Spatio-temporal variability in a mid-latitude ocean basin subject to periodic wind forcing

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Abstract

The mid-latitude ocean's response to time-dependent zonal wind-stress forcing is studied using a reduced-gravity, 1.5-layer, shallow-water model in a rectangular ocean basin. First, the bifurcation diagram is examined for the time-independent forcing case. Subannual and interannual modes of variability are obtained as the wind stress intensity is modified, confirming previous results with this model. The associated spatio-temporal variability is studied in greater detail than before, by using empirical orthogonal function analysis and the scatter plots of potential vorticity vs. streamfunction. For no-slip boundary conditions, variability is strongest in the subpolar gyre even when the subtropical gyre is dominant. Variability under free-slip conditions is found to be strongest in the recirculation region. The effects of changes in the Rossby radius of deformation on the spatio-temporal variability are also studied.

Two aspects of time-dependent forcing that mimics the seasonal cycle are investigated next: (1) a fixed-profile wind-stress forcing with periodically oscillating intensity; and (2) a wind-stress profile with fixed intensity, but north–south migration of the mid-latitude westerly wind maximum. A bifurcation diagram with respect to increasing wind-stress forcing is constructed for the standing-oscillation wind-stress pattern. The intrinsic variability found in the time-independent case is preserved and does not change by including the periodicity in the intensity of the wind forcing. It thus appears that the physics behind the variability is strongly controlled by internal ocean dynamics, although the spatial patterns in the ocean basin are now more complex, due to the interaction between the external and internal modes of variability. The effect of north–south migration of wind forcing on the ocean variability concludes the
investigation. The scatter plots of potential vorticity vs. streamfunction show that the inertial recirculation is substantially suppressed; this suppression increases with the amplitude of north–south migration in the wind-stress forcing.
1 Introduction

The wind-driven, basin-scale circulation in the mid-latitude upper ocean exhibits variability on different time scales. This variability could be due to changes in the external atmospheric forcing or to the system's intrinsic instability and nonlinearity. The latter explanation was first put forward by Veronis (1963, 1966), but the former has enjoyed greater popularity in the oceanographic community until fairly recently. A systematic application of the methods of dynamical systems theory to the wind-driven circulation problem has yielded several physical mechanisms for the observed low-frequency variability, on the time scale of several months to several years.

This circulation has been studied in the last decade with a hierarchy of models that include rectangular-basin, as well as more realistic configurations. Pedlosky (1996) reviewed work that used time-independent, single-gyre forcing. Single-gyre results of particular interest since then include Meacham and Berloff's (1997a, 1997b) work that emphasizes the role of basin size in this problem and Sheremet et al.'s (1997) classification of instabilities for the single gyre. Chang et al. (2001) reviewed double-gyre results and compared the physical mechanisms for intrinsic variability found in this case with those obtained in the single-gyre problem. We only review here, therefore, the results that are most relevant to the present work. These emphasize the double-gyre problem.

Jiang et al. (1995; JJG hereafter) studied the double-gyre wind-driven circulation in a mid-latitude rectangular basin on a $\beta$-plane using a 1.5-layer, reduced-gravity, shallow-water (SW) model. They used a time-constant zonal-wind profile, symmetric about the basin's mid-latitude axis. Their results showed that when nonlinear processes come into play, multiple steady solutions satisfying identical boundary conditions can arise for sufficiently strong
wind-stress forcing. Two stable solutions were found to co-exist over a certain range of parameters. One had a stronger subpolar gyre and the other a stronger subtropical gyre, with overshooting of the western boundary currents to the north and south of the symmetry axis, respectively. Their periodic solutions had periods of several years and several weeks.

Following JJG, Speich et al. (1995) analyzed the dependence of the stationary solutions on the SW model's nondimensional parameters such as the amplitude of the forcing, the Ekman number, the Rossby number, the nondimensional $\beta$ parameter, and the bottom drag coefficient. Using pseudo-arclength continuation, they showed systematically how model solutions vary in number, stability and spatial features with the parameters and confronted their time-dependent solutions with observational data.

Using a 1.5-layer quasi-geostrophic (QG) model, Cessi and Ierley (1995) also found multiple equilibria for the double-gyre circulation. They found antisymmetric, as well as asymmetric solutions. Some of their solutions fill the whole basin, while others decay rapidly to zero away from the western shore of the ocean basin. Many features of the bifurcation sequence in the double-gyre problem have since been shown to be independent of model details: QG vs. SW dynamics, domain shape and size etc. (Dijkstra et al., 1999; Dijkstra, 2000; Dijkstra and Ghil, 2004).

Ghil et al. (2002a) used a QG two-mode model and compared its results with those obtained by Dijkstra and Katsman (1997) in a QG model with 1.5 and 2 layers. Ghil and colleagues investigated in more detail the dipole region in the double-gyre problem. They obtained three-dimensional analytical solutions for the flow in the dipole region and found them to agree in their main properties with symmetric and asymmetric solutions obtained numerically in the QG two-mode model, as well as with the corresponding solutions of an SW model.
Simonnet et al. (2003a, 2003b) studied low-frequency variability of the large-scale mid-latitude ocean circulation with a 2.5-layer wind-driven, SW model, using both rectangular and more realistic bathymetry. The bifurcation structure was numerically investigated in rectangular geometry using pseudo-arclength continuation along the bifurcated branches. Simonnet and colleagues found the system to exhibit homoclinic orbits that induce transitions from upper- to lower-branch solutions at high levels of forcing. Simonnet et al. (1998) also performed a set of forward integrations in a rectangular domain that resembles in size the North Atlantic basin between 20° N and 60° N, as well as in a domain that roughly reproduces the shape of the eastern and western coasts of the North Atlantic ocean between these two latitude circles. These simulations were conducted at horizontal resolutions as high as 10 km and yielded two modes of variability, around 6 years and 20 months (see also Simonnet et al., 2003b).

Rhines and Schopp (1991) studied the effect of tilted wind-stress forcing. When increasing the northward tilt of the winds away from the east–west direction they found decreasing total energy of the circulation, great decrease in the penetration length of the eastward-flowing oceanic jet, increased concentration of the circulation in the upper ocean, increased production of cut-off rings near the western boundary, and displacement of the boundary current separation point poleward of the line of vanishing Ekman pumping.

All these studies considered wind-stress forcing that is constant in time. In reality, winds vary in time on seasonal, as well as on subannual and interannual time scales. These variations include north–south migration of the mid-latitude jet axis, as well as changes in the jet's intensity. Liu (1996) investigated the effect of seasonal wind migration on the inertial recirculation using QG models. He showed that the seasonal wind migration can suppress the
inertial recirculation substantially for the barotropic case. According to his study the two key dynamic conditions for the suppression are: (1) the mismatch between the formation time scale of the western boundary current and the recirculation; and (2) the interaction between the two neighbouring recirculation cells, which is related to the intercell transport. Yang (1996), using a simple barotropic, double-gyre model driven by surface wind, also showed the exchange between the subtropical and subpolar gyre to be substantially enhanced when the wind forcing is allowed to migrate north and south.

The aim of the present paper is to study the differences in the mid-latitude ocean response to time-dependent vs. time-independent wind-stress forcing. More specifically, we wish to investigate the different modes of variability and the associated spatial patterns for time-dependent vs. time-independent wind forcing. The model and methodology are described in Section 2. In Section 3, time-independent wind forcing is considered and the temporal and spatial features of the flow are presented in greater detail and physical depth than in previous work. In Section 4, we turn to wind-stress forcing that is allowed to vary periodically in time, first in intensity and then in spatial pattern. The methods applied to the analysis of the flow in the time-independent-forcing case are put to good use in this section too, in order to compare the results in the two cases. The main aspects of this comparison are summarized in Section 5.

2 Model and diagnostic methods

a Shallow-water (SW) model and wind-stress forcing

We use in this study JJG's SW model. It is a 1.5-layer model with an active upper layer of depth $H$ and an inert bottom layer of infinite depth; details of the model can be found in JJG.
The model domain is a mid-latitude rectangular basin on the $\beta$-plane, given by $0 \leq x \leq L$ and $0 \leq y \leq D$.

The dynamics is governed by the reduced-gravity, SW equations in flux form.

