Spatio-temporal variability in a mid-latitude ocean basin subject to periodic wind forcing

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Abstract

The mid-latitude ocean's response to time-dependent zonal wind-stress forcing is studied using a reduced-gravity, 1.5-layer, shallow-water model in two rectangular ocean basins of different sizes. The small basin is 1000 km x 2000 km and the larger one is 3000 km x 2010 km; the aspect ratio of the larger basin is quite similar to that of the North Atlantic between 20°N and 60°N. The parameter dependence of the model solutions and their spatio-temporal variability subject to time-independent wind stress forcing serve as the reference against which the results for time-dependent forcing are compared.

For the time-dependent forcing case, three zonal-wind profiles that mimic the seasonal cycle are considered in this study: (1) a fixed-profile wind-stress forcing with periodically varying intensity; (2) a wind-stress profile with fixed intensity, but north–south migration of the mid-latitude westerly wind maximum; and (3) a north–south migrating profile with periodically varying intensity. Results of the small-basin simulations show the intrinsic variability found for time-independent forcing to persist when the intensity of the wind forcing varies periodically. It thus appears that the physics behind the upper ocean’s variability is mainly controlled by internal dynamics, although the solutions’ spatial patterns are now more complex, due to the interaction between the external and internal modes of variability. The north–south migration of wind forcing, though, does inhibit the inertial recirculation; its suppression increases with the amplitude of north–south migration in the wind-stress forcing.

Model solutions in the larger rectangular basin and at smaller viscosity exhibit more realistic recirculation gyres, with a small meridional-to-zonal aspect ratio, and an elongated eastward jet; the low-frequency variability of these solutions is dominated by periodicities of 14 and 6–7 years. Simulations performed in this setting with a wind-stress profile that involves
seasonal variations of realistic amplitude in both the intensity and the position of the atmospheric jet show the 7-year periodicity in the oceanic circulation to be robust. The intrinsic variability is reinforced by the periodic variations in the jet’s intensity and weakened by those in meridional position: the two effects cancel, roughly speaking, thus preserving the overall characteristics of the 7-year mode.
1 Introduction

The wind-driven, basin-scale circulation in the mid-latitude upper ocean exhibits variability on different time scales. This variability could be due to changes in the external atmospheric forcing or to the system's intrinsic instability and nonlinearity. The latter explanation was first put forward by Veronis (1963, 1966), but the former has enjoyed greater popularity in the oceanographic community until fairly recently. A systematic application of the methods of dynamical systems theory to the wind-driven circulation problem has yielded several physical mechanisms for the observed low-frequency variability, on the time scale of several months to several years (Dijkstra and Ghil, 2005).

This circulation has been studied in the last decade with a hierarchy of models that include rectangular-basin, as well as more realistic configurations. Pedlosky (1996) reviewed work that used time-independent, single-gyre forcing. Single-gyre results of particular interest since then include Meacham and Berloff's (1997a, b) work that emphasizes the role of basin size in this problem, and Sheremet et al.'s (1997) classification of instabilities for the single gyre. Chang et al. (2001) reviewed double-gyre results and compared the physical mechanisms for intrinsic variability found in this case with those obtained in the single-gyre problem. We only review here, therefore, the results that are most relevant to the present work. These emphasize the double-gyre problem.

Jiang et al. (1995; JJJG hereafter) studied the double-gyre wind-driven circulation in a mid-latitude rectangular basin on a $\beta$-plane using a 1.5-layer, reduced-gravity, shallow-water (SW) model. They used a time-constant zonal-wind profile, symmetric about the basin's mid-latitude axis. Their results showed that when nonlinear processes come into play, multiple steady solutions satisfying identical boundary conditions can arise for sufficiently strong
Two stable solutions were found to co-exist over a certain range of parameters. One had a stronger subpolar gyre and the other a stronger subtropical gyre, with overshooting of the western boundary currents to the north and south of the symmetry axis, respectively. Their periodic solutions had periods of several years and several weeks.

Following JJG, Speich et al. (1995) analyzed the dependence of the stationary solutions on the SW model's nondimensional parameters such as the amplitude of the forcing, the Ekman number, the Rossby number, the nondimensional $\beta$ parameter, and the bottom drag coefficient. Using pseudo-arclength continuation, they showed systematically how model solutions vary in number, stability and spatial features with the parameters and confronted their time-dependent solutions with observational data.

Using a 1.5-layer quasi-geostrophic (QG) model, Cessi and Ierley (1995) also found multiple equilibria for the double-gyre circulation. They found antisymmetric, as well as asymmetric solutions. Some of their solutions fill the whole basin, while others decay rapidly to zero away from the western shore of the ocean basin. Many features of the bifurcation sequence in the double-gyre problem have since been shown to be independent of model details: QG vs. SW dynamics, domain shape and size, etc.; see, for instance, Dijkstra et al. (1999), Dijkstra (2005) or Dijkstra and Ghil (2005).

Ghil et al. (2002a) used a QG two-mode model and compared its results with those obtained by Dijkstra and Katsman (1997) in a QG model with 1.5 and 2 layers. Ghil and colleagues investigated in more detail the dipole region in the double-gyre problem. They obtained three-dimensional analytical solutions for the flow in the dipole region and found them to agree in their main properties with symmetric and asymmetric solutions obtained numerically in the QG two-mode model, as well as with the corresponding solutions of an SW model.
Simonnet et al. (2003a, b) studied low-frequency variability of the large-scale mid-latitude ocean circulation with a 2.5-layer wind-driven, SW model, using both rectangular and more realistic bathymetry. The bifurcation structure was numerically investigated in rectangular geometry using pseudo-arclength continuation along the branches. Simonnet and colleagues found the system to exhibit homoclinic orbits that induce transitions from upper- to lower-branch solutions at high forcing levels. Simonnet et al. (1998) also performed a set of forward integrations in a rectangular domain that resembles in size the North Atlantic basin between 20° N and 60° N, as well as in a domain that roughly reproduces the shape of the eastern and western coasts of the North Atlantic between these two latitude circles. These simulations were conducted at horizontal resolutions as high as 10 km and yielded two modes of variability, around 6 years and 20 months; see also Simonnet et al. (2003b).

