$ (w_c values for western Pacific reported in NPH09 have been used) for the
 320 given $\hat{T}$, and then compute the mean of all the conditioned profiles, such that the composite
 321 represents a typical temperature sounding conducive for deep convection. Plume computa-
 322 tions then use these conditional mean profiles. The results can thus be interpreted in terms
 323 of this simple set of T soundings (as opposed to carrying out calculations for all T soundings
 324 and then averaging the results as was done in HN09). Similarly, we wish to have a simplified
 325 profile of relative humidity that maintains essential features of observed moisture variations
 326 but with only two vertical degrees of freedom (characterizing free troposphere and ABL,
 327 respectively) for ease of interpretation. This also allows calculations to be easily continued
 328 into the very high free tropospheric relative humidity range (which in observed profiles has
 329 smaller sample size). The idealized profile is chosen as follows. A typical surface relative
 330 humidity value for convective onset cases (85%) is prescribed, with a linear increase at 6.5%
 331 per 50 hPa up to 950 hPa, approximately matching observations for convecting or close to
 332 convecting cases (HN09). Above this a blending region has an interpolation to a constant
 333 free tropospheric value above 800 hPa. This free tropospheric relative humidity is varied

through a range of 51-99%. Plume calculations are performed for these T and q profiles of the environment, using in turn each of the entrainment cases listed in section 2a.

We wish to compare the convective onset boundary estimated in observations to a reasonable analog for each of these calculations. Neelin et al. (2008) analyzed the dependence of entraining CAPE on CWV (where the entraining CAPE $\int_{z_0}^{z_{LNB}} \bar{T}_v^{-1} g(T_{vp} - \bar{T}_v) dz$ is calculated as for CAPE but replacing an adiabatic parcel virtual temperature by that of an entraining parcel T_{vp} , in this case with a constant mixing of 0.1% of environmental air per hPa, with $\bar{T}_v$ the virtual temperature of the environment). They found a sharp pickup, similar to that seen in precipitation, in the entraining CAPE at sufficiently high CWV corresponding to the onset of deep convective cases. Thus, entraining CAPE can be used as a reasonable measure to characterize deep convective onset, although it comes with the caveat that the w_c value thus estimated might show marginal differences depending on the method of estimation. Figure 2 shows the deep convective onset boundary, using the 100 J/kg entraining CAPE contour as an estimator of the onset, for each of the entrainment cases, as a function of $\hat{T}$ and CWV. Sensitivity to the choice of entraining CAPE contour will be discussed below.

Consider one of the entrainment cases, Deep B, as an example. At a given $\hat{T}$ if one begins at low values of free tropospheric relative humidity as indicated by low CWV values, this entrainment calculation yields convective stability. As one moves towards higher CWV, the onset of convective instability occurs. For each contour, shading indicates which side of the contour has increasing CAPE, i.e., on which side convective instability occurs. At different temperatures, the onset boundary as estimated by the 100 J/kg contour occurs at different column water vapor values. For this entrainment scheme, overall the onset boundary angles towards higher CWV at higher $\hat{T}$ in a manner that roughly parallels the onset as estimated from microwave precipitation observations. As one considers different entrainment cases, however, the onset boundary can shift substantially.

The shifts in the position and slope of the onset boundaries in Fig. 2 as one changes

entrainment cases indicate that the relationship of the critical column water vapor to tropospheric temperature shows strong sensitivity to the entrainment formulation. For the non-entraining case, it can be seen from the figure that deep convection occurs over almost the entire range of $\hat{T}$ and CWV values used in this study. For the constant entrainment cases, the temperature dependence of w_c is found to be a strong function of the mixing coefficient value in the free troposphere. For the no entrainment and C0 cases, lower relative humidity soundings can be slightly more unstable due to the direct effect of water vapor on the density of air, i.e., the virtual temperature effect. These cases are highly unstable under circumstances for which the observations suggest deep convection does not occur. As entrainment increases, the onset boundary rapidly changes to a configuration where instability increases for higher relative humidity because the entrained air has less negative impact on the buoyancy of the parcel.

Although the two deep inflow cases are very similar in design, for a given temperature, onset with Deep A starts at much lower values of free tropospheric CWV than that of Deep B which has stronger entrainment at a given level. In the case of interactive entrainment, the prescribed minimum value plays a significant role in governing the onset characteristics. For I2, deep convection starts at somewhat lower values of $\hat{T}$, as compared to I4. In addition, for a given value of $\hat{T}$, the critical CWV for I4 is found to be much higher than that for I2. An interactive entrainment case with no minimum entrainment (not shown) behaves much like the C0 case (with no free tropospheric entrainment) in terms of onset boundary. This occurs because near the onset of conditional instability, there is little buoyancy to force vertical acceleration that could yield dynamic entrainment.

Overall, the high entrainment cases show a qualitative agreement with the observations, in terms of slope of the onset boundary. In terms of the w_c values estimated from CAPE, the set of C2, I2, Deep A and Deep B, and the set of C4 and I4, bracket the observations. Note that the onset boundary for the high entrainment cases is at an angle to the constant RH line, such that for higher $\hat{T}$, deep convection occurs at lower values of free tropospheric

RH, consistent with the findings from observations and those reported in NPH09. Even at
 100% free tropospheric RH the highly entraining schemes do not show any deep convection
 for colder tropospheric temperatures, approximately reproducing this feature of the obser-
 vations. Interestingly, the observed $w_c-\hat{T}$ relationship can be approximately captured even
 with a simple constant entrainment scheme, with the right choice of mixing coefficient values,
 thus explaining to some extent, the success of some of the cumulus parameterization schemes
 employing such entrainment formulations. We note that even when different entrainment
 formulations look similar on this onset diagram, they can have other measures that distin-
 guish them. For instance, the Deep B case shows considerable difference in the plume-top
 height (discussed below). Thus the constraint placed on entrainment by the criteria used
 here for the match to the observed onset in the temperature-CWV plane is likely compli-
 mentary to constraints on entrainment that use cloud-top data (e.g., Brown and Zhang 1997;
 Bacmeister and Stephens 2010; Luo et al. 2010).

Another measure of deep convective onset diagnosed in the plume model is the updraft
 vertical velocity. In Fig. 3a we show the 400 hPa plume vertical velocity as a measure of
 deep convective transition, for the various entrainment cases. Selection of the 400 hPa level
 to distinguish between the deep cumulonimbus and the cumulus congestus is based on some
 of the recent observational studies (e.g., Luo et al. 2009). It can be seen from Fig. 3a, that
 similar onset dependence of temperature can be reproduced by using plume vertical velocity
 as an alternate measure of deep convection for all the entrainment cases except Deep inflow
 B, which has a lower-tropospheric choke-point for lower values of free tropospheric humidity.
 In this measure, its onset shifts closer to those of C4 and I4. It is worth noting that the
 400 hPa w_u criterion in Fig. 3a more sharply divides the entraining plume models into
 two families, with Deep A, C2 and I2 onsets occurring at lower CWV than observations.
 Similar onset boundaries are obtained using the saturation column water vapor over the
 lower troposphere (550-875 hPa) as an alternate measure of lower tropospheric temperature
 (figure not shown).

415 Since perturbations in the boundary layer temperature can potentially modulate the
 416 deep convective transition characteristics, we perform similar plume computations as above,
 417 but with a 0.5 K temperature perturbation in the boundary layer (1000 - 945 hPa), and a
 418 reduced initial updraft velocity (w_0 in the updraft vertical velocity equation is set to 5 m/s
 419 instead of 10 m/s). Entraining CAPE as a function of $\hat{T}$ and CWV is shown in Fig. 3b,
 420 similar to Fig. 2 above. It can be seen from the figure that the overall characteristics of
 421 the onset boundaries for the various entrainment schemes are similar to Fig. 2. The deep
 422 convective onset simply occurs at slightly lower values of CWV, and at colder tropospheric
 423 temperatures (compared to Fig. 2) for all the cases. Thus, entraining CAPE seems to be a
 424 robust measure of the deep convective transition, at least qualitatively.

