The uncertain future of climate uncertainty

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This paper presents a stochastic model for the evolution in time of the probability distribution of climate sensitivity. The analysis of this model shows that the future trajectory of climate uncertainty may itself be highly uncertain, even when assuming steady progress in climate research. Uncertainties in climate model feedbacks play a key role in these considerations.

1. Introduction and motivation

Strong scientific consensus prevails over the fact that Earth's climate is currently warming and will be warming further over the coming decades, as a consequence of the radiative perturbation caused by anthropogenic greenhouse-gas (GHG) emissions. The conclusions of the IPCC's Fourth Assessment Report (AR4: Solomon et al. [2007], [AR4] hereafter) further buttress this consensus.

There is, however, substantial uncertainty regarding the extent of future warming, as pointed out in the same report and in many of its references. This uncertainty renders decision making on appropriate mitigation and adaptation steps more difficult. In addition, the uncertainty level regarding future climate evolution has not decreased significantly over the past decades, an observation that paves the way for climate-warming naysayers and is sometimes used as an argument to discredit climate science and its findings as a whole. Credence given to the naysayers tends, in turn, to slow down or even stop action on this issue; it also tends to interfere with a healthy, truly scientific debate on the real extent of and on the reasons for the uncertainty. These two reasons motivate us to (a) revisit here a key cause for the persisting uncertainties, in order to potentially reduce them; and (b) try to anticipate their future evolution as research makes further progress.

Uncertainties regarding future climate warming are usually divided into three categories: (i) those regarding the GHG increase scenarios [AR4]; (ii) those arising from the climate system's internal variability [Ghil et al., 2008]; and (iii) those due to model formulation and properties. The relative importance of either type of causes varies considerably according to lead time, spatial scale and geographic location (Hawkins and Sutton [2009]). The contribution of internal variability may vanish after a few decades, while scenario uncertainty is part and parcel of humankind's future course of action. This letter focuses, therefore, on model uncertainty and we thus refer to it henceforth simply as climate uncertainty.

Climate sensitivity is often defined as the change ΔT in global equilibrium surface temperature T associated with a given change Δp_{CO_2} in atmospheric CO_2 concentration p_{CO_2}. The diversity of plausible long-term future climate states is determined, to a large extent, by the range of this sensitivity $\gamma = \Delta T/\Delta p_{\mathrm{CO}_2}$.

A metric widely used as a summary quantification of climate uncertainty consists in the dispersion of the multiple values of climate sensitivity obtained by general circulation model (GCM) simulations and by various other studies. According to [AR4], this range is still as high as $\Delta T = 2°\mathrm{C} - 4.5°\mathrm{C}$ for a doubling of p_{CO_2}. It is critical for socio-economic and political decision making to know whether or not it is reasonable to expect it to decrease in the future (e.g., Stainforth et al [2005]; Knutti and Hegerl [2008]; Hannart et al. [2009]; Zaliapin and Ghil [2010] and references therein).

The purpose of this letter is to contribute to the debate on the question above. We argue that the future trajectory of climate uncertainty may be itself quite uncertain, even when assuming future progress in climate research. Our arguments rely on simple mathematical considerations on the role of feedbacks, within a probabilistic framework.

2. Feedback, sensitivity and uncertainty: A simple framework

The most frequent explanation of high climate uncertainty does indeed rely on the uncertainty in feedbacks and most critically, in the cloud-radiative feedback (Dufresne and Bony [2008]; Soden and Held [2006]; Colman [2003]). This argument can be qualitatively illustrated using a highly idealized feedback-sensitivity equation, cf. Hansen et al [1984]:

$$\Delta T = \frac{\Delta T_0}{1-f}; \tag{1}$$

here ΔT_0 is the temperature change for CO_2 doubling in the absence of feedbacks, $f = 0$.

