Geophysical flows as dynamical systems: the influence of Hide’s experiments

Michael Ghil, Peter L Read and Leonard A Smith recount the many and various ways that Raymond Hide has influenced their life and work in geophysical fluid dynamics, meteorology, climatology and planetary sciences, as well as in developing the study of dynamical systems in general.

Today it is well understood that the theory of dynamical systems, both finite- and infinite-dimensional (Constantin et al. 1989, Temam 1997), provides a powerful way of looking at the nonlinear systems of equations that govern geophysical and other flows and phenomena, from biology to society. But in the early 1950s, when Raymond Hide set out to explain the geodynamo, dynamical systems theory was not well known outside specialized circles of mathematicians. The vision of explaining complex phenomena by “climbing the bifurcation tree”, i.e. proceeding systematically from the simple to the complex, via sudden changes in system behaviour, was only a glimmer in the eye of those in other fields. This was to change.

Geophysical fluid dynamics

In the beginning there was the experiment. Lorenz (1967) devotes chapter six, out of a total of eight, of his book on the general circulation of the atmosphere to “Laboratory models of the atmosphere”. There are four basic approaches to the understanding of the general circulation: observational, theoretical, experimental and numerical. At present, with the advent of satellites on the one hand and computers on the other, the observational and numerical approaches have gained in importance; even more human and material resources are invested in these two approaches. But a deeper understanding of the phenomena cannot be obtained without theoretical and experimental work, which benefit from concentrated efforts by individuals or smaller groups. F Vettin (1857) carried out the first known laboratory experiments on the general circulation, using air as the working fluid and ice in the centre of a cylinder to create a temperature contrast. He obtained a Hadley-type circulation, visualizing the flow using smoke from a cigar. His contemporaries did not appreciate the relevance of these experiments to an understanding of meteorological phenomena, although J Thomson (1892) proposed similar experiments using water instead, but did not carry them out.

After a few more experiments in the first half of the 20th century, D Fultz’s group set out to perform a more systematic series of experiments, with the US Air Force as the sponsor, at the turn of the first into the second half of the century. These
experiments at the University of Chicago used water in a dishpan, mounted on a rotating table. A heat source was provided near the rim, and in some experiments there was a cold source in the centre. The idea was that the rim simulated the equator and the centre was the North Pole. Two flow regimes were observed, one that was nearly axisymmetric, called the Hadley (1735) regime, the other wave-like, called the Rossby regime (Fultz 1951, Fultz et al. 1959). But the dishpan experiments were hard to control precisely and thus to replicate. It was a young graduate student in Cambridge, who was interested in explaining the mechanism of the geodynamo, who set up an experimental apparatus that overcame the difficulties of the Fultz team and was destined to be the main progenitor of modern experimentation on the general circulation to this day. This path-breaking 1950–1953 episode is partially and briefly retold in Hide (2010). The apparatus is sketched in figure 1, whose caption also defines the two main parameters that help characterize the flow regime, namely the Hide number $H$ and the Taylor number $T$.

Raymond Hide’s unusual PhD thesis made at least three fundamental contributions:

- (i) the first documentation of periodic, regular Rossby (1939) waves in a fluid;
- (ii) the discovery of the quasi-periodic vacillation phenomenon; and
- (iii) one of the first, or maybe the first, study of what we call today bifurcation and regime diagrams in a fluid dynamical context.

The first of these three contributions solidly established the pertinence of simple theoretical studies of rotating, stratified flows. The second involved, eventually, the study of amplitude and shape — also called tilted-trough — vacillation. The third is an entirely different, and much broader story, discussed in later sections.

The first contribution is much less trivial than it might appear today, when Rossby waves are taken for granted and widely taught. At the time though, given the observed irregularity of atmospheric flows, the possibility of regular waves in a rotating fluid was far less than obvious (Lorenz 1967). Furthermore, the fact that the transition from the Hadley to the Rossby regime occurred by the recently discovered, truly 3-D baroclinic instability of Charney (1947) and Eady (1949), was a further, deep insight for geophysical fluid dynamics (GFD). We shall treat the second and third contributions below.