Rhines and Schopp (1991) studied the effect of wind-stress forcing that tilts away from the zonal, eastward direction. When increasing the northward tilt of the winds away from this direction, they found that the total energy of the circulation decreases, as does the penetration length of the eastward jet, while more cut-off rings are produced near the western boundary, and the boundary current’s separation point moves poleward of the line of vanishing Ekman pumping.

All these studies considered wind-stress forcing that is constant in time. In reality, winds vary in time on seasonal, as well as on subannual and interannual time scales. These variations include north–south migration of the mid-latitude jet axis, as well as changes in the jet's intensity (Strong et al., 1993). Liu (1996) investigated the effect of seasonal wind migration on the inertial recirculation using QG models. He showed that the seasonal wind migration can suppress the inertial recirculation substantially for the barotropic case. According to his study, the two key dynamic conditions for the suppression are: (1) the
mismatch between the formation time scale of the western boundary current and the recirculation time; and (2) the interaction between the two neighbouring recirculation cells, which is related to chaotic intercell transport. Yang (1996), using a simple barotropic, double-gyre model driven by surface wind, also showed the mass exchange between the subtropical and subpolar gyre to be substantially enhanced when the wind forcing is allowed to migrate north and south.

The aim of the present paper is to study the differences in the mid-latitude ocean response to time-dependent vs. time-independent wind-stress forcing. More specifically, we wish to investigate the different modes of variability and the associated spatio-temporal patterns for time-dependent vs. time-independent wind forcing. The model and methodology are described in Section 2. In Section 3, time-independent wind forcing is considered and the temporal and spatial features of the flow are presented for two different sized basins. In Section 4, we turn to wind-stress forcing that is allowed to vary periodically in time, in intensity only, in spatial pattern only and finally as a combination of the previous two. The methods developed in earlier work for the analysis of the flow in the case of time-independent forcing are put to good use in this section too, in order to compare the results in the two cases. The main aspects of this comparison are summarized in Section 5.

2 Model and diagnostic methods

2.1 Shallow-water (SW) model and wind-stress forcing

We use in this study a 1.5-layer SW model with an active upper layer of depth $H$ and an inert bottom layer of infinite depth; details of the model can be found in JJG. The model domain is a mid-latitude rectangular basin on the $\beta$-plane, given by $0 \leq x \leq L$ and $0 \leq y \leq D$. 
The dynamics is governed by the reduced-gravity, SW equations in flux form:

\[
\frac{\partial U}{\partial t} + \nabla .(Uv) = -g' \frac{\partial H}{\partial x} + fV + \alpha_A A \nabla^2 U - RU + \alpha_\tau \frac{\tau_x}{\rho}, \quad (1a)
\]

\[
\frac{\partial V}{\partial t} + \nabla .(Vv) = -g' \frac{\partial H}{\partial y} - fU + \alpha_A A \nabla^2 V - RV, \quad (1b)
\]

\[
\frac{\partial H}{\partial t} = -\frac{\partial U}{\partial x} - \frac{\partial V}{\partial y}, \quad (1c)
\]

here

\[
Ui + Vj = H\nu = H(\text{ui} + \text{vj}) \quad (2)
\]

is the upper-layer mass-flux vector, while \( u \) and \( v \) represent, respectively, the zonal and meridional velocities in the upper layer. The Coriolis parameter is \( f = f_0 + \beta y \), \( g' = g(\Delta \rho / \rho) \) is the reduced gravity, and \( \tau_x \) is the zonal wind-stress that drives the upper ocean. The system is subject to wind-stress forcing \( \tau_x \), to Rayleigh bottom friction with coefficient \( R \), and to Laplacian lateral diffusion with eddy viscosity \( A \). The system's behavior is mainly determined by \( \alpha_\tau \) and \( \alpha_A \), the two nondimensional parameters that control the strength of external wind forcing and lateral viscosity, respectively. The spatio-temporal complexity of the system's behaviour increases with increasing \( \alpha_\tau \) and decreasing \( \alpha_A \).

The model parameter values used in this study are given in Table 1a. Simulations are performed for two different sized basins, 1000 km x 2000 km (as in JJG and in many other QG and SW model studies) and 3000 km x 2010 km. The grid size we use is \( \Delta x = \Delta y = 12.5 \) km for the smaller basin, and \( \Delta x = \Delta y = 15.0 \) km for the large basin. The time step is \( \Delta t = 10 \) min, for both cases. The horizontal resolution for the smaller basin is higher than in JJG and Speich et al. (1995), who used \( \Delta x = \Delta y = 20 \) km, and comparable to that of recent eddy-resolving simulations with ocean general circulation models (Chao et al., 1996). The viscous
boundary width according to Munk theory is \( \delta_m = (\alpha A / \beta)^{\frac{1}{3}} \) (Pedlosky, 1987); for the values of \( A \) and \( \beta \) used in most of the simulations presented in this study, \( \delta_m \) is approximately 21 km in the mid-latitudes for the small basin, and 27 km for the large one. Hence our grid size of 12.5 km for the smaller basin and 15.0 km for the larger one allows marginal resolution of the Munk layer, as well as of typical mid-latitude ocean eddies. Besides the model parameters that are kept constant and given in Table 1a, we systematically vary the control parameters \( \alpha_t \) and \( \alpha_A \) over the range given in Table 1b.

Most of the small-basin simulations described here use no-slip boundary conditions, while all the larger-basin experiments use free-slip conditions. The wind-stress forcing used in Section 3 is of the form

\[
\tau_x = -\tau_0 \cos \left( \frac{2\pi y}{D} \right)
\]

and it has, therewith, zero net vorticity forcing. In Section 4, we prescribe variations in the wind-stress forcing with time that are informed by the wind data made available on TOGA CD-ROMs. These data sets were obtained from the National Center for Atmospheric Research (NCAR) and are provided on a \( 2^\circ \times 2^\circ \) grid over the entire globe.

We analyzed six years of wind data (1985–1990) and found changes in wind-stress magnitude, as well as a north–south shift in the wind-stress pattern. Based on the analysis of these observational data, we chose three idealized wind-stress patterns that allow: (1) changes in the wind-stress magnitude alone, i.e., a standing oscillation in the wind-stress pattern; (2) a north–south migrating wind-stress pattern; and (3) a combination of (1) and (2) above.