Equation (1) is derived from a simple, linearized energy balance model (EBM). The inverse relationship between

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feedbacks f and sensitivity ΔT in Eq. (1) has two implications. The first one is obvious: the average sensitivity increases faster as the value of the overall feedback parameter f approaches one [Roe and Baker, 2007; Hannart et al., 2009; Zaliapin and Ghil, 2010]. GCM simulations find, on average, $f = 0.65$ and $\Delta T_0 = 1.2°$C; the value of $\Delta T = 3.4°$C obtained by plugging these values into Eq. (1) is consistent with average ΔT values found in numerous studies; hence this linearized-EBM framework may be useful indeed, as long as f stays well below one [Zaliapin and Ghil, 2010].

More relevant to the focus of this letter, Eq. (1) has an interesting implication on the dispersion of sensitivity [Hannart et al., 2009]. Due again to the inverse relationship between ΔT and $1 - f$, when the feedback parameter f is large enough, but not too close to unity, a small fluctuation of f around its average M_f results in a large fluctuation of the sensitivity value. The uncertainty in sensitivity is thus also subject to a feedback amplification effect.

More precisely, following Hannart et al. [2009], we define S_f and $S_{\Delta T}$ to be the feedback spread and sensitivity spread, respectively; they are linked by

$$S_{\Delta T} \simeq \frac{\Delta T_0}{(1 - M_f)^2} S_f \,. \qquad (2)$$

Equation (2) shows that the uncertainty amplification factor associated with $S_{\Delta T}$ is quadratically magnified with respect to the amplification factor for ΔT in Eq. (1). With $S_f = 0.13$ from GCM studies, and the same values for $M_f = 0.65$ and $\Delta T_0 = 1.2°$C as above, Eq. (2) leads to $S_{\Delta T} = 1.25°$C, i.e. an uncertainty range of $2 \times S_{\Delta T} = 2.5°$C, which is consistent with the AR4 range. The implications of Eqs. (1) and (2) appear in Fig. 1.

3. Implications for the future of climate uncertainty

Let us now explore the implications of the linearized-EBM framework of Eqs. (1, 2) and Fig. 1 for the future evolution of climate uncertainty. In doing so, we stick to the central idea that uncertainty in sensitivity is caused merely by uncertainty in the feedbacks, and we project ourselves into the future. At any given future instant $t > t_0$, with t_0 the present time, the state of our knowledge on the climate system can be described by a probability distribution function (pdf) $\mathbb{P}(f \mid t)$ for the feedback parameter f. The time-evolving mean $\mathbb{E}(f \mid t) = M_f(t)$ of this distribution thus represents our best guess on overall feedbacks based on $\mathbb{P}(f \mid t)$, and its spread parameter $S_f(t)$ represents the extent of the uncertainty, resulting from the incomplete understanding and modeling of the physical processes that underlie the feedbacks. In this highly simplified, linear framework, the uncertainty on climate sensitivity ΔT is similarly modeled by a pdf $\mathbb{P}(\Delta T \mid t)$ with spread $S_{\Delta T}(t)$.

Future progress in climate research can be modeled in this framework simply through a change in the feedback spread $S_f(t)$. More specifically, one would expect that progress in climate science will lead to a reduction in $S_f(t)$. One might also hope that an increase in the amount of effort and resources dedicated to climate research would reduce $S_f(t)$ at a faster pace.

Leaving aside the latter speculation on the pace of the decrease, we are now getting to the central point of our argument. Let us assume that some significant improvements in our understanding and modeling of feedbacks will have been obtained and that these improvements translate into a decrease of S_f by one half, say. How will this reduction in S_f affect the sensitivity spread $S_{\Delta T}$, by which we measure here climate uncertainty ?