425 Another sensitivity test was conducted to verify which levels in the free troposphere are
 426 most important in controlling the behavior seen here. One suspects that entrainment in
 427 the lower free troposphere will determine whether the plume reaches deep convective levels.
 428 Thus we ran a case using C4 values in the lower free troposphere but C2 values in the upper
 429 troposphere. Figures comparable to 2 and 3a (not shown) yield 400 hPa vertical velocity
 430 contours almost identical to the C4 case and CAPE contours very close to the C4 case. This
 431 reinforces that, as expected, the lower free troposphere is the key layer.

432 Although the vertical integral of buoyancy is a useful indicator of deep convective onset,
 433 details of the vertical buoyancy profile lends important insights into the potential choke-
 434 points that prevent a plume to undergo deep convection. In Fig. 4a, we show the buoyancy
 435 profiles for the various entrainment schemes for a typical value of free tropospheric humidity
 436 (91%) and tropospheric temperature (271 K). For simplicity, freezing is assumed to occur
 437 rapidly, so the buoyancy increase associated with freezing can be easily read from the plot as
 438 a jump near 500 hPa. While upper tropospheric buoyancy is larger, the key layers are in the
 439 lower troposphere with small buoyancy, where plumes terminate if environmental relative
 440 humidity is reduced. It can be seen from the layer of negative buoyancy around 900 hPa
 441 in Fig. 4a that there is substantial convective inhibition at the top of the ABL for most of

442 the cases. Even in the subsequent layers up to around 700 hPa, the available buoyancy is
 443 very close to zero, and even marginally negative for some of the cases. Thus, the plumes
 444 that have enough vertical velocity to cross this inhibition zone, make it to the congestus and
 445 deep convective levels. Conductive perturbations to the environmental temperature or/and
 446 initial vertical velocity of the plume can thus be important for deep convective onset. Worth
 447 noting is the higher cloud-top height in the Deep B case, as compared to C4 and I4, although
 448 their onset boundaries look similar. Also interesting to note is that, even though the two
 449 deep inflow cases have large differences in their estimated w_c values (see Fig. 2), they are
 450 apparently similar in terms of their buoyancy profiles (see Fig. 4a), which can be attributed
 451 to the lower-tropospheric choke-point present in the case of Deep B plumes.

452 Using plume-top pressure, plume vertical velocity, and convective instability as measures
 453 of deep convection, we examine the dependence of these across the convective onset as a
 454 function of free tropospheric relative humidity (RH_{FT}) for various entrainment assumptions.
 455 For these deterministic calculations, we do not expect to reproduce the power-law pickup in
 456 precipitation with column water vapor reported in PN06 and NPH09, but simply to obtain
 457 a sense of underlying plume properties. It can be seen from Figs. 4b-c, that there is a pickup
 458 in both plume-top pressure and vertical velocity when RH_{FT} exceeds a certain temperature-
 459 dependent critical value, only for the high entrainment cases. The cases with low or zero
 460 entrainment are unstable for the entire range of free tropospheric humidity for the given $\hat{T}$
 461 values. From Fig. 4c, it is to be noted that the 400 hPa vertical velocity lines shown in the
 462 figure are subject to stochastic broadening, as can be seen from the small jump in vertical
 463 velocity for I2, before the occurrence of the actual pickup. Figure 4d shows the entraining
 464 CAPE, and as can be seen from the figure, it does not show a corresponding pickup at higher
 465 values of RH_{FT} . This is primarily due to the fact that we use the level of neutral buoyancy
 466 of the plume to compute the CAPE, which is simply a measure of the accessible amount of
 467 energy if the plume is able to undergo deep convective transition, and not the actual energy
 468 available to the plume.

b. Sensitivity to Temperature Perturbation Vertical Structure

We examine the robustness of these onset characteristics in a simpler context. The ERA40 temperature profiles are replaced by profiles created by adding a set of idealized temperature perturbations to a mean state profile taken from the Department of Energy’s Atmospheric Radiation Measurement (ARM) Program radiosonde soundings (Mather et al. 1998) at Nauru from 1 April 2001 to 16 August 2006, averaged over the upper tercile of column water vapor cases, so that cases far from convective onset do not distort the average. In each case the same set of RH perturbations is examined as in section 3a. Plume stability calculations are carried out for the full set of temperature and relative humidity profiles and results are shown in the temperature-CWV plane as before. In a relevant study, Holloway and Neelin (2007, hereafter HN07) used a variety of observations including Atmospheric Infrared Sounder (AIRS) satellite data, radiosonde observations, and NCEP-NCAR reanalysis over the tropics to investigate the dominant vertical structures of temperature perturbations. They found a significant vertical coherence of temperature perturbations in the free troposphere. In addition, the boundary layer was found to be fairly independent of the free troposphere, for smaller spatio-temporal scales. Since the relationship between the boundary layer and the free tropospheric temperature perturbations depends on the space and time scales, we perform plume computations for both vertically coherent as well as non-coherent perturbation profiles.

1) CONSTANT TEMPERATURE PERTURBATIONS IN BOUNDARY LAYER AND FREE TROPOSPHERE

In this set of plume calculations we assume that the temperature perturbations in the ABL and the free troposphere are vertically coherent, and apply constant temperature perturbations in the entire atmospheric column (50-1000 hPa). In these experiments, the perturbations are applied to the basic state tropical sounding from Nauru, described above.

Vertically constant temperature perturbations were applied, such that the lapse rate of the atmosphere remains unchanged, with the size of the perturbation varying to change the tropospheric mean temperature through a range of approximately 267.4 to 274.2 K by intervals of 0.2 K. Although the temperature perturbation differences in ERA-40 vary with height (see Appendix), they tend not to increase with height as a moist adiabat, thus a vertically constant temperature perturbation seems as suitable an idealization as any. Relative humidity profile is same as that used in the previous set of computations.

The results for each entrainment formulation are presented in Fig. 5. The onset boundaries are somewhat simpler than those estimated using reanalysis temperature profiles in Figs. 2 and 3a, but maintain very much the same overall features. As was seen earlier, the slope of the onset boundary strongly depends on the value of the mixing coefficient for the constant entrainment calculations, and on the value of the prescribed minimum for the interactive entrainment calculations. As in Fig. 2, the observed $w_c\text{-}\hat{T}$ relationship can be approximately captured by the schemes with sufficient entrainment. Overall, this indicates that the behavior is quite robust and can be captured even in simplified conditions.

2) OTHER VARIANTS OF TEMPERATURE PERTURBATIONS IN THE VERTICAL

In order to further probe the sensitivity of deep convection onset characteristics to temperature perturbation profiles, we conduct three more sets of computations. The first case examines the impact of free tropospheric temperature if the ABL temperature does not change. In the second and third case, the environmental temperature variations are assumed to be mimicked by those of a convective plume: in the second case with entrainment and in the third case without. This idealization is relevant to the limit in which the vertical dependence of the large-scale tropospheric temperature is dominated by convective processes (neglecting effects such as wave dynamics or radiation). Together, these cases provide a sense of how the behavior depends on different idealizations of the environmental temperature profile variations.