The wind-stress pattern of the standing oscillation is given by

\[
\tau_x = -\tau_0 \cos \left( \frac{2\pi y}{D} \right) \times \left[ 1 + a \cos \left( \frac{2\pi}{T_{amp}} \right) \right],
\]
while the north–south migrating pattern has the form

\[ \tau_x = -\tau_0 \cos \left( \frac{2\pi}{D} \left[ y - b \sin \left( \frac{2\pi}{T_{\text{mig}}} \right) \right] \right); \quad (5) \]

here \( T_{\text{amp}} \) is the period of the standing oscillation and \( T_{\text{mig}} \) that of the north–south migration. In the present paper, \( T_{\text{amp}} = T_{\text{mig}} = T = 12 \) months, but longer and possibly distinct periods could be investigated in the future. Furthermore, \( a \) is the peak-to-peak change in the magnitude of wind-stress forcing at the latitude of the zero wind-stress curl, while \( b \) is the range of migration of this latitude.

Finally, the north–south migrating profile with periodically varying intensity is given by

\[ \tau_x = -\tau_0 \left[ 1 - \frac{a}{2} \sin \left( \frac{2\pi}{T_{\text{amp}}} \right) \right] \cos \left( \frac{2\pi}{D} \left[ y - b \sin \left( \frac{2\pi}{T_{\text{mig}}} \right) \right] \right), \quad (6) \]

This pattern mimics the one used by Strong et al. (1993), based on their analysis of European Centre for Medium Range Weather Prediction (ECMWF) wind profiles, with the wind-stress intensity at the line of zero wind-stress curl having its minimum when in its northernmost position and maximum in its southernmost position.

### 2.2 Control parameters and analysis methods

As stated in the previous subsection, the main control parameters are \( \alpha_\tau \) and \( \alpha_A \). Most of the study uses no-slip conditions at the lateral boundaries for the small basin, but the effect of free-slip conditions is also examined in several cases. Following Ghil et al. (2002a), we also examine model sensitivity with respect to the Rossby radius of deformation \( R_d = (g' H)^{\frac{1}{2}} / f \) for the small basin, which equals about 80 km according to Table 1a.
Parameter dependence of model solutions is examined by using the bifurcation diagram, in which dependence on one parameter is tracked while all others are kept fixed. Individual solutions, at fixed parameter values and for given boundary conditions, are studied in detail using several dynamical and statistical methods. Following Chang et al. (2001) and Simonnet et al. (2003a,b), we construct the bifurcation diagram by plotting the normalized transport difference $TD$ with increasing wind-stress forcing, where $TD$ is given by

$$TD = \frac{|\psi_{po}| - |\psi_{tr}|}{\max|\psi|};$$

(7)

here $\psi_{po}$ and $\psi_{tr}$ are the negative and positive extrema of the streamfunction in the subpolar and subtropical gyres, respectively. Positive $TD$ points to a stronger subpolar gyre and negative $TD$ to a stronger subtropical gyre, hence the names of subpolar and subtropical branch, respectively. Chang et al. (2001) showed that this choice of $TD$ yields a bifurcation diagram where the upper and lower branches correspond to those of JJG, although JJG constructed their bifurcation diagram by plotting the confluence point, i.e. the merging point of the two separated western boundary currents, with increasing wind-stress parameter $\alpha$. We shall use therefore interchangeably the terms upper, positive or subpolar and lower, negative or subtropical branch.

To obtain the $TD$-vs.-$\alpha$ plot, the system of equations (1a)–(1c) is first integrated forward in time for a given $\alpha$ value from a state of rest until either a steady state or a statistically stationary regime is reached. The transport difference $TD$ is then calculated for that steady state, cf. Eq. (7), from the positive and negative values of the streamfunction that are maximal in absolute value; this steady state is then used as the initial state for the next integration at a slightly higher $\alpha$ value, all other parameters remaining unchanged. When the
asymptotic solution is time dependent, the range of $TD$ values is measured, and an arbitrary solution state within this range is used as the initial state at higher $\alpha$. This “poor man's continuation method” (see Quon and Ghil, 1992, 1995; Chang et al., 2001; Dijkstra and Ghil, 2005) is used here for getting points along each solution branch of a bifurcation diagram.

The type of an individual solution — steady, periodic or chaotic — is determined by examining its $TD$ time series. In particular, we use the SSA-MTM Toolkit (Ghil et al., 2002b) to study the temporal characteristics of the time-dependent solutions. Each time series is prefiltered data-adaptively by projection onto the leading empirical orthogonal functions (EOFs) provided by its singular-spectrum analysis (SSA). The prefiltered time series is then analyzed by the maximum-entropy method (MEM: Penland et al., 1991) and the results are further checked by the multi-taper method (MTM: Thomson, 1982; Mann and Lees, 1996) or the Monte Carlo version of SSA (Allen and Smith, 1996).

For the less viscous regimes investigated in the larger rectangular basin, the asymmetric energy, defined as

$$\Delta E = (H^- - H_0)^2 - (H^+ - H_0)^2,$$

is preferred to $TD$. Here $H^+$ and $H^-$ correspond to values of upper-layer thickness $H$ greater or less than $H_0 = 500$ m, respectively.

To describe spatial circulation patterns, we use the streamfunction $\psi$, rather than the horizontal velocity vector $(u, v)$ of the model's active upper layer. The former is obtained from the latter by solving the Poisson equation

$$\nabla^2 \psi = u_y - v_x,$$

subject to the appropriate boundary conditions that correspond to no slip or free slip.
Spatio-temporal variability of solutions and their dynamics are studied by several methods. Spatial EOF analysis of solutions follows Ghil (1987), Mo and Ghil (1987) and Berloff and McWilliams (1999). A more detailed analysis of the dynamics of different regions in the flow domain relies on the use of scatter plots of potential vorticity ($PV$) vs. streamfunction $\psi$; the $PV$ is given by

$$PV = \frac{\nabla^2 \psi + f_0 + \beta y}{H}. \quad (10)$$

The $PV$-$\psi$ scatter plots (Griffa and Salmon, 1989; Ghil et al., 2002a) allow us to distinguish, in a quantitative and objective way, between the two basin-scale gyres, the western boundary currents, and the recirculation dipole.