**Vortices, radio astronomy and stars**

Michael Ghil recalls doing military service (1967–1971) as an officer in the Israeli Navy, at that time headquartered in Haifa, while also serving as an instructor at the Technion-Israel Institute of Technology. In the latter capacity he worked as an assistant for the last graduate course that Sydney Goldstein taught after his retirement from Harvard; the course was based on Goldstein (1960). Ghil was completing an MSc degree in the Faculty of Mechanical Engineering, under Alex Solan, on heat transfer through a Rankine vortex as a model of the vortices in a Karman vortex street behind a cylinder (Goldstein 1966, Ghil and Solan 1973).

When he could get away from his naval duties, Ghil ran to the Technion library to see whether anybody had solved the MSc problem of his choice in the meantime. It was in the process of rummaging for results on vortices — any vortices — that he first stumbled on the work of Raymond Hide. This geophysical fluid dynamics connection provided later on – while Ghil was a PhD student at the Courant Institute of Mathematical Sciences in New York City, under Peter Lax — part of the motivation for taking a summer job at NASA’s Goddard Institute for Space Studies and losing himself forever in the atmospheric and oceanic sciences.

Shortly after completing his PhD thesis, Ghil met Hide in person at the Summer School on Rotating Fluids in Newcastle upon Tyne (Hide 1977/1978, Roberts and Soward 1978). Hide kindly invited him right after that to the lab he was running at the time at the UK Meteorological Office in Bracknell, to observe some of the experiments (Ghil 1978) and obtain invaluable advice for his research and the life built around that research. As a result, Ghil invited Hide to lecture at a Summer School on Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics in Varenna, Italy (Ghil et al. 1985, Hide 1985) and subsequently dedicated chapter V of his next book (Ghil and Childress 1987) to Hide’s experiments and Ed Lorenz’s analysis thereof (Lorenz 1962, 1963b).

Peter Read joined Raymond Hide’s research group at the Met. Office in 1980, shortly after completing a PhD in radioastronomy with Martin Ryle’s group at Cambridge. This was a particularly interesting and exciting time, very soon after the Voyager spacecraft had flown past Jupiter and returned astonishing images and measurements of Jupiter’s Great Red Spot, its equatorial jet and other features about which Hide had theorized since the 1960s. It was also just at the time when theoretical ideas relating to autonomous dynamical systems were making a transition from the arcane world of pure
mathematics and starting to make a significant impact on a number of applied sciences, notably experimental physics and meteorology.

One of Read's first recollections of Hide's group was making the trip from Bracknell in Berkshire, as a newly arrived post-doctoral scientist, to the Mathematics Institute at Warwick University in early 1980. This was to attend an informal workshop in which leading pure mathematicians from Warwick, such as Christopher Zeeman, Ian Stewart and David Rand, sought to educate curious, though (at the time) somewhat ignorant, physicists and applied mathematicians in the mysteries of low-dimensional attractors and chaos, and to convince them of their significance for complex nonlinear phenomena in the “real world”. Though well-meaning, communication between these disparate groups was not easy, because it seemed to many of the physicists that the mathematicians were expending inordinate amounts of effort proving aspects of phenomena and enumerating the roles of such as the existence and uniqueness (or not!) of solutions to ordinary differential equations, based on truncated partial differential equations (Ghil 1978, Ghil et al. 1985).

At the time, there was much interest in obtaining, from observational or experimental data, a statistical “proof” that a “low-dimensional description” of the dynamics existed; such a description would require only a small number of independent degrees of freedom, a number called the “dimension” d of the system (Grassberger and Procaccia 1983).

Smith conjectured that it required more data to establish that such a description existed than it did to actually provide it (Smith 1997). Indeed, faithful, low-dimensional reconstructions of the dynamics of the rotating annulus were found, and as well as illustrating less generic, but perhaps equally interesting, aspects of nonlinear dynamical systems (e.g. Read et al. 1992, Mullin 1993), the annulus inspired many discussions of the advantages and disadvantages of viewing the dynamics in this way. There are significant theoretical and practical advantages in building low-dimensional models for the evolution of annulus flows, in a phase space with d ~ 10 dimensions, and considering model trajectories in this phase space; still, there is also much information that is lost with respect to working with much higher-dimensional simulations, based on truncated partial differential equations (Ghil 1978, Ghil et al. 1985).