In the first case, we assume that the temperature in the boundary layer and that in the free troposphere vary independently, and investigate the role of free tropospheric temperature (200-950 hPa) in controlling the transition to deep convection. The mean temperature sounding is perturbed in the 50-950 hPa layer, keeping the ABL temperature unchanged. Figure 6a shows the deep convective onset boundaries for this case, for the various entrainment schemes, as a function of $\hat{T}$. The non-entraining case is found to be unstable for the entire range of temperature and relative humidity profiles used, and thus doesn't have an onset boundary in the $\hat{T}$ -CWV domain shown in the figure. Even C0 is unstable for most of the domain, similar to that seen in Fig. 2 and Fig. 5a, except for the high humidity values at lower temperatures. Onset characteristics with the constant entrainment cases C1, C2 and C4 in Fig. 6a are generally consistent with the results from ERA-40 profiles (Fig. 2) and those obtained with constant perturbation in ABL and free troposphere (Fig. 5a). The onset boundaries of the deep inflow and the interactive entrainment cases, however, behave differently than that seen from earlier plume calculations. For higher values of $\hat{T}$, the onset boundary of Deep B has a slope similar to that estimated from observations, while that of Deep A, I2 and I4 are almost parallel to lines of constant relative humidity. However, for lower values of $\hat{T}$, the onset boundaries for all four cases, bend towards much lower values of CWV with decrease in tropospheric temperature. Thus, while the most common behavior encountered in the observed statistics is typified by the case where free troposphere and ABL temperature vary together, when the ABL temperature varies less than that of the free troposphere, it can have significant impacts on the onset boundary. A potential application of this would be to separate such cases in the observations and contrast the onset dependence as a means of providing additional constraints on the entrainment.

In the second case, we perturb the mean temperature sounding with the perturbation profile for a strongly entraining plume, specifically, we use the perturbation profile computed for C4, with $RH_{FT} = 0.75$, and the two $\hat{T}$ values of 270 and 270.2 K (with constant perturbation in the ABL and free troposphere). The onset boundaries for this case are shown

in Fig. 6b. Overall, it looks very similar to the results obtained using ERA-40 temperature profiles, although the deep convective onset occurs at somewhat lower values of $\hat{T}$ for most of the entrainment cases.

In the third case, we add moist adiabatic perturbations to the mean state sounding, and construct a similar plot showing onset boundaries for the various entrainment cases. It can be seen from Fig. 6c that the onset boundaries look very different from the ones seen before. For most of the entrainment cases, the onset boundaries cut across lines of constant relative humidity toward somewhat higher values with increasing $\hat{T}$.

Stronger entrainment cases still come closer to the observed onset line than do weaker entrainment cases, but the angle does not match the observed boundary. This suggests that the idealization in case 2 (in which both environment and parcel are treated as being affected by entrainment; Fig. 6b) is more self-consistent than case 3, in which the environment was idealized as having changes in tropospheric temperature structure corresponding to a non-entraining case. Comparing these cases to earlier sections, typically Deep A, Deep B, C2 and I2 show similar onset behavior as do C4 and I4. The exception is the case where temperature perturbations are applied only to the free troposphere (Fig. 6a). Thus situations where the ABL and the free tropospheric temperature behave differently appear to yield stronger distinctions among schemes with different vertical dependence of the entrainment coefficient.

4. Usefulness for Model Analysis

The observed dependence of critical column water vapor on a bulk measure of tropospheric temperature can potentially be used as a constraint for the cumulus parameterization schemes used in general circulation models (GCMs). Here we provide an example of the comparison to microwave retrievals for a widely used atmospheric GCM. The plume stability calculation can likewise be compared to the GCM to aid interpretation. Figure 7 shows precipitation statistics conditionally averaged by 0.3-mm bins of CWV, for 1-K bins of $\hat{T}$

572 (similar to NPH09), over the tropical eastern and western Pacific ocean basins, from a mod-
 573 erately high-resolution (0.5 degree) simulation with the NCAR CAM version 3.5 (Gent et al.
 574 2009), which is closely related to CAM4. This model version has the ZMNR deep convective
 575 scheme [Neale et al. (2008) modification of the Zhang and McFarlane (1995) scheme]. One
 576 of the most important aspects of the modification is that an entraining CAPE is used in the
 577 closure for the mass flux. As discussed in section 2, the scheme has substantial entrainment
 578 through the free troposphere. Similar to Fig. 1a, power-law fit lines are shown for CWV
 579 values greater than w_c , with an exponent of 1 for the model output, whereas for TMI it
 580 is 0.23 (same as that reported in NPH09). Also overlaid on Fig. 7a are the correspond-
 581 ing values presented in NPH09 (using precipitation and CWV from TMI, and tropospheric
 582 temperature data from ERA-40).

583 There are several important pieces of information in this plot. First, the precipitation
 584 rates from the model reached fairly high values, i.e., at this resolution and for this convec-
 585 tive parameterization, the model is far from the low intensity, overly-constant rainfall that
 586 characterizes some convective schemes. This agrees with the findings of Boyle and Klein
 587 (2010), where they used CAM4 at varying horizontal resolutions, and found that the fre-
 588 quency distribution of rainfall intensity improved at finer resolutions through an increase in
 589 frequency of heavy rainfall events as well as in the occurrence of little or no precipitation
 590 events. Second, the pickup in precipitation has very reasonable dependence on CWV. The
 591 pickup is linear (i.e., a power-law exponent of 1) by the postulates of the convection scheme,
 592 and it may be seen that in the region before the linear pickup, there is a “foot” region due
 593 to the effects of vertical degrees of freedom that are not controlled here, which act like a
 594 stochastic broadening. A similar foot occurs in the microwave retrievals before the sharpest
 595 part of the pickup (and is well known in statistical mechanics analogs, see discussion in PN06
 596 and NPH09).

597 In comparing the model and the microwave retrievals in Fig. 7a, the eye is drawn to a
 598 difference in curvature at high CWV. The model and microwave curves each fit with a form

599 $a(w - w_c)^\beta$ above w_c but they disagree on the value of β . This is left as an open question
 600 here because it is unclear to what extent the calibration of the microwave precipitation
 601 estimates can be trusted in the high precipitation regime (although Peters et al. (2009) find
 602 similar effects in radar retrievals). Rather, we focus on the temperature dependence of the
 603 convective onset. From this point of view, the fit is simply a means of empirically estimating
 604 w_c .

605 For all temperatures and all parts of the curve, the CAM pickup occurs at slightly lower
 606 water vapor than in the observations. However, careful quantification of this and comparison
 607 to plume model results suggests that overall the model is doing reasonably well compared to
 608 what could occur, and that part of this success can be attributed to the stronger entrainment
 609 in the free troposphere. Similar analysis was carried out for the tropical Atlantic and the
 610 Indian Ocean basins, and the corresponding w_c values were estimated. These estimated w_c
 611 values along with the corresponding saturated CWV values for the above ocean basins are
 612 shown as a function of $\hat{T}$ in Fig. 8a. It can be seen from the figure that the dependence of
 613 w_c on $\hat{T}$ is similar across all the basins, with a roughly linear increase in w_c associated with
 614 an increase in $\hat{T}$, but at a rate slower than that of the column saturated value, similar to the
 615 findings of NPH09, and those seen from plume calculations above. For the w_c values shown
 616 corresponding to $\hat{T}=269$ K, the lower value for the western Pacific is due to the smaller angle
 617 of the linear fit, which is even worse for $\hat{T}=268$ K (not shown). If one were to compute $\widehat{q_{sat}}$
 618 using unconditioned temperature profiles, we note that the values for the Atlantic at 273
 619 and 274 K, and for the Indian ocean at 274 K, are substantially higher in the CAM than the
 620 other basins or observations due to occasional occurrence of warm low level conditions that
 621 are also dry and have no chance of convecting (figure not shown). The $\widehat{q_{sat}}$ values shown in
 622 Fig. 8a are computed for CWV values high enough that the temperature profile is relevant to
 623 the convective onset. Specifically, the average is for temperature profiles with CWV greater
 624 than $(38 + 15(\hat{T}-268)/6)$, which roughly approximates $0.8w_c$. The w_c values for the Atlantic
 625 could not be estimated for $\hat{T}$ values of 273-274 K.