### 3 Time-independent wind stress forcing

As mentioned earlier, experiments have been performed for two rectangular basins of different sizes, i.e. 1000 km x 2000 km and 3000 km x 2010 km. Analysis for the small basin will be presented first, followed by that for the larger basin.

#### 3.1 Small rectangular basin

**a. Parameter dependence of solutions**

The bifurcation diagram for $TD$-$vs.-\alpha_4$ is computed at the higher resolution used here and is shown in the central panel of Fig. 1 for fixed $\alpha_A = 1.3$, $R_d = 80$ km, and no-slip boundary conditions. This diagram serves as the reference against which the bifurcation diagram for time-dependent forcing is compared in Section 4. JJG's bifurcation diagram (see Fig. 3 there) differs in the physical quantity plotted on the ordinate — merging-point latitude there vs. $TD$ here. Still, both diagrams represent essentially the same qualitative changes in solution.
behaviour: from a unique steady state to multiple ones, from each steady state to a periodic solution, and from each of these on to an aperiodic one, as briefly discussed below.

For very low wind stress values, $TD = 0$, and a cyclonic subpolar gyre lies to the north and an anticyclonic subtropical gyre to the south of the symmetry axis. At $\alpha^* = 0.47$, i.e., point $SB$ in Fig. 1, the subpolar gyre gains strength and leads to a steady positive branch. The stronger subpolar vortex pulls the subtropical recirculation vortex further north, thereby deflecting the jet from a purely eastward to a northeastward direction. As $\alpha^*$ increases, a first bifurcation to multiple steady states occurs at $\alpha^* \approx 0.7$, marked by $P$ in the diagram. The negative branch is characterized by a stronger subtropical gyre and a jet flowing southeastward. Comparison of the analytic solutions for a low-order QG model with the numerical results of their SW model led JJG to conclude that point $P$ corresponds to a perturbed pitchfork bifurcation.

Both the positive- and negative-$TD$ branch lose their stability to periodic solutions through Hopf bifurcation, at $\alpha^* = 0.865$ for the negative branch and at $\alpha^* = 1.0$ for the positive one; the two bifurcation points are marked by $H^-$ and $H^+$, respectively. Upon increasing the wind stress further, the periodic solutions give way to aperiodic ones. On the negative branch this transition occurs at $\alpha^* = 1.15$ and on the positive branch at $\alpha^* = 1.24$. Both Hopf bifurcations, as well as the transitions to aperiodic solutions, occur at lower $\alpha^*$ values than in JJG. This shift to lower $\alpha^*$ is consistent with the higher resolution used in the present model version, which leads to greater intrinsic variability at the same parameter values.

The time series of transport difference $TD$ are shown for points on the positive and negative branches in the four smaller, top and bottom panels of Fig. 1, respectively. The time
series in the upper-left panel is for a point on the positive branch immediately after the Hopf bifurcation ($\alpha = 1.08$), while the one on the upper-right is for a point just before the aperiodic regime on the same branch ($\alpha = 1.2$). The spectra of the time series calculated using the SSA-MTM Toolkit (Dettinger et al., 1995; Ghil et al., 2002b) show a subannual oscillation period of 8 months for the positive periodic branch. Subannual oscillation periods for the positive branch were also found by JJG, Speich et al. (1995), Dijkstra and Katsman (1997) and Chang et al. (2001).

The negative branch here undergoes two successive Hopf bifurcations. The time series in the bottom-left panel of Fig. 1 is for a point $\alpha = 0.9$ on the negative branch immediately after the first Hopf bifurcation, while the one at bottom-right is for a point just after the second Hopf bifurcation at $\alpha = 1.1$. The first Hopf bifurcation leads to a limit cycle with a period of about 10 months and the second one to a limit cycle with a period of about 21 months. The interaction between the two oscillations, with their incommensurable periods, leads to the fairly irregular, albeit still quasi-periodic solution in the lower right panel of Fig. 1. JJG and Speich et al. (1995) obtained interannual periods on the negative branch of the bifurcation diagram, but found no subannual oscillations on this branch. The physical mechanisms behind the latter oscillations are discussed in Section 3.1b.

b. Spatio-temporal variability of the solutions

(i) Empirical orthogonal function (EOF) analysis. EOF analysis of the streamfunction anomalies was performed to identify the dominant spatial patterns and the underlying instability mechanisms for the upper and lower branches.
The two leading EOFs contribute more than 80% of the solutions' variability on the lower branch, at $\alpha_t = 0.9$, after the first Hopf bifurcation (Figs. 2a, b). Their spatial patterns show strong low-frequency variability far away from the recirculation zone, in parts of the domain where the mean flow itself is less intense. The spectra of the PCs associated with these two EOFs (Figs. 2c, d) are both dominated by a period of 10 months. This subannual variability along the negative branch was not captured before in studies which used the same model with no-slip boundary conditions (JJG; Speich et al., 1995), although it did appear on both branches of Chang et al.'s (2001) QG model. The latter authors showed that the period of this mode increases with $\alpha_t$, up to interannual periods similar to those found in JJG. We suspect that it is the higher horizontal resolution used in the present study that has enabled us to capture this variability at lower $\alpha_t$ values.

The spatial patterns in Figs. 2a, b suggest that the oscillation's physical mechanism is a somewhat localized and shifted version of the gyre mode first identified by JJG and Speich et al. (1995) and described in greater detail by Chang et al. (2001), Simonnet and Dijkstra (2002) and Simonnet et al. (2003b). The vorticity concentration off the western boundary, well within the subpolar gyre, plays the role of the recirculation dipole in generating this oscillation.

A second Hopf bifurcation off this lower branch at $\alpha_t = 1.1$ leads to an interannual oscillation with a period of 21 months. The anomalies in the northern half of the basin still dominate but strong anomalies start to appear in the recirculation region as well. The variability in the recirculating region is associated with changes in the strength and orientation of the vorticity dipole. JJG and Speich et al. (1995) both found this variability in the recirculation region with their lower-resolution version of the same model. Figure 3 displays the spatial patterns of the 4 leading EOFs at $\alpha_t = 1.1$; they contribute over 80% of the total
variability of the solution. EOFs 1 and 2 still resemble those at $\alpha_\tau = 0.9$ (see Figs. 2a, b), while the second EOF pair resembles quite well the internal gyre mode of JJG and subsequent authors. The dominant peak of the spectra of the first pair of PCs (not shown) agrees with that of Figs. 2c, d. The second-pair spectrum is dominated by an interannual peak, as expected from the spatial patterns in Figs. 3c, d.

