Data-based paleoclimate stochastic model

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Introduction

We apply the empirical mode reduction (EMR) methodology (Kraus et al., 2005) to construct a hierarchy of dynamic, stochastically forced models for the analysis and simulation of paleoclimate data. We apply this methodology to a multivariate dataset consisting of Vostok ice-core and marine-core time series, representing proxy records for temperature and global ice-volume, respectively. In addition to residual stochastic forcing, the model is externally forced by orbital periodic forcing. The model’s fit to the proxy records is verified by checking the probability density functions (PDFs) and autocorrelations, of the paleodata and of the reduced model’s simulations. An analytical study of the reduced models suggests a role for stochastically forced internal variability, in addition to the periodic orbital forcing.

Empirical Mode Reduction (EMR)

- Sometimes we have observational data but not a good physical model.
- We want models that are as simple as possible, but not any simpler!

Criteria for a good data-derived model:
- Capture statistics (histograms, correlations, spectra) and relevant dynamics: regimes, oscillations, etc.
- Deterministic dynamics easy to analyze analytically.
- Good noise estimates.
- Describes independent data.

General form of stochastic DE system

\[
\dot{x}_i = (x^T A_i x + b_i^{(o)} x + c_i^{(o)}) dt + r_i^{(0)} dt,
\]

- Matrices \( A_i \), vectors \( b_i^{(o)} \), and scalars \( c_i^{(o)} \) are estimated by least squares.
- Multiple predictors are time series \( r_i \).
- Predictant variables are one-step time differences of predictors; \( \Delta t \) = sampling interval.
- Multi-level modeling of noise \( r_i \) to account serial correlations in the regression residuals.
- The number of levels is such that each of the last-level (L) regression residuals is “white” in time.
- Spatial (cross-channel) correlations of the last-level residuals are retained in subsequent regression-model simulations.
- Model nonlinearity is chosen to optimize the EMR performance (comparison with data).

Multiplicative periodic forcing:
- Modify linear and constant terms on the main (0) level to account for periodic forcing with period \( T \):

\[
\begin{align*}
\dot{b}_i^{(0)} &= b_i^{(0)} + b_i^{(0)} f_p(t, T), \\
\dot{c}_i^{(0)} &= c_i^{(0)} + c_i^{(0)} f_p(t, T).
\end{align*}
\]

EMR for paleoclimate

Oscillatory feedbacks:
- Ice-albedo
- Temperature-precipitation

\[
\dot{T} \approx -V \quad \dot{V} \approx -T
\]

Milankovitch forcing:
- Obliquity (41 kYr), precession (23,19 kYr), eccentricity (100 kYr)

+ Nonlinearity

Results for quadratic 3-L EMR model for \( V \) and \( T \) with quasi-periodic forcing at 19, 23, 41, and 100 kYr.

Data

- Marine core (proxy for ice volume \( V \))
- Vostok core (proxy for surface air temperature \( T \))
  - Both records are interpolated to 1 Kyr resolution

![Normalized proxy records](image)

References


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[http://www.atmos.ucla.edu/tcd/](http://www.atmos.ucla.edu/tcd/)