Considering the linearity of Eq. (2) in S_f, it is tempting to go for the straightforward answer, i.e. that $S_{\Delta T}$ will also decrease by a half. This answer, however, assumes that M_f

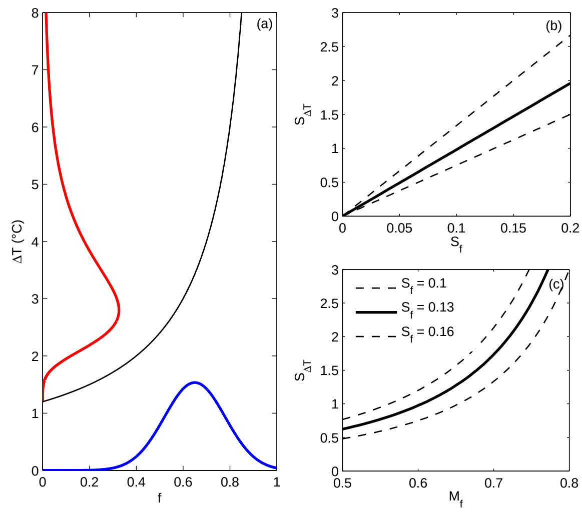

Figure 1. Climate sensitivity ΔT and uncertainty spread $S_{\Delta T}$ as a function of the feedback parameter f, its mean M_f and its spread S_f, respectively. (a) Assumed probability density function (pdf) of f (blue curve), ΔT from Eq. (1) (black curve), and pdf of ΔT (red curve) with current values ($M_f = 0.65$, $S_f = 0.13$); after Roe and Baker [2007]. (b) $S_{\Delta T}$ as a linear function of S_f for several fixed M_f values; and (c) as a nonlinear function of M_f for several fixed S_f values.

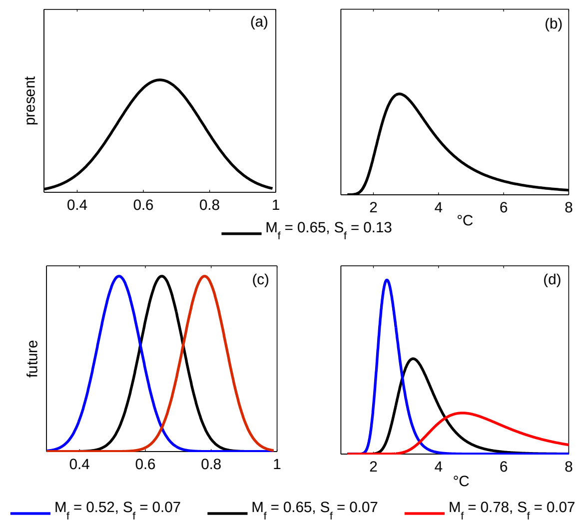

Figure 2. The combined effect of changing the mean M_f and spread S_f of the feedback parameter f. (a) PDF of f; and (b) pdf of ΔT with present values ($M_f = 0.65$, $S_f = 0.13$). (c) Possible future pdfs of f for a halving of S_f (reduction by 50%), and using three updated values of M_f, modified by -20%, 0% and $+20\%$, respectively; and (d) resulting pdfs of ΔT. The curves in panels (c,d) are blue, black and red for the successively increasing values of M_f.

— which reflects our current best guess for the value of f — has not been modified. But it is most likely that the mean M_f of the feedback parameter's pdf, too, has been modified, along with its spread S_f. Furthermore, because the initial uncertainty in f at the present time was large, this change in M_f is expected to be large as well.

This has important consequences for $S_{\Delta T}(t)$ at $t > t_0$, because if M_f is reduced, then the quadratic amplification term in Eq. (2) gets smaller, thus enhancing further the resulting decrease in $S_{\Delta T}$. Conversely, it $M_f(t)$ has increased at $t > t_0$, then the resulting decrease in $S_{\Delta T}$ may be reduced or even changed to a net increase.To illustrate this effect, let us assume for instance that M_f is reduced by 20% while halving S_f. In this case, the 50% reduction of S_f is further amplified and converts into a 75% reduction of $S_{\Delta T}$. Conversely, if M_f is increased by 20%, the same decrease in S_f now results in an increase of $S_{\Delta T}$ by 25%, instead of a decrease. This example is illustrated in Fig. 2.