The model equations are:

\[
\frac{\partial U}{\partial t} + \nabla.(Uv) = -g'H \frac{\partial H}{\partial x} + fV + \alpha_\lambda A \nabla^2 U - RU + \alpha_x \frac{\tau_x}{\rho},
\]

\[
\frac{\partial V}{\partial t} + \nabla.(Vv) = -g'H \frac{\partial H}{\partial y} - fU + \alpha_\lambda A \nabla^2 V - RV,
\]

\[
\frac{\partial H}{\partial t} = -\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y}.
\]

here

\[Ui + Vj = Hv = H(\partial_i + \partial_j)\]

is the upper-layer mass-flux vector, while $u$ and $v$ represent, respectively, the zonal and meridional velocities in the upper layer. The Coriolis parameter is $f = f_0 + \beta y$, $g' = g(\Delta \rho / \rho)$ is the reduced gravity, and $\tau_x$ is the zonal wind-stress that drives the upper ocean. The system is subject to wind-stress forcing $\tau_x$, to Rayleigh bottom friction with coefficient $R$, and to Laplacian lateral diffusion with eddy viscosity $A$. The system's behavior is mainly determined by $\alpha_x$ and $\alpha_\lambda$, the two nondimensional parameters that control the strength of external wind forcing and lateral viscosity, respectively. The spatio-temporal complexity of the system's behavior increases with increasing $\alpha_x$ and decreasing $\alpha_\lambda$.

The model parameter values used in this study are given in Table 1a. The grid size we use is $\Delta x = \Delta y = 12.5$ km and the time step is $\Delta t = 10$ min. This horizontal resolution is higher than in JJJG and Speich et al. (1995), who used $\Delta x = \Delta y = 20$ km, and comparable to that of recent eddy-resolving simulations with ocean general circulation models (Chao et al.,
1996). The viscous boundary width according to Munk theory is \( \delta_m = (\alpha \alpha A/\beta)^{1/3} \) (Pedlosky, 1987); for the values of \( A \) and \( \beta \) used in this study, \( \delta_m \) is approximately 27 km in the mid-latitudes at \( \alpha A = 1.3 \). Hence our grid size of 12.5 km allows marginal resolution of the Munk layer, as well as of typical mid-latitude ocean eddies. Besides the model parameters that are kept constant and given in Table 1a, we systematically vary the control parameters \( \alpha_\tau \) and \( \alpha_A \) over the range given in Table 1b.

Most of the work described here uses no-slip boundary conditions, as was the case in JG and Speich et al. (1995). These authors discussed the use of free-slip or partial-slip conditions and the latter were also implemented by Ghil et al. (2002a) in a QG, baroclinic model. Some of our numerical experiments use, therefore, also free-slip conditions.

The wind-stress forcing used in section 3 is of the form

\[
\tau_x = -\tau_0 \cos \left( \frac{2\pi y}{D} \right)
\]

and it has, therewith, zero net vorticity forcing. In section 4, we prescribe variations in the wind-stress forcing with time that are informed by the wind data made available on TOGA CD-ROMs. These data sets were obtained from the National Center for Atmospheric Research and are provided on a 2° x 2° grid from 0°E to 360°E and from 90°N to 90°S.

We analyzed six years of wind data, from 1985 to 1990, and found changes in wind-stress magnitude, as well as a north–south shift in the wind-stress pattern. Based on the analysis of these observational data, we chose two idealized wind-stress patterns that allow: (1) changes in the wind-stress magnitude alone, i.e., a standing oscillation in the wind-stress pattern; and (2) a north–south migrating wind-stress pattern.

For the standing oscillation the wind stress pattern is given by
\[ \tau_x = -\tau_0 \cos \frac{2\pi y}{D} \times \left[ 1 + a \cos \frac{2\pi}{T_{\text{amp}}} \right], \]  \hspace{1cm} (4)  

while for the north–south migration it is in the form

\[ \tau_x = -\tau_0 \cos \frac{2\pi y - b \sin \frac{2\pi}{T_{\text{mig}}}}{D}. \]  \hspace{1cm} (5)  

In the above equations \( T_{\text{amp}} \) is the period of the standing oscillation and \( T_{\text{mig}} \) is the period of the north–south migration. In the present paper, \( T_{\text{amp}} = T_{\text{mig}} = T = 12 \) months, but longer and possibly distinct periods could be investigated in the future, as suggested for instance by the results of Feliks et al. (2004). Furthermore, \( a \) is the peak-to-peak change in the magnitude of wind-stress forcing at the latitude of the zero wind-stress curl, while \( b \) is the range of migration of this latitude.

**b Control parameters and analysis methods**

As stated in the previous subsection, the main control parameters are \( \alpha_r \) and \( \alpha_A \). Following Ghil et al. (2002a), we also examine model sensitivity with respect to the Rossby radius of deformation \( R_d = (g' H)^{1/2} / f \), which equals about 80 km for the model parameter values given in Table 1a. Several additional values of \( R_d \), at which numerical experiments were carried out, are given in Table 1b. Finally, most of the study uses no-slip conditions at the lateral boundaries, but the effect of free-slip conditions is also examined in several cases.

The key methods of examining solution dependence of parameters are: (i) the bifurcation diagram, in which dependence on one parameter is tracked as all others are kept fixed; and (ii) the regime diagram, in which the loci of bifurcation points are plotted as curves in the plane spanned by two parameters, while all others are kept fixed. Individual solutions, at
fixed parameter values and for given boundary conditions, are studied in detail using several dynamical and statistical methods.

JGJ constructed their model's bifurcation diagram by plotting the confluence point, i.e. the merging point of the two separated western boundary currents, with increasing wind-stress parameter $\alpha$. They found that the approximate position of the confluence point shifted north or south from the position of the zero wind-stress curl line (which is also the mid-axis of the basin) as $\alpha$ increased and that this position could exhibit two distinct values or even oscillate for fixed forcing. The exact position of the confluence point, however, is hard to determine with precision.

Following Chang et al. (2001) and Simonnet et al. (2003a,b) we construct, therefore, a more accurate bifurcation diagram by plotting the normalized transport difference $TD$ with increasing wind-stress forcing. The normalized transport difference $TD$ is given by

$$TD = \frac{|\psi_{po}| - |\psi_{tr}|}{\max|\psi|},$$

where $\psi_{po}$ and $\psi_{tr}$ are the negative and positive extrema of the streamfunction in the subpolar and subtropical gyres, respectively. A perfectly antisymmetric flow pattern, with subtropical and subpolar gyres of equal strength, corresponds to $TD=0$. Positive $TD$ points to a stronger subpolar gyre and negative $TD$ to a stronger subtropical gyre. As shown by Chang et al. (2001), this choice of $TD$ yields a bifurcation diagram where the upper and lower branches correspond to those of JGJ. They are also referred to as the subpolar and subtropical branches, because of the relative strength of the two gyres on either branch (e.g., Simonnet et al., 2003a,b) and we shall use interchangeably the term upper, positive or subpolar and lower, negative or subtropical.
To obtain the $TD$-vs-$\alpha_t$ plot, the system of equations (1a)–(1c) is first integrated forward in time for a given $\alpha_t$ value from a state of rest until either a steady state or a statistically stationary regime is reached. The transport difference $TD$ is then calculated for that steady state, cf. Eq. (6), from the positive and negative values of the streamfunction that are maximal in absolute value; this steady state is then used as the initial state for the next integration at a slightly higher $\alpha_t$ value, all other parameters remaining unchanged. When the asymptotic solution is time dependent, the range of $TD$ values is measured, and an arbitrary solution state within this range is used as the initial state at higher $\alpha_t$. This 'poor man's continuation method' (see Chang et al., 2001; Quon and Ghil, 1992, 1995; Dijkstra and Ghil, 2004) is used here for getting points along each solution branch of a bifurcation diagram. The type of an individual solution—steady, periodic or chaotic—is determined by examining its $TD$ time series. In particular, we use the SSA-MTM Toolkit (Ghil et al., 2002b) to study the temporal characteristics of the time-dependent solutions. Each time series is prefiltered data-adaptively by projection onto the leading empirical orthogonal functions (EOFs) provided by its singular-spectrum analysis (SSA). The prefiltered time series is then analyzed by the maximum-entropy method (Penland et al., 1991) and the results are further checked by the multi-taper method (MTM: Thomson, 1982; Mann and Lees, 1996) or the Monte Carlo version of SSA (Allen and Smith, 1996).

To describe spatial circulation patterns, we use the streamfunction $\psi$, rather than the horizontal velocity vector $(u, v)$ of the model's active upper layer. The former is obtained from the latter from the Poisson equation

$$\nabla^2 \psi = u_y - v_x,$$  

(7)
subject to the appropriate boundary conditions that correspond to no slip or free slip. Sequences of snapshots of $\psi$ are used to describe the time-dependent model solutions.

