THE HYPOTHESIS OF IRREDUCIBLE IMPRECISION
IN ATMOSPHERIC AND OCEANIC SIMULATIONS (AOS)

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1. AOS successes and errors

2. Sources of AOS delicacy, irreproducibility, and imprecision

3. Coping with AOS imprecision

4. Surveying the solution space of an AGCM
1. WHY DO WE LIKE CLIMATE SIMULATION MODELS SO MUCH?

For the realism in emergent structures from first-principle fluid dynamics — mean winds and oceanic currents; overturning circulation; temperature, water, and chemical distributions; storms and eddies; seasonal cycle; low-frequency variability patterns; and long-term climate hindcasts & forecasts —

in spite of the fact that essential material processes, micro-scale transports, & thermodynamic/radiative energy flows are not fundamentally represented and can be shown to be unrealistic in detail even when tuned to be correct in bulk ways.

*Faute de mieux:* What other scientific approach can provide as full and falsifiable a characterization of real and imaginary worlds?

*[cf., theory, measurement, regression, & low-order (bulk) dynamical systems]*
Iso-surfaces of vorticity magnitude at several magnifications in 3D homogeneous turbulence with large $Re$. $L$, $\lambda$, $\eta$ = forcing, Taylor, dissipation scales. (Kaneda & Ishihara, 2005)
Vorticity in a subdomain for a barotropic fluid in turbulent equilibrium with large $Re$.
(Bracco & McWilliams, 2009)
Instantaneous SST [$^\circ$C] in a regional oceanic simulation with fine resolution ($dx = 0.75$ km): coastal upwelling, mesoscale filaments & eddies, and unstable submesoscale fronts.

(Capet et al., 2008)
Measured radar reflectivity in a mid-American summertime squall line on 10 June 2003 (left) and in a 36-hour forecast from a cloud-resolving, regional atmospheric model (right). (Klemp, 2005)
Mean surface wind, zonally averaged over the oceans: observed (ERA40) and simulated with different coupled climate models.

(IPCC, 2007)
Centennial Climate Hindcasts

Global mean surface $T$: observed (black) and simulated with 58 realizations from 14 different climate models and their ensemble mean (red). Gray lines are volcanic eruptions. (IPCC, 2007)
2. AOS MODEL DELICACY AND IMPRECISION

What kind of mathematical object is an AOS model and its solution space?

What is the nature of the meta-model space?
Sources of Simulation Discrepancy with Nature

Why are there discrepancies between observations and simulations and between different simulations?

- Intrinsic variability of simulations and nature
  [dynamical sensitive dependence $\Rightarrow$ predictability limits & sampling errors]

- Model bias and/or phenomenological deficiency
  [model design $\Rightarrow$ e.g., multi-scale computations]

- Irreducible imprecision of simulations
  [dynamical structural instability $\Rightarrow$ model families and solution ensembles]
Propositions

1. "[Almost] all chaotic dynamical systems are structurally unstable."
   (Leonard Smith, in conversation)

   “Structurally stable systems are not dense”
   (Stephen Smale, 1966)

LOOSE DEFINITION: A small or seemingly innocuous change in a model, either for a parameter value or an equation form, can lead to appreciable change in the long-time solution behavior (i.e., the attractor), either topological or metrical.

Structural instability is untestable in general because there is no meaningful limit for the types of model changes that could be made, only particular specified types.
Generic behaviors for chaotic dynamical systems with dependent variables $\xi(t)$ and $\eta(t)$: (Left) *sensitive dependence*, small changes in initial or boundary conditions imply limited predictability with (Lyapunov) exponential growth in phase differences, and (Right) *structural instability*, small changes in model formulation alter the long-time probability distribution function, PDF (*i.e.*, the attractor).

**SENSITIVE DEPENDENCE**

[ $\varepsilon$ in i.c. or b.c.]

**STRUCTURAL INSTABILITY**

[ $\varepsilon$ in model parameter or equation set]
Low-order Chaotic Dynamical Systems

Lorenz (JAS, 1963) is a 3rd-order ODE system derived by Galerkin truncation of mid-latitude atmospheric and/or convective dynamical equations.