Beyond this illustrative example, we now proceed to derive the so-called *elasticities* that measure the relative influence of M_f and S_f on $S_{\Delta T}$. The elasticity $\varepsilon_{Y|X}$ of $Y = Y(X)$ with respect to X is defined as the logarithmic derivative $\partial(\log Y)/\partial(\log X) = (X/Y)(\partial Y/\partial X)$, and we get:

$$\varepsilon_{S_{\Delta T}|M_f} = \frac{2\,M_f}{1 - M_f} \quad \text{and} \quad \varepsilon_{S_{\Delta T}|S_f} = 1. \tag{3}$$

Since the ratio of elasticities $\varepsilon_{S_{\Delta T}|M_f}/\varepsilon_{S_{\Delta T}|S_f}$ obtained from Eq. (3) is equal to 3.7 for $M_f = 0.65$, the spread $S_{\Delta T}$ is thus much more sensitive to a change in the mean M_f than it is to a change of similar magnitude in the spread S_f.

Finally, we need to examine the relative magnitude of simultaneous changes in M_f and S_f, as a result of improved knowledge of feedbacks f. For this purpose, we formulate a Bayesian model of pdf evolution that is described in Appendix A. Given certain hypotheses on the progress of climatic research that are explicitly stated there, one obtains two coupled relationships between M_t and S_t at times t and $t+1$, which also involve the mean μ_t and spread σ_t of the knowledge acquisition process:

$$M_{t+1} = \frac{\sigma_t^2 M_t + S_t^2 \mu_t}{\sigma_t^2 + S_t^2}, \qquad S_{t+1}^2 = \frac{\sigma_t^2\, S_t^2}{\sigma_t^2 + S_t^2}. \tag{4}$$

This model enables us to generate a set of plausible trajectories of M_f for a given trajectory of decrease in S_f, from which we then obtain a set of trajectories for $S_{\Delta T}$ by using Eq. (2). The main results of this probabilistic modeling of uncertainty evolution are illustrated in Figs. 3(a–c).

From the results in this figure, it follows that — given the present state of our knowledge on feedbacks — the envelope of plausible future trajectories of climate uncertainty is very broad with respect to its current width: it ranges from an enhanced decrease to a substantial increase in width, cf. Fig. 3(c). In probabilistic terms, while climate uncertainty is more likely to decrease as we learn more about the system, there is still a significant chance that it may paradoxically increase even though we improve our understanding of the climate system; see Figs. 3(e,f). For instance when feedback uncertainty is halved, there is still a relatively high probability, $p \simeq 20\%$, to obtain an increase in climate uncertainty. Thus, the effect illustrated in the preceding example, in connection with Eq. (3), appears to be both realistic and worrisome. The slightly paradoxical aspects of this effect are further explained in Appendix B.

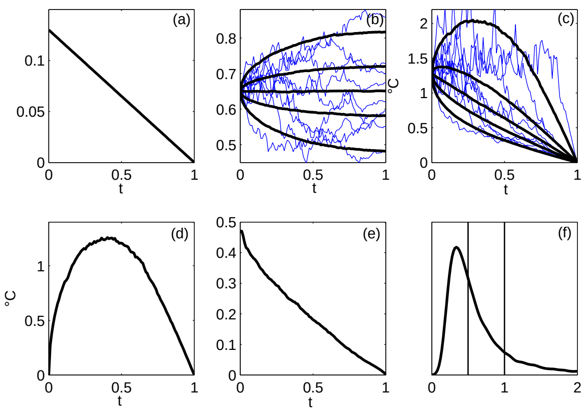

Figure 3. Sample scenarios of the evolution in time of the spread $S_f(t)$ and the mean $M_f(t)$ of the feedback factor f, and resulting scenarios of the evolution of the climate uncertainty $S_{\Delta T}(t)$. (a) Linear decrease in S_f; (b) stochastic trajectories of M_f (light blue lines) and 10%, 30%, 50%, 70% and 90% quantiles (heavy black lines); and (c) stochastic trajectories of $S_{\Delta T}$ (light blue lines) and 10%, 30%, 50%, 70% and 90% quantiles (heavy black lines). (d) Trajectory of the 90% range of future values of $S_{\Delta T}$; (e) trajectory of the probability that $S_{\Delta T}$ is larger than its initial value; and (f) pdf of the ratio of $S_{\Delta T}$ to its initial value for a halving of S_f (heavy line), and partitioning between an enhanced decrease ($S_{\Delta T} < 0.5$), a weaker decrease ($0.5 < S_{\Delta T} < 1$) and an increase ($1 < S_{\Delta T}$) of the sensitivity spread $S_{\Delta T}$.