Bifurcations, dynamical systems and chaos

The origins of the modern theory of differentiable dynamical systems lie in the pioneering work of Henri Poincaré in the late 19th century, while in the first half of the 20th century G.D. Birkhoff in the United States and A.M. Lyapunov, A.A. Andronov and colleagues in Russia made many key contributions. Several references can be found, for instance, in Guckenheimer and Holmes (1983). But the interest of physicists for differentiable dynamical systems theory had to wait for connections with some striking experimental evidence to be established.

The authors of this tribute are convinced that Raymond Hide’s experiments were certainly among the first and most insightful, and we shall now elaborate. Figure 2a shows a detailed regime diagram, in which the various flow types are separated by sharp boundaries. For a fixed apparatus of height D and gap width L = b − a, and a given fluid with expansion coefficient α and viscosity ν, one can change the rotation rate Ω and the temperature difference ΔT = T₁ − T₂. The latter involves the two heat baths, outside and inside the fluid gap (see figure 1e), and their
temperatures $T_h$ and $T_e$ require a relatively long time to stabilize, while the former only depends on the motor that makes the apparatus rotate. Because we have $H - \Delta T Q_q^2$, while $T - Q_q^2 v^2$, setting $\Delta T$ as constant and gradually increasing $\Omega$ corresponds to moving along a given straight, downward-slaning diagonal, to the right and down, in a log-log diagram of $H$ vs $T$, such as that in figure 2a.

Figure 2b illustrates the main flow regimes found in the rotating, differentially heated annulus of figure 1. There are essentially five distinct qualitative regimes, which are further described in figure 5b below: (H) the axisymmetric Hadley regime (figure 2a), in which the fluid rises close to the outer rim, sinks close to the inner rim, and is deflected by the Coriolis force as it flows near the surface from the outer to the inner rim; (Rj) the purely periodic Rossby regime (figure 2b and figure 2c, with azimuthal wave number $k = 2$ and $k = 3$, respectively), in which waves characterized by a low wavenumber $k$ (roughly $2 \leq k \leq 5$) and by a fixed shape and amplitude travel around the annulus; (Rq) the quasi-periodic Rossby regime (figure 2d), in which either the amplitude or the shape of the waves changes in a periodic fashion (see figures 3 and 4 below); (T) a transitional regime (figure 2e), in which the principal wavenumber is no longer an integer or even constant in time and the motion is somewhat irregular; and (Tq) a turbulent regime (figure 2f), in which waves are still recognizable, but the flow is quite irregular. The last two regimes look more strikingly like large-scale atmospheric flows in the Earth’s atmosphere, though we now know that the second and third regimes bear a strong resemblance to similar large-scale flows in the atmosphere of Mars.

Figure 3 shows a sequence of flow patterns that illustrate the so-called “amplitude vacillation” regime, in which the flow is dominated by a single wave that drifts around the annulus and whose amplitude grows and decays periodically in regular cycles. The period of the amplitude cycle is distinct from that of the drift cycle. When the two periods are rationally unrelated, the motion is termed quasi-periodic in differentiable dynamical systems (DDS) theory.

The other main kind of “vacillation” is illustrated in figure 4, which shows a sequence of flow patterns during a “tilted trough” or “structural” vacillation. In this case, the amplitude of the main wave doesn’t change much, but its structure – in particular its tilt in the radial direction – changes back and forth with time. As indicated before, the discoveries of the amplitude and tilted-trough vacillation associated with the quasi-periodic Rossby regime ($R_q$) were very important discoveries in GFD, and they had major implications for the theory of the general circulation of atmospheres and oceans. The former was shown to be largely due to baroclinic phenomena associated with the modulation of the transport of heat by the waves, and hence the rate of release of potential energy; the latter is mainly due to the nonlinear interaction of barotropic waves and thus entails direct exchanges of kinetic energy. Both changes in amplitude and tilt of atmospheric waves play a key role in the eddy transport of heat and momentum across latitudes, as well as in the establishment of “persistent anomalies” or “blocking” in the mid-latitudes.