As a function of its forcing amplitude $F$, it shows successive bifurcations from steady to transient and chaotic states.

For a certain range in $F$, it is famous as a prototype for sensitive dependence and limited predictability.

It also is structurally unstable for a range in $F$ (Williams, 1979); e.g., very small $\delta F$ causes shifts between chaotic strange attractors and periodic limit cycles that are densely intermixed.

It is structurally unstable to truncation level (Curry, 1978) — as are many LES (Stevens et al., 2001 & 2005).

This type of structural instability is parametric and topological.
Low-order Random Dynamical Systems

Phase portraits of the stochastic Lorenz system with smaller (left) and larger (right) multiplicative noise magnitudes. Note persistence of fine-structure on the attractor, not just a “fuzzing” of deterministic chaos. (Simonnet et al., 2009)
Fluid Dynamics at High Reynolds Number, $Re$

The Navier-Stokes equations and a well-resolved dissipation range $\rightarrow$ sensitive dependence but **structural stability** with respect to $Re$.

But with alternative choices for "monotone" advection schemes that preserve shape and effect minimal dissipation at a given grid resolution $\rightarrow$ sensitive dependence & **structural instability**.

Examples of late-time vorticity fields in 2D turbulence using utopia and two varieties of flux-corrected transport (Shchepetkin & McWilliams, 1998).

Numerical algorithms, subgrid-scale parameterizations, and opting for "exciting" (non-smooth) solutions all can induce structural instability of an equation-set and metrical type.
Propositions (cont):

2. Structural instability is untestable in general because there is no meaningful limit for the types of model changes that could be made, only for particular specified types.

3. AOS models — turbulent fluid dynamics plus ... — are chaotic, hence almost certainly structurally unstable.

4. Nevertheless, AOS solutions are remarkably like nature in many ways, both qualitatively and semi-quantitatively.

⇒ AOS models can teach us about nature and make predictions that can be partially right but will remain partially uncertain.
Precipitation Change Under Global Warming

Predicted average DJF precipitation change [mm day$^{-1}$] between 1961-1990 and 2070-2090 from different climate models with similar mid-range emission scenarios (from Neelin et al., 2003).

Note agreement in the broadest aspects but substantial disagreement in specific patterns and magnitudes.
Sources of AOS Irreducible Imprecision

- AOS solution fields are non-smooth near the space-time discretization scales (i.e., the “resolution” of the model) imposed on the known governing principles expressed mostly as partial differential equations.

- AOS models contain essential parameterizations for unresolved or highly simplified processes whose formulations are not at a fundamental level of known governing principles, hence non-unique.

- AOS models are open-ended in their scope for including and dynamically coupling different physical, chemical, biological, and even societal processes.

Inter-model irreproducibility is a common experience for many solution aspects (e.g., ∼ 25 years of non-shrinking global warming forecast spreads).

Structural instability — manifested as delicacy of late-time solution measures — is a plausible concept for understanding this behavior.

Model delicacies & irreducible imprecision levels are not yet well quantified.
3. STRATEGIES FOR COPING WITH AOS IMPRECISION

Use models to study processes and phenomena apart from precise comparisons with nature.

Deliberately design model ensembles (not merely inadvertent and opportunistic, as in IPCC, AMIP, CMIP, etc.).

Reframe comparisons with nature and climate forecasts in terms of model-ensemble distributions, and document “tuning” to available observational constraints.

Expand the characterization of AOS model attractors.

Search for more robust discretization, parameterizations, and couplings (e.g., differentiable parameterizations?, stochastic PDEs?), as part of continuing model improvement.

(McWilliams, 2007)
Frequency distributions for climate sensitivity (i.e., mean $\Delta T$ for doubled atmospheric CO$_2$) in an atmosphere + ocean mixed-layer GCM with different parameter choices: (black) all model runs; (red) excluding perturbations to rain threshold; (blue) excluding perturbations to convective entrainment coefficient. (Stainforth et al., 2005, based on climateprediction.net)
4. SURVEYING THE MODEL SPACE OF AN AGCM
(with Annalisa Bracco and David Neelin)

Goal 1: describe the model solution space w.r.t. parameter variations pro tem (algorithms and parameterizations later).