To further quantify the comparison of the observed results and those from the plume calculations with the ZMNR scheme, offline calculations using ZMNR with similar profiles of temperature and moisture to those used for the plume calculations were carried out. Entraining CAPE contours of 70 and 100 J/kg are shown in Fig. 8b. Note that much of the information is repeated from Fig. 2 and Fig. 8a for comparison. The 100 J/kg contour runs very close to w_c values estimated from the precipitation onset statistics in CAM3.5 for all of the temperatures except the lowest bin. The 70 J/kg contour parallels this, shifted a few millimeters toward lower CWV. Combining this with information from the CAPE and precipitation pickup curves in Figs. 4 and 7, we can infer some factors affecting quantitative comparison to observations. The model convective onset inferred from the precipitation statistics in most respects agrees very well with the model onset inferred from the ZMNR plume calculation, supporting the usefulness of quantitative comparison between these. A caveat may be seen at the lowest temperature, where the model w_c estimate is noticeably affected by the flattening by stochastic effects in the “foot” of the $\hat{T}=269$ curve in the western Pacific. Indicators of this behavior include the lack of distinction between the upward curving and linear portion of the $\hat{T}=269$ curve in Fig. 7b, the difference in slope between this curve and those for other $\hat{T}$ values, and the disagreement in w_c values for different basins at $\hat{T}=269$ in Fig. 8a.

The precipitation-estimated onset in CAM3.5 is more closely matched by the 100 J/kg contour in plume calculations than by the 70 J/kg contour despite the fact that in ZMNR cloud-base mass flux increases linearly above a CAPE threshold of 70 J/kg. This likely reflects differences in details of the CAM3.5 vertical structure compared to the ERA-40 based profiles used in the plume calculations (possibly combined with differences in the parameterized precipitation pickup from that of CAPE). This serves as a reminder that differences between model and observed onset boundaries need not be purely due to factors within the deep convection scheme. We note that for other entrainment schemes that are reasonably close to matching the observed onset, C2, C4 and Deep B, the offset between

70 and 100 J/kg is smaller (Fig. 8b). The results suggest that the increased entrainment of ZMNR relative to the standard Zhang-McFarlane scheme has been highly beneficial in yielding results close to observations in these measures. However, the differences between the steepness of the onset in upper tropospheric vertical velocity compared to the slow onset in CAPE hint that changes in the form of the parameterization may also be worth considering.

From these results, it is tempting to suggest that an alternation of the entrainment assumptions might further improve the model’s match to the location of the onset in the temperature-water vapor plane. However, even if one can deliver the correct convective response to the temperature/humidity states (in terms of the critical value of onset) as a function of entrainment in offline mode, there is no guarantee that, in the model, this will deliver the correct frequency of a particular temperature/humidity state. For instance, feedbacks that tend to humidify the environment are encountered when entrainment is further increased in CAM3.5. Thus, it is worth underlining that these statistics are among the many that must be used to constrain a model.

5. Discussion

The ability of a simple plume model to reproduce and explain conditional mean characteristics of the observed transition to deep convection is examined using various entrainment assumptions. Results for the convective onset in a relatively high-resolution GCM with parameterized convection are then compared both to observations and to the plume calculations. In the plume model, two measures of the onset of deep convection via conditional instability are examined: entraining CAPE (from a very standard parcel computation) and plume vertical velocity diagnosed from buoyancy via the simple plume model.

The behavior is examined in a temperature-water vapor thermodynamic plane, using column water vapor and tropospheric vertically averaged temperature $\hat{T}$ because these can be compared to the onset of strong precipitation as previously estimated in satellite retrievals.

Plume calculations using ERA-40 temperature profiles yield deep convective onset characteristics that are strongly affected by entrainment assumptions. The threshold value of CWV above which there is an abrupt increase in precipitation, referred to as the critical CWV, is found to increase with mean tropospheric temperature, but at a rate slower than the column saturation, as in observations, only for the cases with high or moderately high fractional entrainment rates. Low entrainment rates yield a curve for the onset of conditional instability that is very different from the observed curve for the onset of strong precipitation.

This result has a number of consequences.

- i) First, it confirms the assumption in PN06 and NPH09 that certain basic features of the observed statistics for the transition to strong tropical precipitation are associated with the onset of conditional instability through a deep convective layer, and that these can be captured by conventional parcel or plume representations as initially examined in HN09.
- ii) Second, this permits exploration of the factors that set the empirically determined onset boundary as a function of temperature and moisture.
- iii) Third, it places additional observational constraints on the entrainment representation in the convective schemes, and helps to quantify the dependence of the onset of the convection on free tropospheric moisture, providing further evidence that this arises by entrainment in the lower free troposphere.
- iv) Finally, it provides additional validation metrics for climate model performance on the onset of deep convection.

We elaborate on each of these below.

Regarding (i), one should distinguish between properties that one does or does not expect to be captured by parcel or plume calculations. The dependence of the onset boundary on temperature and moisture is an important property that can be captured by individual plume calculations. More challenging properties of the set identified in observations (PN06, NPH09), such as power law spatial correlation, cannot. The plume results here are consistent

705 with the conjecture (Neelin et al. 2008) that convective plumes may be suitable microscale
 706 entities for calculations that include neighbor plume-plume interactions and stochastic inter-
 707 actions with microphysics and turbulence to capture the additional properties in convective
 708 schemes. The form of the pickup in convective onset variables (such as precipitation, or
 709 buoyancy-based measures) should depend on both the form of the pickup for an individ-
 710 ual plume, and the neighbor or stochastic interactions (NPH09, Muller et al. 2009). Here,
 711 dependence of the plume-top height and the 400 hPa-vertical velocity on free tropospheric
 712 humidity, show a sharp pickup similar to that seen in observational radar and microwave
 713 precipitation retrievals, provided entrainment is sufficiently high. The sharp transition is
 714 associated with a rapid change from plumes that terminate at trade cumulus or conges-
 715 tus levels to deep convection. Entraining CAPE, however, shows an approximately linear
 716 pickup, since the integrated buoyancy (from the surface to the level of neutral buoyancy of
 717 the plume) evolves relatively smoothly as free tropospheric moisture increases for a given
 718 temperature profile. The quantitative nature of the pickup in models may thus be expected
 719 to depend fairly sensitively on parameterized dependence of precipitation microphysics on
 720 vertical velocity, mass flux, or buoyancy.

721 Regarding (ii), in addition to the strong dependence on properties that can be regarded
 722 as internal to the convective elements, such as entrainment, the convective onset does nat-
 723 urally have a dependence on how the large-scale temperature profile varies as its vertical
 724 average (or other bulk measure) increases in observations. Sensitivity tests using simplifying
 725 assumptions regarding the perturbation profile (i.e., the departure of environmental temper-
 726 ature from a typical mean temperature sounding) may be summarized as follows. Convective
 727 onset dependence on temperature and moisture similar to those found in the ERA-40 case
 728 can be obtained for cases where the environmental temperature changes are applied to the
 729 boundary layer and the free troposphere and have little vertical structure (e.g., are constant
 730 in the vertical or mimic the perturbation structure of a strongly entraining plume). If the
 731 environmental temperature structure instead increases with height like a moist adiabat, the

angles of the onset boundary change substantially in the temperature-moisture plane (under
 this simplifying circumstance parts of the onset boundary become closer to lines of constant
 relative humidity, although they still depart significantly from this for cases with convective
 onset at lower relative humidity). When the environmental temperature changes in the free
 troposphere but not in the boundary layer, onset boundaries change substantially (relative to
 ERA-40 cases) for some entrainment schemes but not others, indicating a greater sensitivity
 to the vertical structure of the assumed entrainment. This suggests that it should be possible
 to find regional or seasonal dependence in the onset boundary that reflect this sensitivity.
 Controlling for additional vertical degrees of freedom, especially boundary versus lower free
 troposphere, in the temperature and water vapor while estimating the deep convective onset
 boundary may then provide additional constraints on entrainment assumptions.