We now turn our attention to the upper branch. The spatial patterns of the first four leading EOFs (contributing 80% of the total variability) for $\alpha_\tau = 1.08$, after the Hopf bifurcation $H^+$, are shown in Fig. 4. The two leading EOFs of this upper-branch solution are dominated by an elongated wave pattern in the northeast–southwest direction, resembling those obtained by JJG and Speich et al. (1995) for their subtropical branch in the SW model.

To check the robustness of the spatial patterns associated with the periodic solutions we also carried out integrations with free-slip boundary conditions. These runs necessitated lower values of $\alpha_\tau$ and higher values of $\alpha_A$, in order to stay within the periodic regime. The spatial patterns of the first four EOFs for a solution on the lower branch, just past the first Hopf bifurcation, are shown in Fig. 5, for $\alpha_\tau = 1.0$ and $\alpha_A = 3.5$. They are clearly different from the analogous EOFs obtained with no-slip boundary conditions in Figs. 2a, b and 3: for free-slip boundary conditions, the variability is mostly concentrated in the recirculation region, along both the subpolar and subtropical (i.e., upper and lower) branches (not shown for the former). For the no-slip case along the lower branch, the variability is strongest in the northern part of the basin, away from the recirculation zone. This does not, however, seem to be the case in the North Atlantic or the North Pacific, where low-frequency variability is mostly observed in and near the meandering-jet region of the Gulf Stream or Kuroshio. Since the details of the flow near the boundary are not really resolved by such a simple model, or even
by most ocean general circulation models (Ghil et al., 2002a; Dijkstra and Ghil, 2005), the
greater realism of the overall simulation suggests that free-slip or “partial-slip” boundary
conditions might be preferable for our model (see also Appendix A of JJJ).

(ii) Relation between potential vorticity (PV) and streamfunction. Ghil et al. (2002a) showed
that, in the recirculation region, the points of the $PV$-vs.-$\psi$ scatter plot cluster very close to a
straight line, with regression coefficients larger than 0.99 in absolute value, independently of
the flow regime (steady, periodic or aperiodic). Points outside the dipole region were found to
be spread over a finite area of the $PV$–$\psi$ plane. Griffa and Salmon (1989) also noted this
distinction.

The long-term average $PV$-vs.-$\psi$ scatter plots for points on the upper and lower
branches, for steady, periodic and aperiodic cases, are shown in Fig. 6, in the top and bottom
panels respectively. In each plot the points from the dipole region lie very close to a curve
whose tangent has a negative slope at each of its points. This confirms the dominance of the
inertial and $\beta$ terms in the dynamic balance of the recirculation zone and the fact that the flow
in this region is well approximated by a free mode, with a one-to-one relationship $PV = F(\psi)$
(Stern, 1975; Larichev and Reznik, 1976; Salmon, 1998). As shown in greater detail in Fig. 12
of Ghil et al. (2002a), points from the Sverdrup region outside the dipole fill the two
curvilinear triangles between the $PV = F(\psi)$ curve (red both here and there) and the nearly
vertical curve associated with points that lie near the western boundary (blue in both papers).

The difference between the scatter plots for the two branches is mainly in the extent of
the downward-sloping curve that represents the recirculation zone. The strong recirculation
cell in the subpolar part of the basin is characterized by the extended, slightly curved line in
the upper-left part of the scatter plots (top panels), while the subtropical cell is represented by
the extended line in the lower-right part of the plots (bottom panels). For the upper branch, the upper-left part of the straight line extends further due to the presence of the stronger subpolar gyre; similarly for the lower branch the straight line extends further down and right.

In the top panels of Fig. 6, the strength of the subpolar recirculation cell is much higher (note the $\psi$ values) than that of the southern recirculation cell, especially in the aperiodic regime. Ghil et al. (2002a) showed that, for steady-state solutions at least, the average slope of the line that represents the recirculation region in the $PV$-vs.-$\psi$ plot decreases as the wind stress increases and the solutions become more nonlinear.

The effect of Rossby radius of deformation on the $PV$-vs.-$\psi$ scatter plot is shown in Fig. 7. The scatter plots correspond to Rossby radii of deformation, $R_d = 80, 100, 120$ and 140 km (see Table 1b), for $\alpha_t = 1.0$ and $\alpha_A = 1.3$ on the lower branch. The subtropical gyre is stronger, and the flow is quasi-periodic for all four cases. An increase in the recirculation is noted with increasing $R_d$. The points in the dipole region fall approximately on a straight line, the slope of which decreases in absolute value as $R_d$ increases.

3.2 Large rectangular basin

To better compare our numerical results with the observed circulation of an ocean basin, we performed a numerical integration of system (1a)–(1c) for the larger rectangular basin of 3000 km x 2010 km, whose aspect ratio is quite similar to that of the North Atlantic between 20°N and 60°N. Free-slip boundary conditions and a reduced lateral viscosity of $A = 200 \text{ m}^2\text{s}^{-1}$ were used to further increase the realism of the simulated fields. As indicated in Section 2, $\Delta x = \Delta y = 15.0$ km for this larger-basin simulation, with all other parameters given in Table 1.

The model was run for 500 yr from a state of rest for $\alpha_t = 0.3$ and $\alpha_A = 1.0$. The long-
term average streamfunction (Fig. 8a) exhibits a fairly smooth Sverdrup interior in the eastern part of the basin and an elongated jet with small meridional-to-zonal aspect ratio. The separation and meandering of the jet, as well as the size and strength of the subpolar versus the subtropical gyre, are much more realistic with the configuration of this larger basin compared with the small basin. The long-term average $PV$-$\psi$ scatter plot is shown in Fig. 8b, with points corresponding to the recirculation region lying along a smooth, almost rectilinear curve, as discussed in Section 3.1b.