Lorenz (1962, 1963b) chose a highly simplified model of Hide’s experiment, in which a periodic channel in Cartesian coordinates replaced the annular gap, and the flow was governed by the two-layer quasi-geostrophic equations. Furthermore, these partial differential equations were projected onto an orthonormal basis of sine and cosine functions. Lorenz made the ingenious choice of limiting the truncated set of basis functions to a total of six components: zonal flow and a single wave (represented by a sine and a cosine function, in quadrature), of arbitrary wavenumber $n$ in the zonal direction $x$, and two wave numbers, called modes, with $m = 1, 2$, in the meridional direction. His analysis is reviewed, in the now widely accepted language of successive bifurcations, in chapter 5 of Ghil and Childress (1987), and the results are sketched in figure 5a here.

The sharp transitions in flow regime obtained by Hide in his thesis and the many follow-up experiments with similar apparatus can indeed be explained in this language. The transition from the steady, axisymmetric Hadley regime ($H$) to the purely periodic, steady-wave regime ($R_j$) corresponds to a Hopf bifurcation, in which a stationary, equilibrium solution transfers its stability to a periodic one, while the transition from ($R_j$) to the vacillation regime ($R_q$) is a secondary Hopf bifurcation, from a simply to a doubly periodic solution. Finally, the transitional regime ($T_e$) corresponds to a form of what we now call deterministic chaos or weak turbulence, while ($T_q$) is fully developed, albeit quasi-geostrophic, turbulence.

The first two bifurcations are so-called local bifurcations, whose analysis only requires suitable linearization about the equilibrium – called a fixed point in DDS theory – or the simply periodic solution, called a limit cycle, and they are well captured by Lorenz’s (1963b) model. The
latter two transitions, though, are not so well captured, for two reasons. First, they are non-local bifurcations, which involve the presence of so-called homoclinic or heteroclinic orbits; such orbits are structurally unstable, i.e. small perturbations of the differentiable dynamical systems destroy them. Second, they require an increase in the number of basis functions – i.e. in the model’s dimension – in order to capture the more complex spatio-temporal behaviour. Based on the theoretical considerations above, we sketch in figure 5b a simplified regime diagram of flow in the rotating, differentially heated annulus.

Variations on the rotating annulus

Ed Lorenz emphasized – in his well-known monograph on the general circulation of the atmosphere (Lorenz 1967) – that a key contribution of laboratory experiments such as the rotating annulus was to enable one to distinguish the most fundamental factors determining the properties and behaviour of the atmosphere and climate from factors that are of lesser significance. Hide (2010) further highlights this contribution in the present issue. The simple experiments first studied in detail by Raymond Hide and Dave Fultz in the 1940s and 1950s clearly show that cyclones, anticyclones, jet streams and fronts do not require for their existence the presence of continents and oceans, moisture, mountains or even the spherical curvature of the Earth: only the basic ingredients of an equator-to-pole contrast in heating and cooling and sufficiently rapid background rotation of the planet are truly of the essence.

In later research following on Hide’s pioneering experiments, however, much work has been devoted to investigating the impact of reintroducing at least some of these less essential factors – in particular those that can be emulated in the laboratory fairly easily – as well as unravelling some of the quantitative details of the bifurcations and flow regimes of the original experiments. Some of the low-dimensional flow regimes predicted by simple models, such as those of Lorenz (1962, 1963b), turn out to be producible in small areas of the parameter space in the laboratory, though not all. The latter shortcoming is because the simple models expand the spatial structure of the flow into a set of orthonormal basis functions, and then truncate this set fairly aggressively to result in a tractable set of ordinary differential equations. The retention of only a small subset of modes implicitly assumes that the rest of the spectrum will either be damped out by friction or that the behaviour of the heuristically neglected modes will be dominated by (or “enslaved to”) the behaviour of the dominant, retained modes. Laboratory experiments have shown that this will happen only if the flow is not too strongly baroclinically unstable; still, it is of of major interest that low-dimensional, quasi-periodic and chaotic flows do occur in a real fluid, thus validating the modelling approach of B Saltzman (1962), Lorenz and others. For more unstable flows, however, in the transitional region (\( T_\text{r} \)), the behaviour is no longer low-dimensional but energy begins to spread across many more spatial modes, and thus the transition to (quasi-)geostrophic turbulence eventually energizes a wider and wider range of spatial scales to produce a highly complex flow in the fully irregular (\( T_\text{ir} \)) regime.