Regarding (iii), the sensitivity to entrainment strongly suggests that low entrainment
 cases are inconsistent with observations in the deep convective onset measures used here.
 This reinforces HN09 results as a function of CWV from Nauru data and is consistent with
 findings that intraseasonal variability is better simulated if a minimum entrainment rate is
 enforced (Tokio et al. 1988; Lee et al. 2003). Different vertical structures of the entrainment
 rate can match the observed onset boundary, provided they have sufficient entrainment in the
 lower free troposphere, which is the key layer here. In terms of practical applications, this has
 the benefit that one does not have to fully settle the question of vertical dependence of the
 entrainment coefficient to match this important feature of observations. On the other hand,
 it does imply that other criteria would have to be added to constrain the vertical dependence
 of the entrainment. The onset boundary constraint is sufficiently independent of criteria such
 as cloud top that combining these constraints could yield additional information. It is worth
 underlining that there is no need for an explicit dependence of the entrainment coefficient
 on moisture to match the observed sensitivity of the convective onset to free tropospheric
 moisture in the measures used here. It is sufficient that the entrainment rate be large enough
 in the lower free troposphere. The dependence on free tropospheric moisture arises simply

by entrainment of lower free tropospheric air; if this layer is too dry the resulting loss of buoyancy produces trade cumulus or congestus instead of deep convection.

An additional simplifying consideration comes from examination of a simple interactive entrainment scheme, in which a prescribed turbulent component of total entrainment is augmented by a dynamic component (associated with buoyancy-induced vertical acceleration). The onset of deep convection proves to be strongly governed by the minimum (i.e., turbulent) entrainment. The contribution of the dynamic component to the total entrainment in most cases makes no significant difference to the temperature dependence of the onset. The buoyancy near onset is simply too small to drive much dynamic entrainment; for the large values of the turbulent entrainment coefficient required to match the observed onset boundary, this remains true through most of the domain. Contrary to initial expectations, investment in dynamic entrainment schemes thus does not appear to be a high priority.

Regarding (iv), analysis of data from a moderately high-resolution (0.5 degree) model integration using the NCAR CAM3.5 suggests that the model does well enough in these rainfall statistics for quantitative comparison to be useful. As expected, the simulated pickup shows a linear dependence on CWV, unlike the power law pickup seen in microwave retrievals, due to the assumptions in the convective closure, but simulated rainfall rates do reach high values, unlike the low intensity drizzle associated with some cumulus schemes. The temperature dependence of the precipitation pickup exhibits encouragingly good agreement between the model and observations. The critical CWV for the precipitation pickup is slightly lower for the model than that estimated from observations. There is some dependence on the method for estimating the critical value, given the differences in the form of the pickup, but the tendency for the model to yield a given value of conditionally averaged precipitation at a lower CWV can also be seen directly by comparing the pickup plots. The model output also permits verification that there is a reasonable agreement in the temperature-water vapor dependence of the convective onset obtained from precipitation calculations with that obtained for buoyancy in the offline simulations performed using

the convection scheme of the model. Overall, the level of agreement suggests that these precipitation statistics, including the deep convective onset curves in the temperature-water vapor plane, can provide a strong constraint for high resolution GCMs.

Acknowledgments.

This work was supported in part by National Science Foundation Grant AGS-1102838, National Oceanic and Atmospheric Administration Grant NA11OAR4310099 and Department of Energy grant DE-SC0006739. We thank J. E. Meyerson for graphical assistance.

ERA-40 Temperature Profile Differences and Entrainment Profiles

A significant vertical coherence in tropical temperature perturbations in the free troposphere was reported in Holloway and Neelin (2007), using observations from various datasets, including AIRS satellite data, radiosonde observations, and the NCEP-NCAR reanalysis. They found that the temperature perturbations in the boundary layer are fairly independent of that in the free troposphere, however, this relationship was found to have a spatio-temporal scale dependence. In order to investigate the vertical structure of temperature perturbations over the tropical western Pacific (region of interest), we use the ERA-40 temperature profiles. These temperature profiles were first binned at 1 K intervals of $\hat{T}$, and then the mean profile for each bin was computed. In order to calculate the vertical temperature perturbation profile for each of the bins, the mean for the 270-K $\hat{T}$ bin was chosen as a base profile, and was subtracted from the corresponding mean profile of the other bins. The vertical profile of temperature perturbations, thus computed, are shown in Fig. 9, and it can be seen from the figure that, while for some of the bins the boundary layer and the free troposphere are vertically coherent (e.g., the 269-K bin), for other bins they appear to be fairly independent (e.g., the 274-K bin). Thus, both scenarios of vertical temperature perturbation dependence need to be explored in any such sensitivity study of the type discussed in section 3b of the text.

In Fig. 10, we show the mixing coefficient profiles for the different entrainment schemes. The constant entrainment schemes (C0, C1, C2 and C4) have a high value (0.18 hPa^{-1}) in the ABL, and a much lower value (0, 1, 2, 4, respectively, in units of 10^{-3} hPa^{-1}) for the rest of the vertical column. For the deep inflow schemes, Deep A has an inverse dependence on

height [computed in z coordinates, see (2)] at all levels. Deep B, likewise has approximately
 a z^{-1} dependence at the lowest levels but with a larger coefficient, so it drops off more
 slowly, thus leading to stronger entrainment in the lower troposphere, and tapers to zero at
 mid levels. The interactive scheme is shown for $\hat{T}=271$ K, using the case of 0.5 K added
 in the ABL (which illustrates the largest low level entrainment) and two extreme values of
 free tropospheric humidity, $RH_{FT} = 51\%$ and 99% . Cases with no minimum entrainment
 are shown, where the mixing is purely dynamic, driven by the updraft vertical acceleration.
 When a minimum entrainment is included, the dynamic contribution can change, but tends
 to follow a similar vertical structure. The mixing coefficient exhibits strong vertical variations
 which also change depending on the environment. For the $RH_{FT} = 0.51$ case, there is no
 dynamic entrainment in the convective inhibition layer near 950 hPa at the top of the ABL,
 since there is no positive vertical acceleration. Substantial entrainment occurs in the 930-
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 on the parcel. Thus, specifying a minimum entrainment is important for the interactive
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 sensitive to the environmental conditions.