4. Conclusions

Feedbacks f are recognized as important, yet uncertain factors in determining climate sensitivity $\gamma = \Delta T/\Delta p_{CO_2}$; we have considered here, instead of γ and according to the custom in the more applied climate change literature, that ΔT for CO_2 doubling is an indicator for γ. As a consequence of the inverse relationship in Eq. (1) between ΔT and the feedback parameter f, the uncertainty in climate sensitivity is amplified as f increases.

As time goes by, we hope and expect to improve our knowledge of feedbacks, thus reducing the uncertainties in f. And yet, if one wishes the feedback parameter f in the present, linearized-EBM setting to capture the behavior of fully nonlinear models with internal variability [Ghil et al., 2008; Zaliapin and Ghil, 2010], one also expects the best estimate of M_f to fluctuate within the spread S_f of the currently known range of f.

These fluctuations in M_f will affect in turn the value of the amplification factor $(1 - M_f)^{-2}$ and thus the spread $S_{\Delta T}$ of climate sensitivity. A simple probabilistic model is introduced in Appendix A here to represent the simultaneous changes in $M_f(t)$ and $S_f(t)$, and the resulting evolution in time of the sensitivity spread $S_{\Delta T}(t)$.

Using this model, we find that the future trajectory of $S_{\Delta T}(t)$, i.e. of climate uncertainty, is influenced partly by the future decrease of S_f — an outcome of our collective research effort — but also to a large extent by the unknown future evolution of M_f. In other words, even when assuming steady, deterministic progress in climate research, climate uncertainty may have an uncertain future ahead. The implications for socio-economic and political responses to such an uncertain evolution of $S_{\Delta T}(t)$ may be quite complex, depending on whether future societies find high uncertainty more worrisome than the current range or less so [Hillerbrand and Ghil, 2008].

In any case, it follows that surprises may occur in the future evolution of our assessment of climate sensitivity — even when only considering linear effects in a climatic-equilibrium, EBM-type setting. This being said, the surprises associated with nonlinear behavior may be even greater, as discussed by Zaliapin and Ghil [2010]. In fact, the currently accepted estimate of mean sensitivity, $M_f = 0.65$, is not that far from the point at which linearized-EBM approximations break down. It would thus be interesting to set up a stochastic model for estimates of the distance between the current climatic equilibrium — or, more precisely, the current mean of natural climatic variability — and the closest bifurcation [Ghil, 2001] or "tipping point" [Lenton et al., 2002].

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Appendix A:

A stochastic model of uncertainty dynamics.

We introduce here a stochastic-process model for the evolution in time t of the probability distribution $\mathbb{P}(f \mid t)$ of the climate feedback parameter f in Eqs. (1)–(3) of the main text. Given the mean $M_f(t)$ and spread $S_f(t)$ of f at a given instant $t > t_0$ — where t_0 is the present time, at which we (think we) know M_f and S_f — one can obtain the climate sensitivity spread $S_{\Delta T}(t)$ and elasticity ratio $\varepsilon_{S_{\Delta T}|M_f}/\varepsilon_{S_{\Delta T}|S_f}$ at that time t from Eqs. (2) and (3), respectively.

For the sake of convenience and simplicity, we let the pdf $\mathbb{P}(f \mid t) \equiv \mathbb{P}(f \mid M_t, S_t)$ that summarizes the information available on f at instant t be Gaussian; here M_t and S_t are the mean and standard deviation of this distribution, and the index f is henceforth omitted. In our model, the gain of information on f between time t and time $t+1$ is represented by a change in the distribution $p(f \mid M_t, S_t)$ that is obtained by updating it with a set of new, independent informations acquired at time t. This new set of informations is assumed to have a Gaussian distribution $p(f \mid \mu_t, \sigma_t)$ with mean μ_t and standard deviation σ_t. We note merely that the information content of a Gaussian distribution, measured by its Shannon entropy, is equal to minus the logarithm of its standard deviation.