Spatio-temporal variability of solutions and their dynamics is studied by several methods. Spatial EOF analysis of chaotic solutions follows Ghil (1987), Mo and Ghil (1987) and Berloff and McWilliams (1999). A more detailed analysis of the dynamics of different regions in the flow domain relies on the use of scatter plots of potential vorticity ($PV$) vs. streamfunction $\psi$; the $PV$ is given by

$$PV = \frac{\nabla^2 \psi + f_0 + \beta y}{H}.\tag{8}$$

The $PV$ vs. $\psi$ scatter plots (Griffa and Salmon, 1989; Ghil et al., 2002a) allow us to distinguish, in a quantitative and objective way, between the two basin-scale gyres, the western boundary currents, and the recirculation dipole.

### 3 Time-independent wind stress forcing

a Parameter dependence of solutions

We start by recomputing the bifurcation diagram of JJG (see Fig. 3 there) at the higher resolution used here. The bifurcation diagram for $TD$ vs. $\alpha_T$ is shown in the large central panel of Fig. 1 for fixed $\alpha_A = 1.3$, $R_d = 80$ km, and no-slip boundary conditions. For very low wind stress values, flow in the basin is exactly antisymmetric ($TD = 0$), with a cyclonic subpolar gyre to the north and an anticyclonic subtropical gyre to the south of the symmetry axis. This antisymmetry breaks at $\alpha_T = 0.47$, at the point marked by $SB$ in Fig. 1. Intense recirculation starts to appear, close to the confluence point and on either side of the mid-basin.
axis, for slightly higher $\alpha_\tau$ values. The meeting of the two western boundary currents generates a strong eastward-flowing jet along this axis.

For moderately low wind-stress values, there is still only one steady-state branch, the positive TD branch. Along this branch, the flow in the basin is asymmetric, as the recirculation vortex to the north of the mid-basin axis is stronger than the one to the south of the axis. The stronger subpolar vortex pulls the subtropical recirculation vortex further north. As the subtropical vortex wraps around the subpolar one's east end, it deflects the jet from a purely eastward to a north–east direction.

As the wind forcing $\alpha_\tau$ increases, a first bifurcation to multiple steady states occurs, i.e., a pair of stable branches, one positive, the other negative, coexist; this bifurcation occurs at $\alpha_\tau \approx 0.7$ and is marked by $P$ in the diagram. The negative branch is characterized by a stronger subtropical gyre and a jet flowing south-east ward. Comparison of the analytic solutions for a low-order QG model with the numerical results of their SW model led JJJG to conclude that it is a perturbed pitchfork bifurcation. This conclusion has been confirmed since by careful pseudo-arclength continuation methods across a hierarchy of models (Simonnet et al., 2003a,b; Dijkstra and Ghil, 2004). For the negative branch, equilibrium states are found to occur only for values of $\alpha_\tau > 0.7$. Multiple stable equilibria coexist in a range of $\alpha_\tau$ from 0.7 to 0.865.

Both the positive and negative TD branch lose their stability to periodic solutions through Hopf bifurcation. The bifurcation occurs at $\alpha_\tau = 0.865$ for the negative branch and at $\alpha_\tau = 1.0$ for the positive branch; the two bifurcation points are marked by $H^-$ and $H^+$, respectively. On increasing the wind stress further, the periodic solutions give way to
aperiodic solutions. On the negative branch this occurs at $\alpha_\tau = 1.15$ and on the positive branch at $\alpha_\tau = 1.24$.

A direct comparison with JJG's bifurcation diagram would not be fully correct since the physical quantity plotted on the ordinate in the two diagrams is different. Still, both diagrams represent essentially qualitative changes in solution behavior: from antisymmetric to asymmetric, from unique to multiple steady states, from steady to periodic, and from periodic to aperiodic. We thus expect the differences, if any, to be due mainly to the increased resolution of the present model version. The stable steady-state branches are consistent, here as there, with a perturbed pitchfork bifurcation. This bifurcation occurs somewhat later, i.e. at higher $\alpha_\tau$, than in Fig. 3 of JJG.

The negative steady-state branch undergoes Hopf bifurcation much earlier than the positive branch. However, both Hopf bifurcations, as well as the transitions to aperiodic solutions, occur at lower $\alpha_\tau$ values than in JJG. This shift to lower $\alpha_\tau$ is consistent with the higher resolution, and hence greater intrinsic variability, of the present model version. As in JJG, the negative branch enters the aperiodic regime prior to the positive branch. The unstable steady solutions that coexist with the stable time-dependent ones cannot be determined numerically by the present forward integration method and are not shown in our bifurcation diagram here.

The time series of transport difference $TD$ are shown for points on the positive and negative branches in the four smaller, top and bottom panels of Fig. 1, respectively. The time series in the upper-left panel is for a point on the positive branch immediately after the Hopf bifurcation ($\alpha_\tau = 1.08$) and the one on the upper-right is for a point just before the aperiodic regime on the same branch ($\alpha_\tau = 1.2$). The oscillation amplitude is higher in the weaker,
subtropical gyre than in the stronger subpolar gyre for the positive branch (not shown), a result that agrees with those of Chang et al.'s (2001) in a barotropic QG model.

The spectra of these four time series were calculated using the SSA-MTM Toolkit (Dettinger et al., 1995; Ghil et al., 2002b) and are shown in Fig. 2. The positive periodic branch has a subannual oscillation period of 8 months. The other peak (Figs. 2a, b) has a period of 4 months and is the first harmonic of the dominant peak. Subannual oscillation periods for the positive branch were also found by JJG, Speich et al. (1995) and Dijkstra and Katsman (1997). Chang et al. (2001) obtained oscillations with a period of 126 days as the result of Hopf bifurcation off the two symmetric branches in their QG model. Similar periods of approximately 4–6 months have been found in the Gulf Stream region, for two versions of the Modular Ocean Model run at JPL (Chao et al., 1996) and at UCLA (Chassignet et al., 2000), as well as in an observational study of Topex/Poseidon altimetric data (Ide et al., 1997).

The negative branch here undergoes two successive Hopf bifurcations. The time series in the bottom-left panel of Fig. 1 is for a point $\alpha_T = 0.9$ on the negative branch immediately after the first Hopf bifurcation, while the one at bottom-right is for a point just after the second Hopf bifurcation at $\alpha_T = 1.1$. The spectra of these two time series are shown in Figs. 2c, d. The first Hopf bifurcation leads to a limit cycle with a period of about 10 months and the second one to a limit cycle with a period of about 21 months. The interaction between the two oscillations, with their incommensurable periods, leads to the fairly irregular, albeit still quasi-periodic solution in the lower right panels of Figs. 1 and 2. The oscillation amplitude is now higher for the weaker subpolar gyre associated with the negative branch. JJG and Speich et al. (1995) obtained interannual periods on the negative branch of the bifurcation diagram, but found no subannual oscillations on this branch. Dijkstra and Katsman (1997) in their 2-layer
QG model obtained two Hopf bifurcations, with periods of about 11 months and 1.5 years, respectively. The physical mechanisms behind these oscillations are discussed in Section 3b.

The time evolution of the basin's average kinetic energy $KE$, for the positive and negative branches, in the aperiodic regime are shown in Fig. 3. Both solutions have almost the same range of variability in $KE$. The time series for the positive branch also exhibits pronounced interdecadal variability with an irregular pattern: low-amplitude oscillations about a high-energy state with $KE$ approximately $1.35 \times 10^{12}$ Jm\(^2\) alternate with episodes of high-amplitude oscillations about a low-energy state with $KE$ approximately $1.25 \times 10^{12}$ Jm\(^2\).

This behavior resembles very well the one found in the chaotic regime by Chang et al. (2001) and Simonnet et al. (2003b). It is due there to the presence of homoclinic orbits that connect the two solution branches shown here via the unstable steady-state branch not shown in Fig. 1 (see, in particular, Figs. 1, 9 and 15 in Chang et al., 2001 and Fig. 16 in Simonnet et al., 2003b). Additional information on the role of homoclinic orbits in the double-gyre problem can be found in Meacham (2000) and Nadiga and Luce (2001). Chang et al. (2001) found that the aperiodic low-frequency variability arises in a double-gyre configuration due to the interaction between intense subtropical and subpolar vortices. This interaction in physical space corresponds to proximity or interpenetration of the attractors associated with the two separate branches in phase space.