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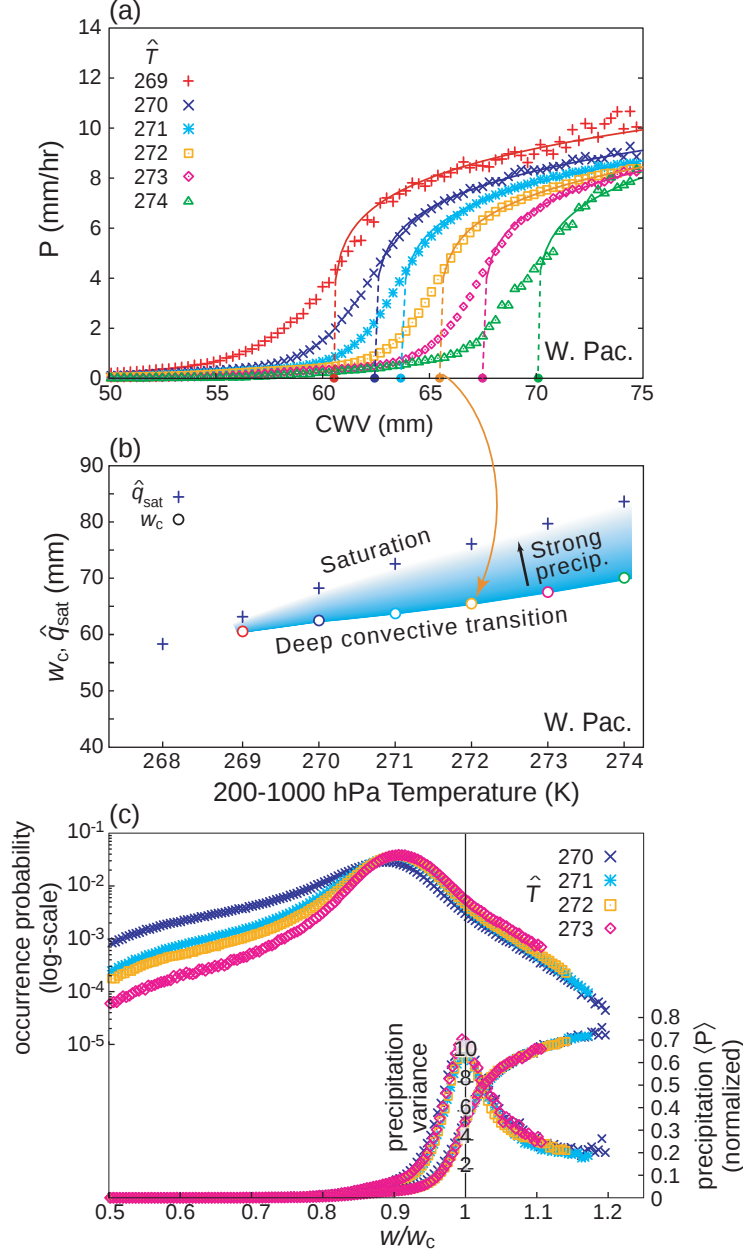


FIG. 1. (a) Conditionally averaged precipitation as a function of CWV for different bins of bulk tropospheric temperature $\hat{T}$, over the tropical western Pacific. Power-law fit lines (solid curves) of the form $a(w - w_c)^\beta$ are shown above the critical value w_c , where precipitation undergoes a rapid pickup (dashed lines connect the fit curves to the values of w_c on the axis). (b) Estimated values of critical CWV w_c , defining an empirical estimate of the deep convective transition, as a function of $\hat{T}$. Also shown is the vertically integrated (200-1000 hPa) saturation specific humidity $\hat{q}_{sat}$. (c) Probability of occurrence of precipitating points, variance of precipitation, and normalized precipitation for the most populous $\hat{T}$ bins, as a function of rescaled CWV (w/w_c).

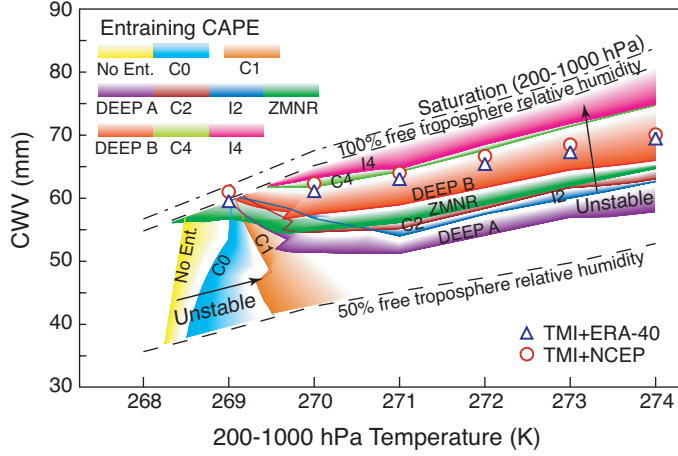


FIG. 2. The onset of conditional instability from parcel/plume calculations (see text) for different entrainment cases as a function of tropospheric temperature $\hat{T}$ is shown. Each line corresponds to an entraining CAPE contour of 100 J/kg, shown as a measure of the convective onset boundary analogous to the deep convective transition line shown in Fig. 1b. The critical column water vapor values w_c from Fig. 1b (estimated from TMI with NCEP temperatures) are shown as open circles for comparison (triangles show corresponding values using ERA-40 temperatures). For each entrainment scheme, shading indicates on which side of the contour convective instability increases (CAPE typically increases all the way to saturation, the shading is faded out to permit onset boundaries for other entrainment schemes to be shown). The constant entrainment cases C0, C1, C2, C4 have coefficients of 0, 1, 2, 4 in units of 10^{-3} hPa^{-1} in the free troposphere. The interactive entrainment cases I2 and I4 have a minimum entrainment in the free troposphere corresponding to C2, C4. The Deep A and Deep B cases have a vertical structure of the entrainment coefficient that has an approximate z^{-1} dependence in the lower troposphere.