Several other, less essential, factors have been successively introduced in recent variations on the theme of the rotating, differentially heated annulus experiment (e.g. Hide and Mason 1975). These factors include the addition of radially sloping lower or upper boundaries, to emulate some of the effects of the spherical curvature of a real planet, variations in the radial and vertical distributions of heating and cooling, and the addition of non-axisymmetric topography; the latter includes full and partial radial barriers, to emulate the effects of mountain ranges, valleys and even coastlines on large-scale atmosphere and ocean circulation systems. These experiments have confirmed that such additional effects often only modify the details of the main baroclinic waves, though in extreme cases they can change the precise parameter values under which the Rossby wave regimes occur in the experiment.

After more than 50 years, it is remarkable that this simple lab analogue of large-scale atmospheric motion still challenges and surprises us.
Rossby wave in a rotating annulus filled with two immiscible fluids: an organic oil in the lower layer and water in the upper. In this case, baroclinic instability shows up as large-scale waves (with $k = 2$) on the interface between the two layers. But the visualization also shows some much smaller-scale ripples in the troughs of the long “planetary” Rossby wave. Further investigation (Williams et al. 2005) demonstrates that these shortwave ripples have actually a completely different origin: they owe their existence to the gravitational restoring forces in the fluid and are hence known to meteorologists and oceanographers as “gravity waves”.

Such short-wavelength, high-frequency waves do occur in the atmosphere and oceans, and are responsible for phenomena such as clear air turbulence, well known to air travelers, and lee wave clouds associated with mountains and mountain ranges. They are also being increasingly recognized as a significant source of uncertainty in weather forecasting, and may play a major role in the dissipation of energy in the oceans as well. It has been known for many years that atmospheric convection and flow over mountains and valleys can excite these waves in the atmosphere, but it is only recently that meteorologists have discovered that similar gravity-wave trains can be excited by complex nonlinear interactions within large-scale baroclinic cyclones and anticyclones.

These gravity waves are difficult to capture in numerical models because of the large difference in scale between them and the Rossby waves, but high-resolution models are now able to do this: figure 7 illustrates such an example. The resemblance between the “chevron-shaped” waves in the model simulation (figure 7, lower panel) and the annulus experiment (see again figure 6) demonstrates that such beautiful laboratory experiments can provide valuable sources of insight, even for the latest generation of meteorologists!

An important issue addressed by a slightly different type of annulus experiment is the interaction of the large-scale atmospheric flow with topography. Charney and DeVore (1979) formulated and analysed a low-order model for this interaction, in a mid-latitude channel; blocked and zonal flow patterns arise as two stable fixed points of their model. The S-shaped bifurcation diagram associated with this model can be seen in chapter 6 of Ghil and Childress (1987, figure 6.5). Legras and Ghil (1983) carried out a detailed study of a more highly resolved model on the sphere, in which – for reasonable values of the parameters – no stable fixed points were present. Instead, zonal and blocked flow patterns were organized into distinct flow regimes, surrounding an unstable fixed point and an unstable limit cycle, respectively.

The latter results were put to the test first in the traditional, “baroclinic” annulus (Bernar-
et al. 1990) of Météo-France in Toulouse. Unfortunately, the very active baroclinic waves that dominated this experimental set-up did not allow a direct confrontation with either the theoretical results of Legras and Ghil’s (1985) model or with atmospheric observations. Indeed, it was already well known in the early 1980s that so-called low-frequency atmospheric variability, in the frequency band of 10–100 days, is predominantly barotropic (Wallace and Blackmon 1983).

It seemed, therefore, more appropriate to use the “barotropic annulus” of Harry Swinney’s research group at the University of Texas in Austin (Sommeria et al. 1989). In this annulus, the flow is driven by injection of fluid through a set of holes in the bottom, arranged equidistantly on an outer circle of radius $r_3$ with $L_a < r < L_3$, and by the suction of the same fluid through another set of holes, lying on a concentric, inner circle of radius $r_2$, with $L_a < r < r_2$; this pumping of fluid replaces the differential heating of the baroclinic annulus we have discussed so far.