The oscillations about the negative branch, while also quite irregular, have fairly constant amplitude and are centered on $KE \approx 1.2 \times 10^{12}$ Jm\(^2\). The oscillations about the positive branch and the negative one thus seem to overlap in $KE$, although not in $TD$ (see again Fig. 1) and suggest the coexistence of two stable attractors at $\alpha_t = 1.3$. Since the main purpose of the present paper is to explore the effects of time-periodic forcing on the double-gyre problem, we
do not dwell further on the finer points of the bifurcation structure for time-independent forcing. Given the high resolution of the present calculations, we do examine, though, the spatial flow patterns associated with the aperiodic solutions.

The long-term averages of the streamfunction and $PV$ fields are shown in Fig. 4 for $\alpha = 1.3$. In the aperiodic regime, for the positive branch, the intensity of the subpolar recirculation cell increases very much in comparison to the strength of the subtropical cell as the wind-stress forcing increases (Figs. 4a, b). The opposite holds for the negative branch at the same $\alpha$ (Figs. 4c, d), but the $PV$ pattern is much more complex in this case.

### i. Effect of Rossby deformation radius.

We conducted runs with $\alpha = 1.0$ for six different Rossby radii of deformation, $R_d = 40$, 55, 80, 100, 120 and 140 km (see Table 1b). The subtropical gyre is stronger in all cases with $R_d \geq 55$ km and the flow is quasi-periodic for $R_d = 55$, 80 and 100 km, while it is periodic for $R_d = 120$ and 140 km. The period of oscillation is of the order of 10 years for $R_d = 120$ km and 30 years for 140 km. For $R_d = 55$ km, variability is present in the dipole region, while variability in the northern part of the basin dominates. With increasing $R_d$, the spatial patterns fill the entire domain.

For $R_d = 40$ km, only one asymmetric steady state, with a strong subpolar gyre is found to exist, while at $R_d = 80$ km there are two, which are approximate mirror images of each other. Dijkstra and Katsman (1997) and Ghil et al. (2002a) showed that, for low $R_d$ values, the transition from an antisymmetric stable steady state to periodic solutions, via Hopf bifurcation, precedes the pitchfork bifurcation to multiple asymmetric equilibria. Both these studies used QG models, in which the pitchfork bifurcation is perfect. In the present SW model, the single asymmetric steady state found at low $R_d$ corresponds to the QG model's antisymmetric stable
equilibrium. Our results thus indicate, once more, the robustness of the basic bifurcation tree across a hierarchy of models.

\textit{ii. Effect of boundary conditions}. To check the robustness of the variability present in the subpolar gyre, the model was also run with free-slip boundary conditions. This model version requires, as expected, much higher values of $\alpha_A$ to get either steady or periodic solutions than in the case for no-slip boundary conditions (see Appendix A of JJG).

For free-slip boundary condition, the regime diagram in the two parameters $\alpha_A$ and $\alpha_\tau$ is shown in Fig. 5. Simulations were carried out for $\alpha_A$ values falling in the range 2.5–3.5. The solid curve represents the locus of the perturbed pitchfork bifurcation, the dash-dotted curve represents that of the Hopf bifurcation off the negative branch and the dashed curve corresponds to the Hopf bifurcation off the positive branch.

As can be seen from the figure, the Hopf bifurcation on the subtropical branch occurs prior to the Hopf bifurcation on the subpolar one. Multiple equilibria occur for $\alpha_\tau$ and $\alpha_A$ that fall in the domain between the loci of the pitchfork bifurcation and the Hopf bifurcation that lies closer to it. This regime diagram is thus in excellent agreement with the one obtained by JJG for no-slip boundary conditions (see Fig. 4 there), except for its being shifted to considerably higher $\alpha_A$ values. Such a good agreement between the two regime diagrams reinforces the view that major qualitative features of the oceans' wind-driven circulation do not depend on the details of the simple models used to explore these features.

\textit{b Spatio-temporal variability of the solutions}
Empirical orthogonal function (EOF) analysis. From the time series of TD and KE (Figs. 1 and 3), as well as their spectral analysis (Fig. 2), it appears that the physical mechanisms behind the modes of oscillation are different for the two branches. We carried out an EOF analysis of the streamfunction anomalies for the two branches to identify the dominant spatial patterns and the underlying instability mechanisms.

Snapshots of streamfunction anomalies (i.e., of the difference between the instantaneous fields and its long-term average; not shown) for the lower branch of the bifurcation diagram indicate that the largest variability occurs in the northern half of the basin for parameter values immediately after the first Hopf bifurcation at $\alpha = 0.865$. The point on the negative branch of the bifurcation diagram with $\alpha = 0.9$ is picked for detailed analysis; the other control parameters are those used in Fig. 1. The instantaneous snapshots of the $\psi$ field (Fig. 6) show that the weaker subpolar gyre wraps around the subtropical gyre's eastern end. Small secondary vortices move southward along the western boundary and eventually get absorbed into the subpolar recirculation region ($PV$ plots not shown).

The two leading EOFs contribute more than 80% of the periodic solution's variability. Their spatial patterns are plotted in Figs. 7a, b and show strong low-frequency variability far away from the recirculation zone, in parts of the domain where the mean flow itself is less intense. The spectra of the PCs associated with these EOFs are shown in Figs. 7c, d and both have period of 10 months.

This subannual variability along the negative branch was not captured before in studies which used the same model with no-slip boundary conditions (JLG, Speich et al., 1995) although it did appear on both branches of the QG model of Chang et al. (2001). These authors showed that the period of this mode increases with $\alpha$, up to interannual periods similar to
those found in JG. We suspect that it is the higher horizontal resolution used in the present study that has enabled us to capture this variability. The spatial patterns in Figs. 7a, b suggest that the oscillation's physical mechanism is a somewhat localized and shifted version of the gyre mode first identified by JG and Speich et al. (1995) and described in greater detail by Chang et al. (2001), Simonnet and Dijkstra (2002) and Simonnet et al. (2003b). The vorticity concentration off the western boundary, well within the subpolar gyre (see Figs. 4b, d; PV pattern not shown here) plays the role of the recirculation dipole in generating this oscillation.

A second Hopf bifurcation off this lower branch at $\alpha_r = 1.1$ leads to an interannual oscillation with a period of 21 months. The anomalies in the northern half of the basin still dominate but strong anomalies start to appear in the recirculation region as well. The variability in the recirculating region is associated with changes in the strength and orientation of the vorticity dipole. JG and Speich et al. (1995) both found this variability in the recirculation region with their lower-resolution version of the same model. Figure 8 displays the spatial patterns of the 4 leading EOFs at $\alpha_r = 1.1$; they contribute over 80% of the total variability of the solution. EOFs-1 and -2 hence still resemble those at $\alpha_r = 0.9$ (see Figs. 7a, b); while the second pair resembles quite well the internal gyre mode of JG and subsequent authors. The spectra of the PCs associated with the four leading EOFs are shown in Fig. 9. The dominant peak of the first pair agrees with that of Figs. 7c, d. The second-pair spectrum is dominated by an interannual peak, as expected from the spatial patterns in Figs. 8c, d.

We now turn our attention to the upper branch. Figures 10 and 11 show the average streamfunction and streamfunction anomaly fields for the positive branch, after the Hopf bifurcation $H^+$ that occurs at $\alpha_r = 1.0$. The figures correspond to the periodic solution at $\alpha_r = 1.08$ which has 8 month oscillation period. The splitting of smaller vortices off the main
vortex dipole, their interaction with each other and eventual merging back into the dipole can be followed in the \( \psi \) field snap-shots (Fig. 10).

The spatial patterns of the first four leading EOFs are shown in Fig. 12; these EOFs contribute 80% of the total variability. Both the \( \psi \)-anomaly fields (Fig. 11) and the two leading EOFs of this upper branch solution are dominated by an elongated wave pattern in the northeast–southwest direction. This pattern resembles those obtained by JJJG and Speich et al. (1995) for their subtropical branch in the SW model, as well as by Chang et al. (2001) on both branches of the symmetric bifurcation diagram of their QG model in a larger, square basin.

To check the robustness of periodic solution behavior, we also carried out integrations with free-slip boundary conditions, along the lower branch. These runs necessitated lower values of \( \alpha_4 \) and higher values of \( \alpha_1 \), in order to stay within the periodic regime, as explained in Section 3a (see again Fig. 5). A strong subtropical gyre and hence a southeastward-flowing jet still dominate the periodic solutions along this branch. The streamfunction snapshots (not shown) indicate pinching off of cyclonic rings or eddies to the north of the jet and of anticyclonic ones to the south.