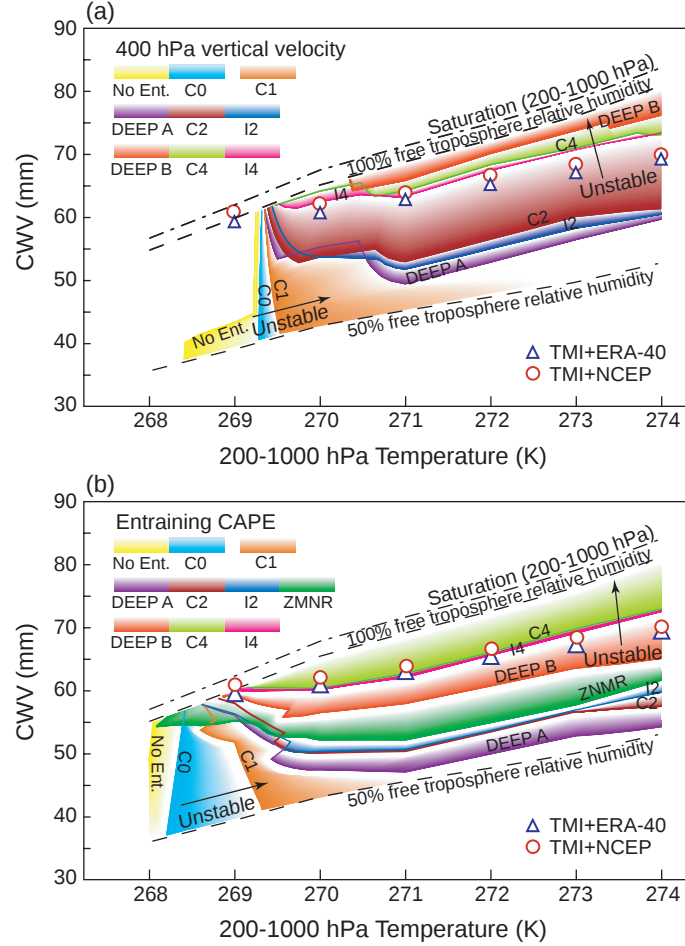


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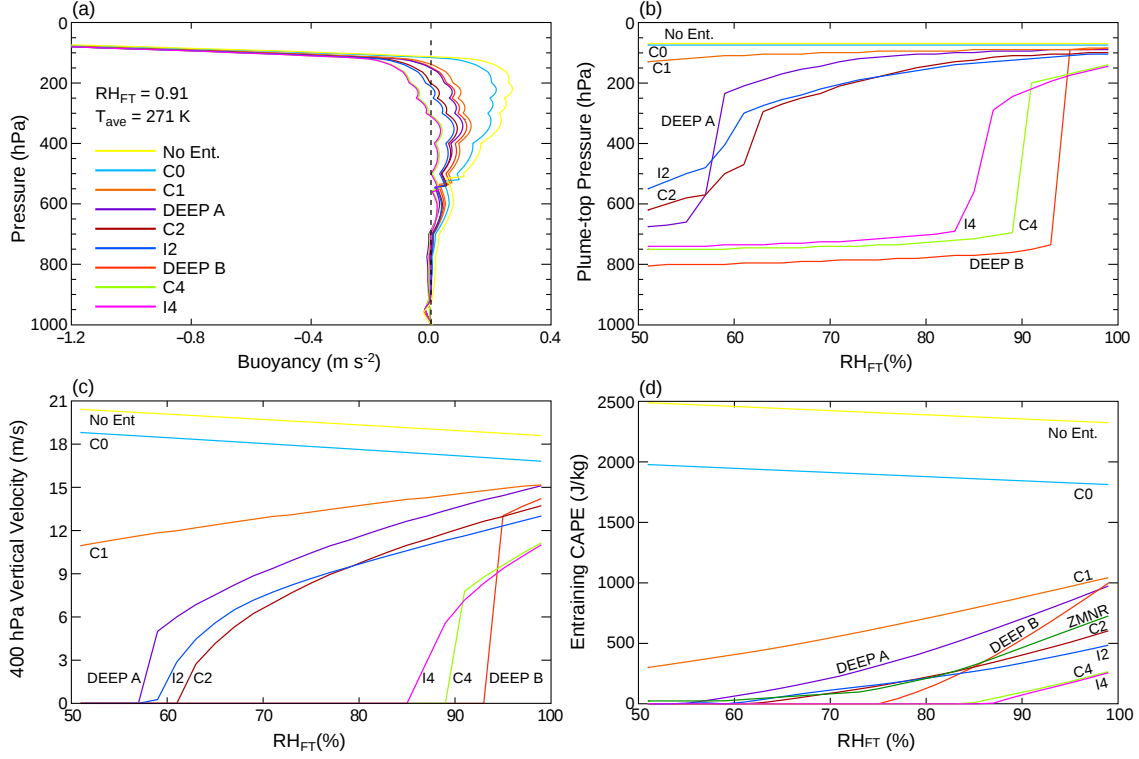


FIG. 4. (a) Parcel buoyancy profiles with various entrainment schemes for free tropospheric $RH = 91$ %, and $\hat{T} = 271$ K. (b) Plume-top pressures for different values of free tropospheric RH , and $\hat{T} = 271$ K. (c) Same as (b), but showing 400 hPa vertical velocity. (d) Same as (b) but showing entraining CAPE in J/kg (also includes the computations for ZMNR).

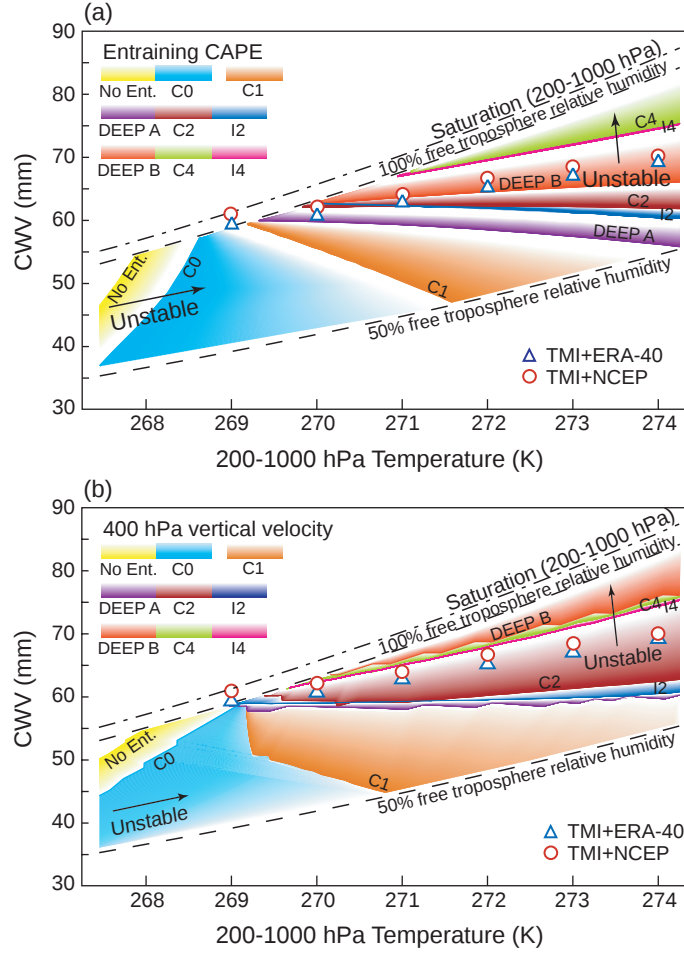


FIG. 5. Onset boundaries for plume calculations as in Fig. 2 and Fig. 3a, but with constant temperature perturbations applied to the mean state temperature sounding over Nauru (see text), in the boundary layer as well as free troposphere. (a) Entraining CAPE contours of 100 J/kg, (b) 400 hPa vertical velocity contours of 5 m/s.