The time series of asymmetric energy $\Delta E$ of the system in Fig. 8c leaves out the first 20 yr of the 500 yr of simulation and varies quite irregularly. SSA was applied to this series using a window width of 48 yr = 576 months and Fig. 8d shows the maximum-entropy spectrum of the series projected onto the sum of its 12 leading reconstructed components (Ghil et al., 2002a); the two peaks correspond to oscillation periods of 7 and 14 yr, respectively. Eight equally spaced phases of the 7-yr oscillation, reconstructed with the leading pair of modes, appear in Fig. 9. The plots show the anomalous upper-layer thickness $H$, i.e. the difference between the field in the corresponding phase and the mean over the oscillation; the compositing of these eight phases follows the methodology of Moron et al. (1998, see their appendix). Similar compositing of eight equally spaced phases of the 14-yr oscillation is presented in Fig. 10. A southward propagation can be noticed in the figures.

The 7-yr oscillation was reported in several earlier model studies, while Moron et al.’s (1998) observational study found both 7-yr and 14-yr modes in the North Atlantic’s sea surface temperatures (SSTs). Simonnet et al. (2003b) noted a 7-yr oscillation in their simulations using a 2.5-layer SW model in a North-Atlantic-shaped basin. Speich et al. (1995) already found a 7-yr peak in the variability of the Gulf Stream axis position, as deduced from 40 yr of Cooperative Ocean-Atmosphere Data Set (COADS) data. Figure 9 suggests that the
7-yr oscillation here could also be linked to the variability of the meridional position of the jet axis.

4 Time-dependent wind-stress forcing

We have thus verified the results for time-independent forcing of the present model version against JJG and subsequent models of different types, domain sizes and resolutions, and found them in good overall agreement. We now turn to the effect of time-periodic forcing on double-gyre flows. As in the previous section, analysis related to the smaller domain will be presented first, followed by those for the larger domain.

4.1 Small rectangular basin

Except for varying $\alpha$, other parameter values are as in Fig. 1; no-slip boundary conditions were used in general. However, free-slip boundary conditions were also used in a few simulations; in these cases $\alpha = 3.5$, to stay in the same regime as the no-slip simulations. In Section 4.1.1 we use the idealized wind-stress pattern with time-periodic wind magnitudes given by Eq. (4), while the effect of north–south migration of the wind-stress forcing on the flow is studied in Section 4.1.2 using the time-varying wind-stress pattern of Eq. (5).

4.1.1. Standing oscillation in the wind-stress forcing

As mentioned in Section 2.1, we take the oscillation period of the magnitude of the wind-stress forcing in Eq. (4) to be $T_{\text{amp}} = T = 12$ months. We take the amplitude factor, in Eq. (4), $a = 0.1$, based on our analysis of observed wind data from NCAR. The bifurcation diagram as
the mean wind-stress forcing $\alpha$ increases is constructed and discussed in the following subsection.

### a. Solution dependence on mean wind-stress intensity

The bifurcation diagram with increasing $\alpha$ is plotted for the normalized transport difference $TD$ and no-slip boundary conditions in the central panel of Fig. 11. For very low values of $\alpha$, only a positive subtropical branch is found, as in the case of time-independent wind-stress forcing (see Fig. 1). The solution here is periodic, however, due to the time-dependent wind forcing. Hence the maximum and minimum values of $TD$ for each $\alpha$ are plotted in the bifurcation diagram. The period of oscillation for these low values of $\alpha$ is the same as the periodicity of the wind forcing, i.e. one year.

At $\alpha = 0.7$, a negative $TD$ branch emerges due to a perturbed pitchfork bifurcation, as in Fig. 1. The location of this bifurcation does not seem to be affected by the periodicity of the forcing, but we still denote it by $\tilde{P}$, to distinguish the bifurcation points in Fig. 11 from those in Fig. 1. As before, the negative branch has a stronger subtropical and a weaker subpolar gyre, but the area covered by each gyre is in the opposite ratio: larger for the subpolar and smaller for the subtropical gyre (not shown). The oscillation period is still 1 yr on both branches. The periodic solutions on the two branches coexist in the range of $\alpha = 0.7–0.85$.

On the upper branch, the solution is quasi-periodic for $\alpha$-values between 0.95 and 1.05, and it becomes aperiodic for $\alpha$ greater than 1.05. The corresponding time series, for periodic and quasi-periodic behaviour, are shown in the two left upper panels of Fig. 11. Spectral analysis results for the upper branch indicate that, throughout the quasi-periodic regime, in addition to the forced oscillation with its 12-month period, there exists another
prominent oscillation, with a period of about 4 months. A 4-month oscillation was also obtained in the steady wind-stress forcing case for the upper branch and it is intrinsic to the system. The mean position of the jet axis, as well as the amplitude of the meanders in it, change over the 4-month period, as they do in the steady-forcing case (not shown).

Two modes of internal variability arise along the lower branch, as for the steady-forcing case: one has a 10-month period, the other one a 6-yr period; the 10-month oscillation was noted in the steady-forcing case as well. At $\alpha = 0.95$ the lower branch solutions become chaotic. The $TD$ time series for the periodic and quasi-periodic regimes on the lower branch (two left bottom panels of Fig. 11) are more complex than for the upper branch, due to the presence of the additional mode of internal variability. The 6-yr mode is clearly apparent in the quasi-periodic regime (bottom-middle panel), as an amplitude modulation of the seasonal cycle.

The rightmost top and bottom panels of Fig. 11 display the aperiodic regime associated with either branch, while Fig. 12 shows the aperiodic behaviour for $\alpha = 2.0$, on the lower branch; the corresponding $TD$ and $KE$ time series appear in the top and middle panel of the latter. The instantaneous $PV$-vs.-$\psi$ scatter plot for points A and B are associated with positive and negative $TD$ values, which in turn correspond to low and high $KE$ values; they are shown in the bottom panel of Fig. 12. These scatter plots appear distorted with respect to those in Fig. 6, due to the several secondary vortices that arise close to the western boundary (not shown).