The spatial patterns of the first four EOFs for a solution on this branch, just past the first Hopf bifurcation, are shown in Fig. 13. They are clearly different from the analogous EOFs obtained with no-slip boundary conditions in Figs. 7a, b and 8: for free-slip boundary conditions, the variability is mostly concentrated in the recirculation region, along both the subpolar and subtropical (i.e., upper and lower) branches (not shown for the upper branch). For the no-slip case along the lower branch, the variability is strongest in the northern part of the basin, away from the recirculation zone. This does not, however seem to be the case in the North Atlantic or the North Pacific, where low-frequency variability is mostly observed in and
near the meandering-jet region of the Gulf Stream or Kuroshio. Since the details of the flow near the boundary are not really resolved by such a simple model, or even by most ocean GCMs (Ghil et al., 2002a; Dijkstra and Ghil, 2004), the greater realism of the overall simulation suggests that free-slip or partial slip boundary conditions might be preferable for our model (see also Appendix A of JJG).

**ii. Relation between potential vorticity (PV) and streamfunction.** Ghil et al. (2002a) showed that, in the recirculation region, the points in a $PV$-$\psi$ scatter plot cluster very close to a straight line, with regression coefficients larger than 0.99 in absolute value, independently of the flow regime (steady, periodic or aperiodic). Points outside the dipole region were found to be spread over a finite area of the $PV$–$\psi$ plane. This distinction was also noted by Griffa and Salmon (1989).

The $PV$-$\psi$ scatter plot for points on the upper and lower branches are shown in Fig. 14, in the top and bottom panels respectively. Panels (a)–(c) of Fig. 14 show the long-term average $PV$-$\psi$ plot for the steady ($\alpha = 0.95$), periodic ($\alpha = 1.08$) and aperiodic cases ($\alpha = 1.3$) for the positive branch, while panels (d)–(f) correspond to a steady ($\alpha = 0.8$), periodic ($\alpha = 0.9$) and aperiodic case ($\alpha = 1.3$) for the negative branch.

In each plot the points from the dipole region lie very close to a curve whose tangent has a negative slope at each of its points. This confirms the dominance of the inertial and $\beta$ terms in the dynamic balance of the recirculation zone and the fact that the flow in this region is well approximated by a free mode, with a one-to-one relationship $PV = F(\psi)$ (Stern, 1975; Larichev and Reznik, 1976; Salmon, 1998). As shown in greater detail in Fig. 12 of Ghil et al. (2002a), points from the Sverdrup region outside the dipole fill the two curvilinear triangles.
between the red $PV = F(\psi)$ curve and the nearly vertical curve associated with points that lie near the western boundary. The difference between the scatter plots for the two branches is mainly in the extent of the downward-sloping curve that represents the recirculation zone. The strong recirculation cell in the subpolar part of the basin is characterized by the extended, slightly curved line in the upper-left part of the scatter plots in the top panels, while the subtropical cell is represented by the extended line in the lower-right part of the plots in the bottom panels. For the upper branch, the upper-left part of the straight line extends further due to the presence of the stronger subpolar gyre; similarly for the lower branch the straight line extends further down and right.

In the top panels, the strength of the northern recirculation cell is much higher (note the $\psi$ values) than that of the southern recirculation cell, especially in the aperiodic regime, in agreement with the visual evidence of the contour plots for the field (see Figs. 4a, c here and previous work). Ghil et al. (2002a) showed that, for steady-state solutions at least, the average slope of the line that represents the recirculation region in the $PV$-vs.-$\psi$ plot decreases as the wind stress increases and the solutions become more nonlinear.

We study now the variability in the basin by observing the changes in time within the $PV$-vs.-$\psi$ scatter plot for a periodic solution, immediately after the first Hopf bifurcation, on the lower branch. The root-mean-square (rms) of the streamfunction anomalies was calculated at each grid point and the points having a rms anomaly above 90% of the maximum rms value were identified and are shown in red in all five panels of Fig. 15. It can be seen from the central panel of the figure and the scatter plots in Figs. 15a–d that, while the mean intensity of the subtropical gyre is greater on the lower branch (see also Fig. 6), the variability is strongest in the subpolar gyre.
4 Time-dependent wind-stress forcing

We have thus verified the results for time-independent forcing of the present model version against JJG and subsequent models of different types, domain sizes and resolutions, and found them in good overall agreement. It is time to turn to the effect of time-periodic forcing on double-gyre flows. Except for varying $\alpha$, other parameter values are as in Figs. 1 and 2; no-slip boundary conditions are used. An idealized wind-stress pattern with time-periodic wind magnitudes, as given by Eq. (4), is used in subsection 4a to study the response of the mid-latitude ocean to such forcing. The effect of north–south migration of the wind-stress forcing on the flow is studied using another idealized model of the wind-stress pattern, cf. Eq. (5), in subsection 4b.

a Standing oscillation in the wind-stress forcing

As mentioned in Section 2a, we take the oscillation period of the magnitude of the wind-stress forcing in Eq. (4) to be $T_{\text{amp}} = T = 12$ months. The amplitude $a$ is taken as 0.1, based on our analysis of observed wind data from NCAR. The bifurcation diagram as the mean wind-stress forcing $\alpha$ increases is constructed and discussed in the following subsection.

i. Solution dependence on mean wind-stress intensity. The bifurcation diagram with increasing $\alpha$ is plotted for the normalized transport difference $TD$ in the central panel of Fig. 16. For very low values of $\alpha$ only a positive subtropical branch is found, as in the case of time-independent wind-stress forcing (see Fig. 1). The solution here is periodic, however, due to the time-dependent wind forcing. Hence the maximum and minimum values of $TD$ for each $\alpha$
are plotted on the bifurcation diagram. The period of oscillation for these low values of $\alpha$ is the same as the periodicity of the wind forcing, i.e. one year.

At $\alpha = 0.7$, a negative $TD$ branch emerges due to a perturbed pitchfork bifurcation, as in Fig. 1. The location of this bifurcation does not seem to be affected by the periodicity of the forcing, but we still denote it by $\tilde{P}$, to distinguish the bifurcation points in Fig. 16 from those in Fig. 1. As before, the negative branch has a stronger subtropical and a weaker subpolar gyre, but the area covered by each gyre is in the opposite ratio: larger for the subpolar and smaller for the subtropical gyre (not shown). The oscillation period is still 1 year on both branches. The periodic solutions on the two branches coexist in the range of $\alpha = 0.7–0.85$.

For the upper branch, the solution is quasi-periodic for $\alpha$-values between 0.95 and 1.05, and it becomes aperiodic for $\alpha$ greater than 1.05. The corresponding time series, for periodic and quasi-periodic behavior, are shown in the two left upper panels of Fig. 16. Spectral analysis results for the upper branch indicate that, throughout the quasi-periodic regime, in addition to the forced oscillation with its 12-month period, there exists another prominent oscillation, with a period of about 4 months. A 4-month oscillation was also obtained in the steady wind-stress forcing case for the upper branch (see Fig. 2) and it is intrinsic to the system.

The mean position of the jet axis, as well as the amplitude of the meanders in it, change over the 4-month period, as they do in the steady-forcing case (not shown). This is in good agreement with the results of Lee and Cornillon's (1995) observational study. In the latter, both the meridional position of the Gulf Stream axis and its meandering intensity change with a subannual period that is clearly distinct from the seasonal cycle of the wind stress, although the period is 9 months, rather than 4.
Two modes of internal variability arise along the lower branch, as for the steady-forcing case: one has a 10-month period, the other one a 6-year period, as in the steady-forcing case. At $\alpha = 0.95$ the lower branch solutions become chaotic. The $TD$ time series for the periodic and quasi-periodic regimes on the lower branch are shown in the two left bottom panels of Fig. 16. The top and bottom rightmost panels are the time series for the aperiodic regime off both branches.

A comparison of these time series shows that the oscillations are more complex for the lower than for the upper branch, due to the presence of the additional mode of internal variability. The 6-year mode is clearly apparent in the quasi-periodic regime (bottom-middle panel), in the form of an amplitude modulation of the seasonal cycle.

**ii. Spatio-temporal variability.** The interaction between the externally forced variability and the modes of internal variability gives rise to fairly complex solution behavior. We study first the upper branch, which is simpler.