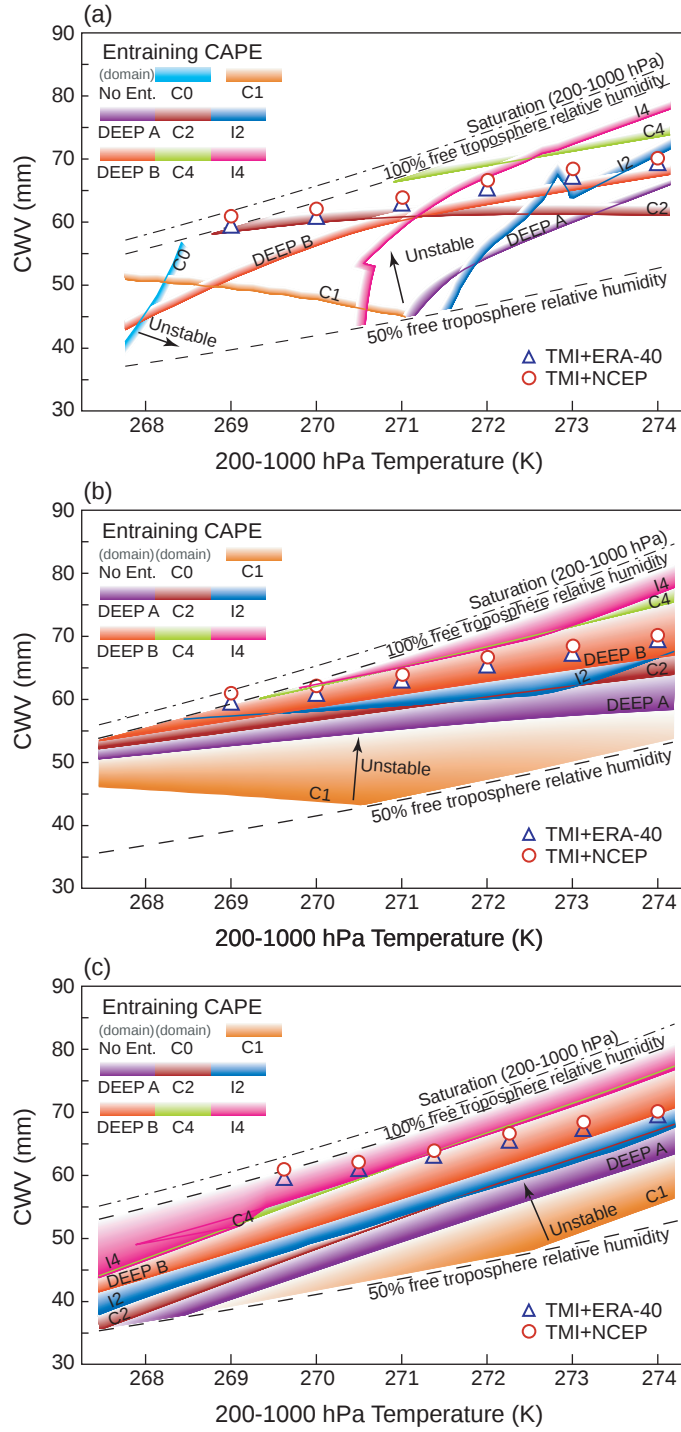


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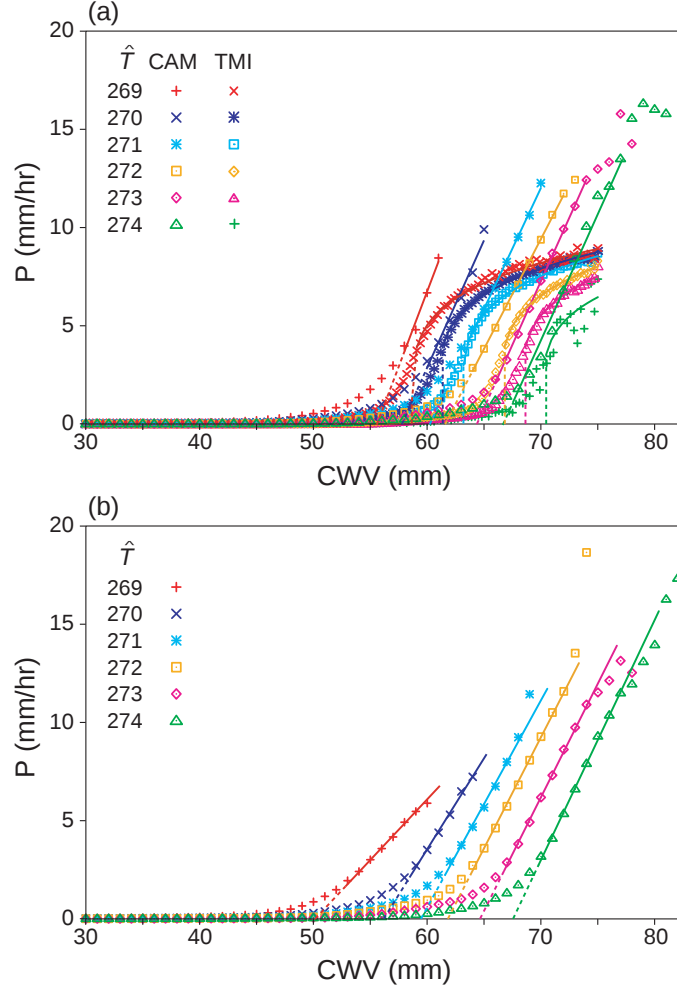


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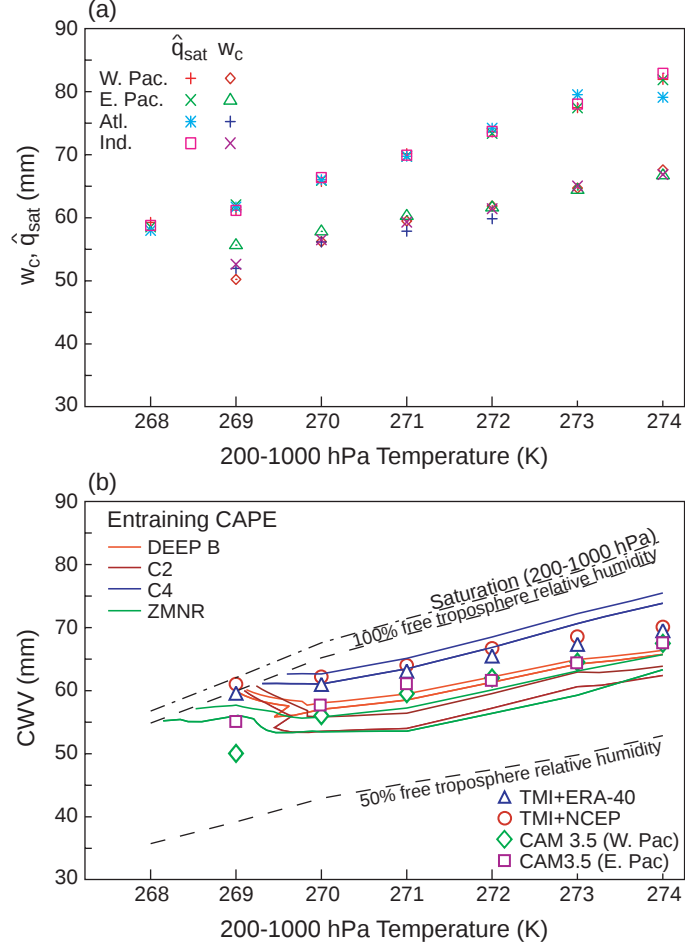


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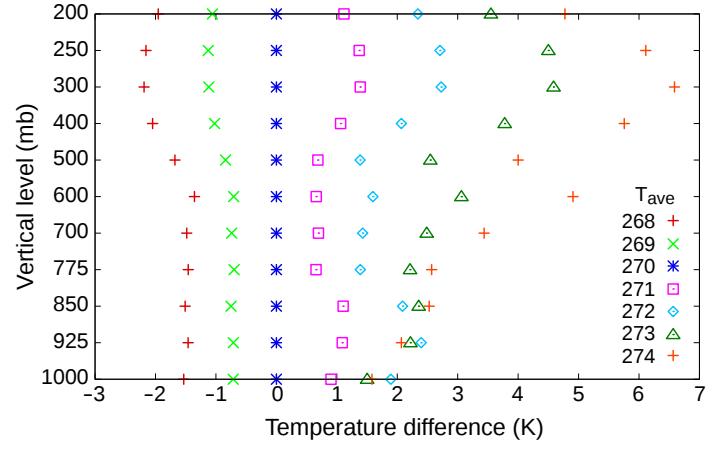


FIG. 9. Vertical temperature perturbation profile over the tropical western Pacific, using ERA-40, for the period 1998-2001, using $\hat{T} = 270$ K as the base profile from which the differences are computed.

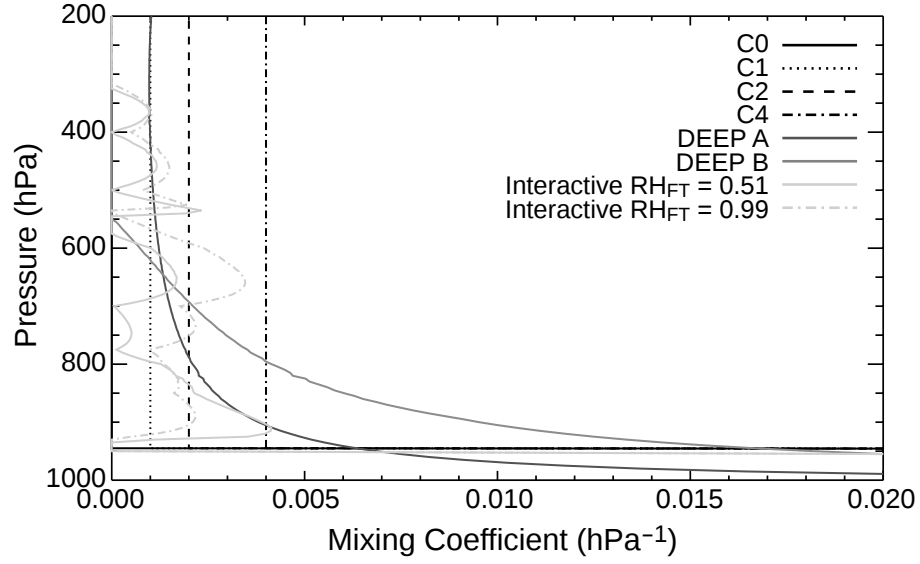


FIG. 10. Vertical profile of mixing coefficient for the various entrainment schemes. The mixing coefficient for the interactive scheme is illustrated for a temperature profile from ERA-40 with $\hat{T}=271$ K, using the case where 0.5 K was added in the ABL (see text), and for two extreme values of free tropospheric relative humidity used in this study (51% and 99%).