**Free-slip boundary conditions.** Simulations were also performed with free-slip boundary conditions, at $\alpha_A = 3.5$ (see above). We explore the nonlinear interaction of the internal mode with the annual forcing for increasing $\alpha$. Along the lower branch, the model follows the universal quasi-periodicity route to chaos as the nonlinearity $\alpha$ is increased; see Fig. 13. The
KE–PE phase-space plots are shown in the figure’s left panels, for selected values of $\alpha$ in the range 0.995 to 1.033, while the right panels show the corresponding time series of $TD$.

For small values of $\alpha$ ($\alpha \leq 0.995$), the $TD$ time series is perfectly periodic, having the seasonal period of forcing. As $\alpha$ increases, a second frequency arises, which is rationally unrelated with the annual frequency; the superposition of the two incommensurable frequencies creates a quasi-periodic time series ($\alpha = 0.999$), which covers the torus (in red in the left-top panel) densely. For several strong nonlinearities, i.e at $\alpha = 0.997$ (not shown) and at $\alpha = 1.01$ (green in the left-top panel), the $TD$ series is mode-locked to simple rational multiples of the driving frequency, so that the inherent frequency equals 1/2 and 1/3, respectively (i.e., periods of 2 and 3 years). The further results at $\alpha = 1.02$ (black), $\alpha = 1.03$ (red), and $\alpha = 1.033$ (green), shown in the bottom panels, are all consistent with the presence of a “Devil’s staircase” (Jin et al. 1994, 1996; Tziperman et al. 1994) for this periodic wind-stress forcing of the standing oscillation type.

b. Spatio-temporal variability of the solutions

The interaction between the externally forced variability and the modes of internal variability gives rise to fairly complex solution behaviour. We study first the upper branch, for the no-slip case, which is simpler.

The spatial patterns of the two leading EOFs are shown in the top panel of Fig. 14 for this branch in the purely periodic, forced regime. These patterns cover a larger area of the basin than the corresponding ones in Fig. 4, where the periodicity is internally generated. Otherwise, though, the patterns are similar, with several closed features aligned in a wave pattern whose tilt changes from EOF-1 to EOF-2. While the overall patterns are more basin-
wide, the strongest features here cluster more closely near the separation and merger points of the two western boundary currents. The transport difference $TD$ reaches its maximum when $|\psi_{tr}|$ is at a minimum and $|\psi_{po}|$ is of intermediate strength (bottom panel of Fig. 14). The minimum of the basin's kinetic energy occurs when the meandering of the jet axis is at its maximum, as in the case for steady forcing (McCalpin and Haidvogel, 1996; Berloff and McWilliams, 1999; Ghil et al., 2002a).

The spatial patterns for the two leading EOFs in the quasi-periodic regime on the upper branch (Fig. 15) are also basin-wide, as in Fig. 14, but the variability is concentrated even more strongly in and near the recirculation zone. Some of the features are inherited from the internal variability. The transport difference $TD$ is again a maximum when $|\psi_{tr}|$ is a minimum, while $|\psi_{po}|$ is now close to its maximum value (not shown). In the aperiodic regime (not shown), the variability fills the rectangular basin, but its features have even smaller scales.

The more complex lower negative branch is studied next. When the oscillation is purely forced, and hence has a one-year period, the spatial patterns obtained from the EOF analysis (Fig. 16) are most intense in the northern part of the basin, as in the steady-forcing case (see Figs. 2 and 3), but extend much further across the basin.

The spatial patterns of the four leading EOFs (Fig. 17) on this branch in the quasi-periodic regime all show dominant variability in the poleward part of the basin. The strengths of the subpolar and subtropical gyre increase and decrease together, and the amplitudes of their respective oscillations are roughly the same. In this case, both the externally forced and the internally generated variability leave their imprint on the total variability: the seasonal forcing is manifest in the first pair of EOFs (Figs. 17a and 17b), which resembles Fig. 16 to a
certain extent, while the internal oscillation is clearly visible when comparing the second pair of EOFs in Fig. 17 with the first pair in Fig. 3.

4.1.2 Migrating wind-stress forcing

In this section we study the response to a smooth, periodic shift of the wind-stress pattern in the north–south direction, according to Eq. (5) of Section 2.1. For simplicity, we assume here that there is no change in the wind-stress magnitudes, i.e., constant $\alpha_r = 0.85$, with $\alpha_A = 1.3$ and $R_d = 80$ km, so that both branches of the steady-forcing diagram in Fig. 1 yield a stable steady solution.

Strong et al. (1993) have reported a shift by $20^\circ$ in the line of zero wind-stress curl, which coincides with the subtropical jet axis, from their analysis of the ECMWF climatology for 1980–86. This line is allowed to migrate in Eq. (5) by an amount equal to $b$, symmetrically to the north and to the south of the mid-basin axis. We have used three values of $b$, so that the total extent of the migration is $2b = 800$ km, 1600 km, and 3200 km. The period $T_{mig}$ of this pattern modulation is assumed to be 1 yr, $T_{mig} = T = 12$ months, as in Section 4.1.1.

We examine here the way that the migratory periodic forcing modifies the flow along the upper branch. Figure 18 shows the long-term average $PV$ and $\psi$ fields (top panels), as well as the $PV$-vs-$\psi$ scatter plots (bottom panels) for the three different oscillation amplitudes. The recirculation cells are still pronounced for small excursions of the atmospheric jet (see top panel of Fig. 18a) but are greatly weakened, to the point of total disappearance, for larger excursions of the jet (see top panels of Figs. 18 b, c, respectively). The visual evidence for this effect in the $\psi$ and $PV$ fields is confirmed by the scatter plots in the bottom panels of the figure. The intensity and penetration length of the oceanic jet is also found to decrease with
the migration amplitude, to the point where the eastward jet is totally absent in the long-term mean of the $PV$ field.

The $\psi$ and $PV$ fields indicate that wind-pattern modulation strongly affects the flows' spatio-temporal behaviour, especially in the recirculation region, where nonlinear effects are strong, in agreement with the earlier results of Liu (1996) and Yang (1996). The most striking effect of the imposed north–south shifts in the atmospheric jet is to greatly enhance $PV$ mixing. As the line of zero wind-stress curl moves north of the mid-basin axis, the subpolar gyre collapses, while the subtropical gyre grows. For a southward excursion of the prescribed atmospheric jet, the subtropical gyre is distorted and the subpolar gyre starts building up. In general, the recirculation cells are much weaker and smaller in extent than when the flow is driven by either a steady or an amplitude-modulated wind stress.