The spatial patterns of the two leading EOFs are shown in the top panel of Fig. 17 for this branch in the purely periodic, forced regime. These patterns extend further out over the basin than the corresponding ones in Fig. 12, where the periodicity is internally generated. Otherwise, though, the patterns are similar, with several closed features aligned in a wave pattern whose tilt changes from EOF-1 to EOF-2. While the overall patterns are more basin-wide, the strongest features here cluster more closely near the separation and merger points of the two western boundary currents. The transport difference $TD$ reaches its maximum when $\psi_u$ is at a minimum and $\psi_p$ is of intermediate strength (middle and bottom panels of Fig. 17). The minimum of the basin's kinetic energy occurs when the meandering of the jet axis is
at its maximum, as in the case for steady forcing (McCalpin and Haidvogel, 1996; Berloff and McWilliams, 1999a; Ghil et al., 2002a).

The spatial patterns for the two leading EOFs in the quasi-periodic regime (Fig. 18) are also basin-wide, as in Fig. 17, but the strongest variability is concentrated even further in and near the recirculation zone. Some of the features are inherited from the internal variability: compare especially the second EOF here with the second one in Fig. 12. The transport difference $TD$ is again a maximum when $\psi_{tr}$ is a minimum, while $\psi_{po}$ is now close to its maximum value (not shown). In the aperiodic regime, the variability has even more small-scale features and fills the rectangular basin.

The more complex lower negative branch is studied next. When the oscillation is purely forced, and hence has a 1-year period, the spatial patterns obtained from the EOF analysis (Fig. 19) are most intense in the northern part of the basin, as in the steady-forcing case (see Figs. 6 and 7), but extend much further across the basin.

The spatial patterns of the four leading EOFs (Fig. 20) on this branch in the quasi-periodic regime all show dominant variability in the northern part of the basin. The strengths of the subpolar and subtropical gyre increase and decrease together, and the amplitude of their respective oscillations are roughly the same. In this case, both the externally forced and the internally generated variability leave their imprint on the total variability: the seasonal forcing is manifest in the first pair of EOFs (Fig. 20(a) and (b)), which closely resembles Fig. 19, while internal oscillation is clearly visible when comparing the second pair of EOFs in Fig. 19 with the first pair in Fig. 7.

b Migrating wind-stress forcing
In this section we study the response of the mid-latitude ocean to a smooth, periodic shift of the wind-stress pattern in the north–south direction, according to Eq. (5) of Section 2. For simplicity, we assume here that there is no change in the wind-stress magnitudes, i.e., \( \alpha = \text{const.} \), while \( \alpha_\tau = 0.85 \), \( \alpha_A = 1.3 \) and \( R_d = 80 \text{ km} \), so that both branches of the steady-forcing diagram in Fig. 1 yield a stable steady solution.

Strong et al. (1993) have reported a shift by 20° in the line of zero wind-stress curl, which coincides with the subtropical jet axis, from their analysis of the European Centre for Medium Range Weather Prediction (ECMWF) climatology for 1980–86. This line is allowed to migrate in Eq. 5 by an amount equal to \( b \), symmetrically to the north and to the south of the mid-basin axis. We have used three values of \( b \), so that the total extent of the migration is \( 2b = 800 \text{ km}, 1600 \text{ km}, \) and \( 3200 \text{ km} \); the middle value corresponds most closely to the observed. For the given parameter settings and no-slip boundary conditions, all the solutions obtained are purely periodic, with period \( T_{\text{mig}} = T = 1 \text{ year} \). The period \( T_{\text{mig}} \) of this pattern modulation is assumed to be 1 year, \( T_{\text{mig}} = T = 12 \text{ months} \).

We examine here the way that the flow is modified by the migratory periodic forcing along the upper branch. Figure 21 shows the long-term average \( PV \) and \( \psi \) fields (top panels), as well as the \( PV-v_s-\psi \) scatter plots (bottom panels) for the three different oscillation amplitudes. The recirculation cells are still pronounced for small excursions of the atmospheric jet (see top panel of Fig. 21a) but are greatly weakened, to the point of total disappearance, for larger excursions of the jet (see top panels of Figs. 21 b, c, respectively). The visual evidence for this effect in the \( \psi \) and \( PV \) fields is confirmed by the scatter plots in the bottom panels of the figure. The intensity and penetration length of the oceanic jet is also found to decrease with the migration amplitude, to the point where the eastward jet is totally
absent in the long-term mean of the $PV$ field (Fig. 21 c). These features can also be noticed in Fig. 22, which shows the variation of the streamfunction along $x = 180$ km, for the three oscillation amplitudes studied.

The $\psi$ and $PV$ fields suggest that wind-pattern modulation strongly affects the flows' spatio-temporal behavior, especially in the recirculation region, where nonlinear effects are strong. The most striking effect of the imposed north–south shifts in the atmospheric jet is to greatly enhance $PV$ mixing. As the line of zero wind-stress curl moves north of the mid-basin axis, the subpolar gyre collapses, while the subtropical gyre grows. For a southward excursion of the prescribed atmospheric jet, the subtropical gyre is distorted and the subpolar gyre starts building up. In general, the recirculation cells are much weaker and smaller in extent than when the flow is driven by a steady wind stress or an amplitude-modulated wind stress.

For small meridional shifts in the wind-stress pattern, the separation latitude of the jet depends strongly upon the position of the maximum wind stress curl (not shown). Ozgokmen and Chassignet (1998) also observed such a dependency while investigating the effect of wind forcing on the eastward jet's separation in a basin with an oblique coastline, using a two-layer QG model.

We also carried out a preliminary study on the effect of different oscillation periods in the forcing. Simulations with periods of 2 and 0.5 years were performed. Results so far indicate that the flow patterns are not very sensitive to the oscillation frequency in comparison with sensitivity to its amplitude.

5 Concluding remarks
This work focused on the differences in mid-latitude ocean response to different types of wind forcing, using a 1.5-layer, reduced-gravity, shallow-water (SW) model with fairly high spatial resolution $\Delta x = \Delta y = 12.5$ km. It thus complements previous studies of the wind-driven, single- and double-gyre circulation and its multiple flow regimes (Cessi and Ierley, 1995; Jiang et al., 1995; Speich et al., 1995; Dijkstra and Katsman, 1997; Sheremet et al., 1997; Meacham and Berloff, 1997a, b; Berloff and McWilliams 1999). Both the time-independent and time-periodic forcing we considered were of the double-gyre type, cf. Eqs. 3–5.

The bifurcation diagram of the flow with increasing wind stress forcing $\alpha_t$ was first constructed for the time-independent forcing, in the no-slip case. The diagram was plotted for $TD$, the normalized transport difference between the subpolar and the subtropical gyre, vs. $\alpha_t$ (Fig. 1). This diagram is characterized by two distinct branches. The upper branch has a stronger subpolar gyre, which is however smaller in size than the subtropical one; it is also referred to as the positive ($TD > 0$) or subpolar branch. The lower branch exhibits the opposite characteristics, with a weaker but larger subpolar gyre and is also called negative ($TD < 0$) or subtropical.

The upper branch of the bifurcation diagram exhibited subannual (8-month) variability and is in agreement with recent work of Chang et al. (2001), using an equivalent-barotropic, quasi-geostrophic (QG) model with a larger domain. The lower branch of the bifurcation diagram undergoes two Hopf bifurcations: the first one gives rise to subannual variability with a 10-month oscillation period and the following one to an interannual oscillation period with a 21 month period (Fig. 2). The associated spatial patterns reveal that the subannual variability is localized in the northern part of the basin. The higher horizontal resolution has enabled us to capture this variability, not found so far in lower-resolution SW or QG models. The
interannual oscillation period is associated with the gyre mode, arising in the dipole region and fairly well known by now (JJG; Speich et al., 1995; Chang et al., 2001; Simonnet and Dijkstra, 2002; Simonnet et al., 2003a, b).

The variability in the case of free-slip boundary conditions is found to be concentrated near the dipole region for both upper and lower branches. The circulation in our fairly small, rectangular basin resembles the real ocean better in this respect while using free-slip boundary conditions. Given the large viscosities that have to be used for the free-slip boundary conditions to still yield relatively simple solutions, the western boundary layers are wider and the numerical scheme becomes more reliable in such a case.

The effect of the Rossby radius of deformation, $R_d$, on the flow pattern in the basin was studied by running the model at various values of this parameter. For high values of the deformation radius the variability is more basin wide, while for low values it is more localised. Multiple states cease to exist at low Rossby radius of deformation, in agreement with the results of Ghil et al., (2002a) using a baroclinic QG model.