Goal 2: assess potential techniques and outcomes of model tuning (tuning later).

We analyze a preliminary ensemble based on

- the ICTP AGCM (Molteni, 2003; Bracco et al., 2004, etc.) — formerly known as SPEEDY — a computationally-efficient, coarse-resolution, IPCC-class model.
- 10 realizations of a 25-year integration with specified SST (1979-2003) vs. NCEP reanalysis
- 9 parameter values for each of 4 “sensitive” parameters across “reasonable ranges” centered on “tuned” values:
  - cloud albedo $a$: $0.40 \pm 0.12$
  - relative humidity threshold for convection $r$: $0.7 \pm 0.2$
  - damping time for horizontal diffusion $t_d$: $7 \pm 5$ hr
  - subgrid wind gustiness for surface heat flux $V_g$: $5 \pm 2$ m $s^{-1}$
- 9 monthly-mean output fields: $(u, v, T)_{10}$, $P$, $(\Phi, \Omega)_{5}$, $(u, v, T)_{2}$. 
SPEEDY has a typical AGCM skill cf., CCM3 and CMAP: e.g., \( \Phi^5 \) and \( P \) in DJF. (F. Kucharski)
Parameter Dependences (1)

Correlation coefficient $C$ and normalized RMS error $E$ for mean JJA precipitation $P$ between the model ensemble with different $V_g$ and NCEP reanalysis.

$x =$ individual realization;
$\square =$ ensemble mean;
filled $\circ =$ maximum correlation.
Parameter Dependences (2)

Correlation coefficient $C$ for mean JJA precipitation $P$ between the model ensemble with different parameters and NCEP reanalysis.

$x = \text{individual realization};$

$\square = \text{ensemble mean};$

filled $\circ = \text{maximum correlation}.$
Parameter Dependences (3)

Correlation coefficient $C$ for various mean JJA fields with different $a$ and $t_d$ between the model ensemble and NCEP reanalysis.

$x$ = individual realization;
□ = ensemble mean;
filled o = maximum correlation.
Parameter Dependences (4)

$(a, t_d, V_g)$ space locations of ensemble mean maximum correlations with NCEP.
Monthly EOF #1 for DJF $\Phi_5$ in the North Pacific sector.
Monthly EOF #1 for DJF $P$ in the North Pacific sector.
Realization- and parameter-composite Taylor diagrams for monthly EOF1 vis a vis NCEP: DJF Φ5 (left) and P (right) in the North Pacific sector.
Realization and Parameter Dependences (6)

Ensemble-mean, parameter-composite Taylor diagrams for monthly EOF1 vis a vis NCEP: DJF Φ5 (left) and P (right) in the North Pacific sector.
Preliminary Conclusions for the AGCM Survey

For SPEEDY with the parameter variations considered and the metrics examined so far,

- Intrinsic variability is large.
- Error levels are not small for many quantities.
- Parameter sensitivities are not large.
- Optimization tensions are high between different metrics.
- The "fitness landscape" appears not too rough (obscured by intrinsic variability); hence, structural instability is not evident.

⇒ model formulations need improvement and tuning is problematic.

...but there certainly is a need to look more broadly at different parameters, metrics, and model formulations.
Equilibrium precipitation as a function of $\sin[\text{latitude}]$ in an aqua planet simulation.

Two variants of the Tiedke convective cloud parameterization are shown. The topology differs between a single, equatorial ITCZ and a double, off-equatorial ITCZ.

(From B. Stevens)
SUMMARY

- Robust patterns emerge out of fluid dynamics in many AO regimes.

- Climate models are close to nature in many measures but appear to have an irreducible imprecision.

- Model families and ensembles can be used to expose the structure and structural instabilities of the attractor.

- Global climate change is surely happening; models are our best tool for envisioning futures; probably our present vision of the climate attractor is falsely narrow.

- Climate change still has a wide scientific frontier.