First, some preliminary “shoelace-and-wax” experiments – to quote Sydney Goldstein’s advice to Michael Ghil, years before – with clumps of clay on the bottom, did yield blocked and zonal flows in the Austin annulus. Next, a smooth wavenumber-two topography and a more powerful pump were installed, and a careful series of experiments were conducted to generate a regime diagram in the parameter plane of suitably defined Ekman and Rossby numbers, $E$ and $R$. The results included the coexistence of zonal and blocked flow regimes (figures 8a and 8b, respectively), with residence times of the flow in either regime that changed with the nondimensional parameter values, as predicted by Legras and Ghil (1985). The changes in azimuthal velocity (at fixed $E$ and $R$) and in residence times, as $R$ changes at fixed $E$, are shown in figures 8c and 8d, respectively.

We have seen so far that the rotating annulus and its variants have enabled, and continue to enable, the discovery and understanding of a wide variety of dynamical phenomena that are reminiscent of flows in the atmosphere and oceans. Even more fascinating is the possibility of generalizing the results of such experiments to other planetary circulations. As Raymond Hide himself wrote in 1969: “The experiments have emphasized the necessity for truly quantitative considerations of planetary atmospheres.”

9: Schematic regime diagram for the rotating, differentially heated annulus. The up-left to down-right slanting diagonals indicate isolines similar to those in figures 2a and 5b, for $\Delta T = 10K$, (characteristic of the meridional gradient in Earth’s mid-latitudes) and $\Delta T = 1K$ (characteristic of Earth’s tropics), respectively. The heavy dots indicate the approximate position of the following planetary circulations: Earth’s atmosphere, tropical and mid-latitude, along with Mars, Venus and Titan.

Chaos and predictability

One of the main promises of deterministic chaos is that it’s more predictable than purely random phenomena. Figure 10 illustrates the increasing difficulty of predicting phenomena that are (a) constant in time, (b) purely periodic, (c) quasi-periodic, and (d) truly aperiodic. In the latter case, while true randomness may be hard to distinguish visually from deterministically chaotic phenomena, many statistical ways of telling the difference have been developed. Furthermore, the presence of coarse graining or of limit cycles that are only slightly unstable in the system’s phase space can further increase the predictability (Ghil and Robertson 2002).

The early 1990s saw the advent of “ensemble forecasts” in operational weather forecasting on both sides of the Atlantic (Molteni et al. 1996, Toth and Kalnay 1993); distinct methodologies were used to generate the initial ensemble of weather states (Legras and Vautard 1996). Questions surrounding how to evaluate the skill of large, complex, nonlinear simulation models are still with us (e.g. McSharry and Smith 2004), but ensemble forecasts of flow in the annulus provided a “neutral” setting, as well as better statistics, which allows the comparison of competing operational forecast methods (e.g. Gilmour 1998). Shadowing of trajectories provides a valuable approach for evaluating the level of detail with which a given model can reflect an actual physical system; this approach is robust in the face of chaos or other nonlinear instabilities that disqualify the use of traditional prediction scores. Once again, the annulus is providing a test bed for comparisons between full-blown numerical simulations and laboratory observations (Young and Read 2010, Young et al. 2010). This work continues at the London School of Economics and Oxford and will be using the annulus to test methods of model evaluation that could help us determine the space and time scales on which full-scale general circulation models of the atmosphere and oceans might reliably inform economic and political decision making, and those on which they cannot.

As Raymond Hide said in 1953: “The possibility of solving problems of dynamical meteorology experimentally is an important one in view of the great theoretical difficulty involved.”
(Hide 1953). Contrasting models and observations of the rotating annulus make it possible to go beyond the classical problems of dynamic meteorology into the highly topical realm of problems that involve the entire climate and even full Earth system, with the atmosphere, ocean and other components interacting across space and time scales. This possibility has become much more significant in view of the great difficulties involved in observing and numerically simulating the climate system, let alone performing experiments on it. Some 50 years ago, the rotating annulus provided concrete insights into phenomena not then well-captured by incipient numerical models of the atmosphere. It still offers today an invaluable platform for critical evaluation of model errors, data assimilation and parameter estimation methodology, and probabilistic forecast interpretation. The multifaceted applications of the annulus experiments continue to provide new insights over a wide range of the Earth and planetary sciences, while also continuing to play a key role in the education of students in these fields (Illari et al. 2009).

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