Our model thus agrees in its major features with results obtained across a hierarchy of distinct model configurations, SW and QG, barotropic and baroclinic, at different resolutions and in different domain sizes and shapes. We thus proceed to study the flow in the basin subject to time-periodic wind forcing. The idealized wind stress patterns for the study were chosen based on the analysis of observational data. For simplicity, the periodic variation in the magnitude of wind-stress forcing and periodic north–south migration of wind-stress forcing were considered separately. Multiple periodic flows were found to occur in the case of varying wind stress magnitudes. The bifurcation diagram (Fig. 16) reveals annual and subannual (4-month) variability on the upper branch.
From the analysis of the Gulf Stream axis data, Lee and Cornillon (1995) obtained two dynamically distinct modes of variability. The near-annual variability is associated with shifts of the mean Gulf Stream path (Schmeits and Dijkstra, 2000), caused by changes in the wind forcing over the area (Kelly et al., 1999). The second mode of variability is associated with changes in meandering intensity having a 9-month periodicity and is presumed to be related to internal ocean dynamics. Our study also points out that the 4-month variability is associated with changes in the dipole region and also to the meandering intensity of the jet. For the positive branch, introducing time dependency in the wind-stress forcing did not change the period associated with the internal mode of variability.

The lower branch shows annual, subannual (10-month) and interannual (6-year) variability. Speich et al., (1995) obtained similar modes of variability (6 years and 20 months) of the Gulf Stream axis position in their lower resolution version of the present model as well as from their analysis of the Cooperative Ocean-Atmosphere Data Set (COADS). Simonnet et al. (2003b) using a realistic North Atlantic shoreline in a 2.5-layer SW model obtained 6-year and 20-month variability, which coincide with the variability in the position of the Gulf Stream axis. The spatio-temporal patterns from the EOF analysis clearly show superposition of the externally driven and internally generated modes of variability (compare again Figs. 7, 19 and 20). When seasonal migration of the wind-stress pattern is used for the forcing, the flow in the basin is affected more substantially than for seasonal changes in intensity. A larger number of small-scale features appear in the basin and the recirculation cells are substantially weakened and reduced as the range of excursion of the zero wind-stress curl axis increases. The wind migration seems to be a crucial mechanism in exciting the interaction and mixing of $PV$ between the two neighboring cells (Figs. 21 and 22). Liu (1996) showed that, in a barotropic QG model, the weakening of the inertial recirculation cells is due to two processes:
(i) the mismatch between the fast spin-up time of the Sverdrup interior and the western boundary currents, on the one hand, and the slow spin-up of the inertial recirculation, on the other; and (ii) the chaotic mixing of $PV$ anomalies between the two cells of the recirculation dipole (see also Yang and Liu, 1994; Yang 1996). The first process was neutralized by slower baroclinic Rossby waves replacing the fast barotropic ones in a 1.5-layer version of Liu's (1996) QG model. The second process thus seems more important and has been studied in greater depth by Coulliette and Wiggins (2000).

Our study indicates that the effectiveness of the migrating wind patterns depends quite strongly on the extent of the meridional excursions of the atmospheric jet. Within a realistic range, the effect is still moderate and probably counterbalanced by the presence of partial slip at the western boundary, which tends to concentrate and reinforce the recirculation dipole. For high Reynolds numbers (Berloff and McWilliams, 1999; Simonnet et al., 2003b, 2004), eddy diffusion might play a comparable or greater role than chaotic mixing (Berloff and McWilliams, 2002a, 2002b, 2003), with localized sub- and super diffusive mixing present in different parts of the flow domain; see also Huber et al. (2001) for a discussion of these topics in the atmosphere.

A more realistic study with a wind-stress pattern that incorporates oscillations in magnitude as well as in the north–south position of the atmospheric jet is thus clearly necessary. This will be just one further step in the exploration of the double-gyre problem in a fully coupled, although still flexible, ocean–atmosphere. Feliks et al. (2004) have found that, in a fairly simple atmospheric model forced at the lower boundary by a sharp oceanic front, like those produced by oceanic eastward jets (e.g., the Gulf Stream or the Kuroshio), the jet cannot only change in intensity and position, but also in its meandering properties. There is thus considerable scope for pursuing this avenue of research.
References


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Table 1. Parameter values.
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Figure 1. Bifurcation diagram for time-constant wind forcing $\alpha$. The central panel shows normalized transport difference $TD$ with increasing $\alpha$; all model parameters as in Table 1a, while $\alpha_A = 1.3$ and $R_d = 80$ km. The top and bottom panels illustrate 10-year segments of the $TD$ time series for the solutions indicated by arrows pointing to the middle panel: (top panels) positive branch, $\alpha = 1.08$ (periodic) and $\alpha = 1.20$ (quasi-periodic); (bottom panels) negative branch, $\alpha = 0.9$ (periodic) and $\alpha = 1.1$ (quasi-periodic).

Figure 2. Spectra of the time series shown in Fig. 1, using the SSA-MEM method described in Section 2b. The SSA window width was 24 months and the MEM order was 30. The panels (a) and (b) correspond to the time series in the left and right top panels of Fig. 1 and (c) and (d) to the left and right bottom panels of Fig. 1, respectively.

Figure 3. Evolution of kinetic energy for the positive (a, b) and the negative (c, d) branches, in the aperiodic regime, at $\alpha = 1.3$; same values as in Figs. 1 and 2 for all other parameters.

Figure 4. Long-term average flow fields in the aperiodic regime: (a, c) streamfunction field, and (b, d) $PV$ field for the positive (top) and negative (bottom) branches, at the same parameter values as in Fig. 3. For the $\psi$ field the positive contours are solid, the negative ones dashed.

Figure 5. Regime diagram for free-slip boundary conditions.

Figure 6. Instantaneous snapshots of the $\psi$ field for a periodic solution on the lower branch, at $\alpha = 0.9$ are shown in the bottom panels. The oscillation has a period of 10 months and the panels are plotted every two months. The top panel shows the $TD$ (dashed line) and $KE$ (solid line) time series. The phase corresponding to the snapshots in the bottom panel are identified on the $TD$ series.
Figure 7. The two leading EOFs for the solution in Fig. 6: (a, b) spatial patterns and (c, d) power spectra of the associated PCs.

Figure 8. Spatial patterns corresponding to the four leading EOFs for $\alpha_\tau=1.1$ on the lower branch, after the second Hopf bifurcation; same parameter settings as for bottom-right panel of Fig. 1 and for Fig. 2d.

Figure 9. Spectra of the PCs associated with the EOFs in Fig. 8.

Figure 10. Snapshots of the $\psi$ fields for the positive branch in the periodic regime, same parameter settings as in the top-left panel of Fig. 1 and in Fig. 2a. The oscillation has a period of 8 months and the panels are plotted every two months. The top panel shows the $TD$ (dashed line) and $KE$ (solid line) time series. The phase corresponding to the snapshots in the bottom panel are identified on the $TD$ series.

Figure 11. Snapshots of the $\psi$-anomaly field for the instantaneous $\psi$ fields shown in the bottom panel of Fig. 10.

Figure 12. Spatial patterns of the four leading EOFs of the $\psi$-anomaly fields shown in Fig. 11.

Figure 13. Spatial patterns of the four leading EOFs for a solution with free-slip boundary conditions, $\alpha_\tau = 1.0$, $\alpha_A = 3.5$ and $R_d = 80.0$ km, on the lower branch.

Figure 14. $PV$-vs.-$\psi$ scatter plots for different regimes, along the upper (top, a–c) and lower (bottom, d–f) branches of the bifurcation diagram. The values of $\alpha_\tau$ are given on each panel, while other parameter values are as in Figs. 1 and 2; no-slip boundary conditions were used. (a, d) steady; (b, e) periodic; and (c, f) aperiodic. Points that originate from the dipole region are in red and lie very close to a curve. The blue points correspond to grid points at $0\Delta x$ on the western boundary. Each point in the scatter plots is based on an average of $PV$ and $\psi$ for a given grid point that is taken over 20 years.
Figure 15. $PV$-vs.-$\psi$ scatter plots for a purely periodic, 10-month oscillation at $\alpha_\tau = 0.9$ on the lower branch; all other parameter settings as in Figs. 1 and 2(c). The region having rms above 90% of the maximum is shown in red in the center panel (e). The points that correspond to this region are shown in red in the scatter plots (a)–(d) as well; each of the latter panels corresponds to a single snapshot of $\psi$, at the time indicated.

Figure 16. Bifurcation diagram for intensity-modulated wind-stress forcing is plotted against the seasonally averaged wind-stress intensity $\alpha_\tau$ in the central panel. The 10-year $TD$ time series for the positive branch in the periodic, quasi-periodic and aperiodic regimes are shown in the three top panels and for the negative branch in the bottom panels, respectively. Same symbols and parameter values as in Fig. 1.