With these assumptions, we apply the update equation obtained from Bayes' theorem:

$$p(f \mid M_{t+1}, S_{t+1}) \propto p(f \mid M_t, S_t) \cdot p(f \mid \mu_t, \sigma_t) . \tag{5}$$

From Eq. (5), it is straightforward to obtain the coupled evolution of M_t and S_t. For clarity and completeness, we thus restate here the equation already mentioned in Section 3 of the main text,

$$M_{t+1} = \frac{\sigma_t^2 M_t + S_t^2 \mu_t}{\sigma_t^2 + S_t^2}, \qquad S_{t+1}^2 = \frac{\sigma_t^2\, S_t^2}{\sigma_t^2 + S_t^2}. \tag{6}$$

In order to simulate M_t and S_t from Eq. (6), we need to further specify the evolution of μ_t and σ_t, which jointly describe the knowledge acquisition process. Regarding μ_t, we introduce its prior distribution, denoted by $\pi(\mu_t \mid \bar{\mu}_t, \sigma_{\mu,t})$, which again is assumed to be Gaussian. This distribution represents the a priori range, seen at instant t, of the possible outcomes of the new informations to be acquired between t and $t+1$.

We assume next that the knowledge acquisition process is self-consistent or, in other words, that what we know at any time is true. Mathematically, this simply means that by integrating the distribution of f at the instant $t+1$, as seen at the instant t, with respect to the set of all updates that may plausibly occur between t and $t+1$, we should obtain the same distribution back at instant t:

$$p(f \mid M_t, S_t) = \int p(f \mid M_{t+1}, S_{t+1}) \cdot \pi(\mu_t \mid \bar{\mu}_t, \sigma_{\mu,t})\, d\mu_t. \tag{7}$$

After a bit of algebra, this yields

$$\sigma_{\mu,t}^2 = S_t^2 + \sigma_t^2\, , \qquad \bar{\mu}_t = M_t, \tag{8}$$

where the overbar $\overline{(\cdot)}$ denotes the mean. Hence μ_t is specified by $\mu_t = M_t + (S_t^2 + \sigma_t^2)^{1/2}\eta_t$, with $\eta_t = \mathcal{N}(0,1)$ a standard Gaussian random variable.

To complete our model specification, we need to also describe how σ_t — which characterizes the intensity of the knowledge acquisition process — evolves between the instants t and $t+1$. From Eq. (6), it is clear that the evolution of S_t is entirely determined by that of σ_t and vice-versa,. We choose therefore to control S_t rather than σ_t because it seems more relevant to define future scenarios of uncertainty reduction directly in terms of their outcome rather than in terms of a parameter that indirectly drives them.

By combining Eqs. (6) through (8) and eliminating σ_t and μ_t, our stochastic model boils down to a single equation that yields the evolution in time of M_t, based on a prescribed scenario of uncertainty reduction chosen for $\{S_k : k = 0, 1, 2, \ldots, t\}$:

$$M_t = M_0 + \sum_{k=0}^{t-1} (S_k^2 - S_{k+1}^2)^{1/2} \eta_k\, , \tag{9}$$

where η_k are independent, identically distributed Gaussian variables, $\eta_k = \mathcal{N}(0,1)$.

Finally, we need to define a scenario $S = S_t$ of future uncertainty. For this purpose, we use here a simple power law:

$$S_t = S_0 \left\{1 - \frac{t}{\tau}\right\}^{\alpha}, \qquad t \le \tau, \qquad \alpha > 0, \tag{10}$$

where τ is the time at which S_t is assumed to vanish, and α gives the shape of the decrease. In such a scenario, the amount of new information obtained at instant t, measured by $-\log \sigma_t$, is proportional to the amount of existing information $-\log S_t$.

