Empirical Modeling of Earth’s Outer Radiation Belt

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Abstract

We present application of a novel empirical mode reduction (EMR) methodology for constructing a hierarchy of empirical, stochastically forced models for the analysis and simulation of spatio-temporal variability in radiation belts. It is based on multiple polynomial regression to estimate deterministic propagator from the data and multi-level modeling of the stochastic forcing. This methodology encompasses both linear and nonlinear time-dependent models that aim to best describe the data set's statistics, and has been successfully applied before for ocean and atmosphere data sets. Here we apply this methodology to study acceleration of relativistic electrons during magnetic storms in the Earth's outer radiation belt. As a starting point, we consider a data set of 1MeV electron fluxes from Earth's outer radiation belt. As a starting point, we apply this methodology to study acceleration of relativistic electrons during magnetic storms in the Earth's outer radiation belt. As a starting point, we apply this methodology to study acceleration of relativistic electrons during magnetic storms in the Earth's outer radiation belt. As a starting point, we apply this methodology to study acceleration of relativistic electrons during magnetic storms in the Earth's outer radiation belt.

Empirical Mode Reduction(EMR)

Motivation:
- Sometimes we have spatio-temporal geophysical dataset but not a good model.
- We want models that are as simple as possible, but not any simpler.

Criteria for a good data-derived model:
- Capture statistics (histograms, correlations, spectra) and relevant dynamics: regimes, oscillations, etc.
- Deterministic dynamics easy to analyze analytically.
- Good noise estimates.
- Describes independent data.

Key Ideas:
Given spatio-temporal dataset \(D(t,s)\) to construct the reduced model in terms of predictant variables \(x_1(t), x_2(t), \ldots, x_K(t)\), by diagonalizing MMM covariance matrix \(C\) of the field \(D\):

\[
C = \frac{1}{N} \sum_{s=1}^{S} (D(s) - <D>(s)) (D(s) - <D>(s))^T
\]

Consider the following system of stochastic ODEs:

\[
\begin{align*}
\frac{dx_i}{dt} &= (x_i^T A x_i + b_i^0 + c_i^0) dt + r_i^0 dt, \\
\frac{dr_i^0}{dt} &= b_i^1 [x_i, r_i^0] dt + r_i^{(1)} dt, \\
\frac{dr_i^{(1)}}{dt} &= b_i^2 [x_i, r_i^{(0)}, r_i^{(1)}] dt + r_i^{(2)} dt, \\
\cdots &
\end{align*}
\]

- Multiple predictors: PCs - one-step time differences of predictors; step = sampling interval dt.
- matrices \(A\), vectors \(b_i\), and scalars \(c_i\) are fitted for each \(i\) independently by least squares.
- Multi-level modeling of noise \(r_i\) to account serial correlations in the regression residuals.
- Number \(K\) of PCs is chosen to optimize the EMR model performance (comparison with data).

Synthetic Data:
Radial diffusion equation for phase space density:

\[
\frac{\partial f}{\partial t} = \frac{L_2^2}{L_1^2} \left( L_2^2 D L_1^2 f \right) - \frac{\partial}{\partial r_i} \tau_i \frac{\partial f}{\partial r_i}
\]

\(\tau_i\) is the electron lifetime, \(D\) is the radial diffusion coefficient with nonlinear dependence on \(K_p\). Different parameterizations are used outside/inside plasmapause \(L_p\).

Main Idea: test EMR on the model dataset for which we know the origin ("true") and learn something new about PDE or/and dynamics.

Basic steps:
- Obtain long time integration of the PDE model (1) forced by historic \(K_p\) to obtain dataset for analysis.
- Calculate PCs of log(fluxes) and fit EMR.
- Obtain simulated data from the integration of reduced model and compare with the original dataset.

EMR for externally forced system:
- Modify linear and constant terms on the mean ("0") level to account for explicit forcing:

\[
\begin{align*}
b_i^{(0)} &= b_i^{(0)} + b_i^{(K)} K_p(t), \\
c_i^{(0)} &= c_i^{(0)} + c_i^{(K)} K_p(t)
\end{align*}
\]

- Consider anomalies with respect to the "quite" state: in this study steady solution of (1) for \(K_p=3\).

Conclusions

- EMR is a promising tool for constructing hierarchy of empirical models of radiation belt variability with potential for predictability, data assimilation, etc.

Future Work:
- Cross-validation for testing the model.
- Statistical analysis.
- Dynamical analysis: stability, optimal perturbations.
- Consider observational dataset with alternative external drivers (solar wind parameters).

References


Q: Why for similar level of geomagnetic activity there is a different response in radiation belt variability?

Random realization from continuous integration of EMR model forced by \(K_p\).

- EMR model is constant over the whole time interval.
- Because of stochastic component, the particular EMR integration will not reproduce the original dataset in details.
- Preliminary analysis of the deterministic part of EMR model shows unstable eigenmodes which may explain the origin of storms in terms of optimal perturbations.


- Cross-validation for testing the model.
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- Dynamical analysis: stability, optimal perturbations.
- Consider observational dataset with alternative external drivers (solar wind parameters).

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