Figure 17. The top panel shows spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing. The solution corresponds to the top-left panel in Fig. 16 ($\alpha_\tau = 0.8$, purely periodic oscillation). The middle panel shows $TD$ and $KE$ time series for the last 10 years of a 40 year run and the bottom panel shows $|\psi_{po}|$ and $|\psi_{tr}|$ series for the same period.

Figure 18. Spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing, for the solution corresponding to the top-middle panel in Fig. 16 ($\alpha_\tau = 1.0$, quasi-periodic oscillation).

Figure 19. Spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing, for the lower branch, corresponding to the bottom-left panel of Fig. 16 ($\alpha_\tau = 0.7$, purely periodic oscillation).
Figure 20. Spatial patterns of the leading four EOFs for the lower branch of Fig. 16. The solution corresponds to the bottom-right panel there ($\alpha = 0.9$) and falls in the quasi-periodic regime.

Figure 21. Solution behaviour for a meridionally modulated wind-stress pattern, for $\alpha = 0.85$; all other parameter settings as in Fig. 1. The range of excursions in the line of zero wind-stress curl is (a) $b = 400$ km; (b) $b = 800$ km; and (c) $b = 1600$ km. Long-term average $\psi$ (in red) and $PV$ (in blue) fields are shown on the top panels, while $PV$-vs.-$\psi$ scatter plots are shown in the bottom panels, with points stemming from the recirculation region, when present, in red, and those along the western boundary ($0\Delta x$) in blue.

Figure 22. Variation of streamfunction along $x = 180$ km, along which roughly lies the center of the recirculation cells.
### a) Fixed model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>L, east–west extent</td>
<td>1000 km</td>
</tr>
<tr>
<td>D, north–south extent</td>
<td>2000 km</td>
</tr>
<tr>
<td>$f_0$, basic Coriolis parameter</td>
<td>$5 \times 10^{-5}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\beta$, rate of change of Coriolis parameter</td>
<td>$2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\tau_0$, wind-stress amplitude</td>
<td>0.1 N/m$^2$</td>
</tr>
<tr>
<td>$A$, eddy viscosity coefficient</td>
<td>300 m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$R$, Rayleigh friction coefficient</td>
<td>$5 \times 10^{-8}$ s$^{-1}$</td>
</tr>
<tr>
<td>$H_0$, upper-layer thickness</td>
<td>500 m</td>
</tr>
<tr>
<td>$\rho$, upper-layer density</td>
<td>1022 kg/m$^3$</td>
</tr>
</tbody>
</table>

### b) Control parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$, wind stress intensity</td>
<td>0.4 – 1.3</td>
</tr>
<tr>
<td>$\alpha_A$, lateral-diffusion coefficient</td>
<td>1.3 (no-slip)</td>
</tr>
<tr>
<td></td>
<td>2.5 – 3.5 (free-slip)</td>
</tr>
<tr>
<td>$g'$, reduced gravity</td>
<td>0.03 m$^2$ s$^{-1}$ (most cases)</td>
</tr>
<tr>
<td></td>
<td>0.008 – 0.098 m$^2$ s$^{-1}$ (sensitivity study)</td>
</tr>
<tr>
<td>$R_d$, Rossby radius of deformation</td>
<td>80 km (most cases)</td>
</tr>
<tr>
<td></td>
<td>40, 55, 100, 120 and 140 km (sensitivity study)</td>
</tr>
</tbody>
</table>

Table 1: Parameter values
Figure 1: Bifurcation diagram for time-constant wind forcing $\alpha_t$. The central panel shows normalized transport difference $TD$ with increasing $\alpha_t$; all model parameters as in Table 1a, while $\alpha_A = 1.3$ and $R_d = 80$ km. The top and bottom panels illustrate 10-year segments of the $TD$ time series for the solutions indicated by arrows pointing to the middle panel: (top panels) positive branch, $\alpha_t = 1.08$ (periodic) and $\alpha_t = 1.20$ (quasi-periodic); (bottom panels) negative branch, $\alpha_t = 0.9$ (periodic) and $\alpha_t = 1.1$ (quasi-periodic).
Figure 2: Spectra of the time series shown in Fig. 1, using the SSA-MEM method described in Section 2b. The SSA window width was 24 months and the MEM order was 30. The panels (a) and (b) correspond to the time series in the left and right top panels of Fig. 1 and (c) and (d) to the left and right bottom panels of Fig. 1, respectively.
Figure 3: Evolution of kinetic energy for the positive (a, b) and the negative (c, d) branches, in the aperiodic regime, at $\alpha_t = 1.3$; same values as in Figs. 1 and 2 for all other parameters.
Figure 4: Long-term average flow fields in the aperiodic regime: (a, c) streamfunction field, and (b, d) PV field for the positive (top) and negative (bottom) branches, at the same parameter values as in Fig. 3. For the $\psi$ field the positive contours are solid, the negative ones dashed.
Figure 5: Regime diagram for free-slip boundary conditions.
Figure 6: Instantaneous snapshots of the $\psi$ field for a periodic solution on the lower branch, at $\alpha_\tau = 0.9$ are shown in the bottom panels. The oscillation has a period of 10 months and the panels are plotted every two months. The top panel shows the $TD$ (dashed line) and $KE$ (solid line) time series. The phase corresponding to the snapshots in the bottom panel are identified on the $TD$ series.
Figure 7: The two leading EOFs for the solution in Fig. 6: (a, b) spatial patterns and (c, d) power spectra of the associated PCs.
Figure 8: Spatial patterns corresponding to the four leading EOFs for $c_t = 1.1$ on the lower branch, after the second Hopf bifurcation; same parameter settings as for bottom-right panel of Fig. 1 and for Fig. 2d.
Figure 9: Spectra of the PCs associated with the EOFs in Fig. 8.
Figure 10: Snapshots of the $\psi$ fields for the positive branch in the periodic regime, same parameter settings as in the top-left panel of Fig. 1 and in Fig. 2a. The oscillation has a period of 8 months and the panels are plotted every two months. The top panel shows the $TD$ (dashed line) and $KE$ (solid line) time series. The phase corresponding to the snapshots in the bottom panel are identified on the $TD$ series.
Figure 11: Snapshots of the $\psi$-anomaly field for the instantaneous $\psi$ fields shown in the bottom panel of Fig. 10.
Figure 12: Spatial patterns of the four leading EOFs of the $\psi$-anomaly fields shown in Fig. 11.
Figure 13: Spatial patterns of the four leading EOFs for a solution with free-slip boundary conditions, $\alpha_t = 1.0$, $\alpha_A = 3.5$ and $R_d = 80.0$ km, on the lower branch.
Figure 14: $PV$ vs. $\psi$ scatter plots for different regimes, along the upper (top, a–c) and lower (bottom, d–f) branches of the bifurcation diagram. The values of $\alpha_t$ are given on each panel, while other parameter values are as in Figs. 1 and 2; no-slip boundary conditions were used. (a, d) steady; (b, e) periodic; and (c, f) aperiodic. Points that originate from the dipole region are in red and lie very close to a curve. The blue points correspond to grid points at $0 \Delta x$ on the western boundary. Each point in the scatter plots is based on an average of $PV$ and $\psi$ for a given grid point that is taken over 20 years.
Figure 15: $PV$-vs.-$\psi$ scatter plots for a purely periodic, 10-month oscillation at $\alpha_\tau = 0.9$ on the lower branch; all other parameter settings as in Figs. 1 and 2(c). The region having rms above 90% of the maximum is shown in red in the center panel (e). The points that correspond to this region are shown in red in the scatter plots (a)–(d) as well; each of the latter panels corresponds to a single snapshot of $\psi$, at the time indicated.
Figure 16: Bifurcation diagram for intensity-modulated wind-stress forcing is plotted against the seasonally averaged wind-stress intensity $\alpha$ in the central panel. The 10-year $TD$ time series for the positive branch in the periodic, quasi-periodic and aperiodic regimes are shown in the three top panels and for the negative branch in the bottom panels, respectively. Same symbols and parameter values as in Fig. 1.
Figure 17: The top panel shows spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing. The solution corresponds to the top-left panel in Fig. 16 ($\alpha = 0.8$, purely periodic oscillation). The middle panel shows $TD$ and $KE$ time series for the last 10 years of a 40 year run and the bottom panel shows $|\psi_{po}|$ and $|\psi_{tr}|$ series for the same period.
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