It follows that M_t can be rewritten as:

$$M_t = M_0 + S_0 \sum_{k=0}^{t-1} \left(\frac{2\alpha}{\tau}\right)^{\frac{1}{2}} \left\{1 - \frac{t}{\tau}\right\}^{\alpha - \frac{1}{2}} \eta_k\, . \tag{11}$$

The actual value of τ is irrelevant here because the time unit is arbitrary. We simply assume that the actual feedback value f will eventually be known at some future, unknown date $t = \tau$, no matter how long it will take.

In practice, we arbitrarily chose $\tau = 100$ and we normalized the time so that $t' = t/\tau$ lie in the unit interval $[0, 1]$. For the shape of the decrease, the basic scenario assumes a linear decrease, $\alpha = 1$. We also considered two alternative scenarios, in which $\alpha = 2$ and 0.5, representing a faster decrease at the beginning or at the end of the process, respectively. Note that, in the latter case, M_t is just a pure random walk.

As a final remark, note that the stochastic process given by Eqs. (10) and (11) converges to a distribution with spread $S_\tau = 0$, i.e. a Dirac mass positioned at M_τ that represents complete knowledge of $f = M_\tau$. Because the process is assumed to be self-consistent, the distribution of the value M_τ, seen from $t = 0$, should match the initial distribution, i.e. M_τ should be distributed with mean M_0 and spread S_0. This is easy to verify analytically, as we immediately obtain from Eq. (9) that $\mathbb{E}(M_\tau \mid t = 0) = M_0$, while its variance $\mathbb{V}(M_\tau \mid t = 0) = \sum_{k=0}^{\tau-1} S_k^2 - S_{k+1}^2 = S_0^2$.

Appendix B:

An apparent paradox and its explanation.

The central result of our model may at first glance appear to be contrary to basic common sense. Indeed, how could we possibly obtain more information on the climate system, and yet at the same time increase the uncertainty on a key climate variable such as ΔT ? One way to resolve this paradox is to consider the question from the point of view of information theory.

This convenient framework provides useful metrics to quantify the amount of information gained at each successive step t of the learning process in our model. Let X be any variable of interest of this model and $\mathbb{P}(X \mid t)$ the information on X available at time t; here X will be taken to be either f or ΔT. The information gain on X between t and $t+1$ can be measured by using the Kullback-Leibler divergence (KLD):

$$D_{KL}\,(X \mid t) = \int_x \mathbb{P}(x \mid t+1) \log \frac{\mathbb{P}(x \mid t+1)}{\mathbb{P}(x \mid t)}\, \mathrm{d}x\,. \qquad (12)$$

The KLD is not a true distance metric in the mathematical sense, in particular it is not symmetric with respect to the two pdfs whose distance it measures. Still, it has has two key properties that are of great interest for the present argument. First, it is nonnegative for any two distributions $\mathbb{P}(X \mid t)$ and $\mathbb{P}(X \mid t+1)$. In particular, the information gain is positive even if the spread at $t+1$ is greater than the spread at t, i.e. an increase in spread is not incompatible with an acquisition of new information. Second, the KLD is invariant under a smooth transformation of variables, i.e. $D_{KL}\,(Y \mid t) = D_{KL}\,(X \mid t)$ for $Y = \phi(X)$, where ϕ is a diffeomorphism. Thus the information gain on f and ΔT in our model is equal at any step t of the learning process, even though the evolution of their respective uncertainties may differ.

It follows from the above considerations that the assumed relationship between the information gain and the uncertainty reduction, respectively, for variables f and ΔT is of the essence in explaining the apparent paradox. Indeed, we assumed here that scientific research drives a direct reduction of the uncertainty S_f on feedbacks, which in turn drives — albeit indirectly — the evolution of uncertainty $S_{\Delta T}$ in climate sensitivity. Had we made a symmetric assumption on f and ΔT, the result would be symmetric, too. We do believe, however, that such a symmetric assumption is not realistic. Indeed, in agreement with previous studies, most of the learning concerns climate processes that relate directly to feedbacks, and only indirectly to climate sensitivity.

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