For small meridional shifts in the wind-stress pattern, the separation latitude of the jet depends strongly upon the position of the maximum wind-stress curl (not shown). Ozgokmen and Chassignet (1998) also observed such a dependency while investigating the effect of wind forcing on the eastward jet's separation in a basin with an oblique coastline, using a two-layer QG model.

### 4.2 Large rectangular basin

As in the small basin, we now consider time-dependent wind stress forcings that allow a standing oscillation in amplitude, cf. Eq. (4), or a north–south migration, cf. Eq. (5). In addition, time-dependent zonal-wind forcing that allows both changes in amplitude and north-south migration, cf. Eq. (6), is also studied for the larger basin. All simulations use free-slip boundary condition and all parameter values are as in Section 3.2.
4.2.1 Standing oscillation in the wind-stress forcing

Ten simulations were performed using amplitude modulation of the wind-stress forcing over the large basin, with \( a \) in Eq. (4) varying from 0.1 to 1.0, i.e., a 10% to 100% increase in the wind-stress intensity; \( \alpha = 0.3 \) for all simulations. The long-term average streamfunction has a fairly smooth Sverdrup interior in the eastern part of the basin (not shown), as for the time-independent case. However, instantaneous snap-shots of the streamfunction fields (not shown) reveal complex interactions among multiple vortices in the recirculation zone, near the detachment of the eastward jet from the western boundary. The root-mean-square (rms) plots of the streamfunction fields for \( a = 0.1 \) and 1.0 (Figs. 19b and 19c) clearly indicate maximum variability along the jet axis.

To compare with the time-independent case, SSA was applied to the time series of asymmetric energy \( \Delta E \), cf. Eq. (8) in Section 2.2, using a window width of 576 months, and the series was projected onto the sum of its 12 leading reconstructed components; the MEM spectrum of this data-adaptively prefiltered time series is shown in Fig. 19a for the same two cases discussed above (\( a = 0.1, 1.0 \)). For \( a = 0.1 \) the same two peaks, corresponding to periods of 7 and 14 yr, are obtained as for the time-independent forcing case (see Fig. 8). For \( a = 1.0 \) the annual peak is obviously much stronger, but the two peaks near 14 and 6 yr are still present and significant. Thus, despite the standing oscillation in the wind-stress intensity, the 14-yr and \( \sim 7 \)-yr oscillation periods appear to be quite robust. It should be noted that the wind forcing in the model acts as a body force and thus its effect is exaggerated compared to reality.

As discussed earlier in Section 3.2, Speich et al. (1995) did find a 7-yr oscillation in the Gulf Stream axis position from their analysis of COADS data. The present results therefore let us conclude that the intrinsic gyre mode studied in previous work on the double-gyre problem for steady wind-stress forcing is still a likely source of the observed 7-yr peak in Gulf Stream
axis position and, more broadly, in North-Atlantic SSTs (Moron et al., 1998). Analysis of the jet axis position in our model indicates increased meandering (not shown) with increase in the wind-stress forcing. The role played by north–south migration of the wind-stress forcing is studied in the following subsection.

We also carried out a preliminary study on the effect of different oscillation periods in the forcing. Simulations with periods of 2 and 0.5 years were performed. Results so far indicate that the flow patterns are not very sensitive to the oscillation frequency in comparison with sensitivity to its amplitude.

### 4.2.2 Migrating wind-stress forcing

Three 500-yr long integrations, for a total extent of migration $2b = 200$ km, 400 km, and 1600 km, respectively, were performed with the migrating wind-stress forcing of Eq. (5) for $\alpha_t = 0.3$. The long-term average upper-layer thickness fields are shown in the left panels of Fig. 20, for all three cases: the recirculation cells are weakened with increasing migration, to the point of complete disappearance, as was the case with the small-basin experiments. The rms fields of the upper-layer thickness appear in the right panels. For small migration the variability is concentrated along the jet axis. The basin $KE$ decreases with north-south migration (Fig. 21), in both mean and range, quite the opposite of the increase in basin $KE$ for increasing magnitudes of the standing-oscillation case (not shown).

### 4.2.3 Migrating wind-stress forcing with periodic changes in intensity

A more realistic wind-stress pattern that incorporates oscillations in magnitude as well as in the north–south position of the zero line of wind-stress curl is used here, cf. Eq. (6). Three 500-yr long simulations were performed with the north–south migration range $b$ set to 200 km;
the amplitude $a$ in intensity is set to 1, 2/3 and 1/5, i.e. the intensity when the line of zero wind-stress curl lies at its southernmost location is 3, 2 and 1.2 times the intensity at its northernmost location for the three simulations, respectively.

SSA is applied to the time series of asymmetric energy $\Delta E$, using a window width of 240 months and projecting the series onto the sum of its 12 leading reconstructed components; the power spectra of the records are shown in Fig. 22. The spectra, for all three cases, have oscillations with periods 7 and 14 yr, similar to the time-independent case, as well as the time-dependent standing oscillation case.

It thus appears that seasonal variations in the jet’s intensity reinforce the 7-yr mode of intrinsic variability, while meridional excursions in the position of the atmospheric jet weaken it. For realistic parameter settings of the domain size and forcing, the two effects cancel, roughly speaking, and thus the overall characteristics of the intrinsic 7-year mode are preserved.

5 Concluding remarks

This work focused on the differences in the mid-latitude ocean’s response to different types of wind forcing, using a 1.5-layer reduced-gravity shallow-water (SW) model with fairly high spatial resolution. Both the time-independent and time-periodic forcing we considered were of the double-gyre type, cf. Eqs. (3)–(6). Experiments were performed for two basin sizes; the small basin is 1000 km x 2000 km, while the larger one is 3000 km x 2010 km and has an aspect ratio very similar to that of the North Atlantic between 20°N and 60°N.

For the case of time-independent forcing, our model agrees in its major features with results obtained across a hierarchy of distinct model configurations, SW and QG, barotropic
and baroclinic, at different resolutions and in different domain sizes and shapes (Dijkstra and Ghil, 2005). To study the effects of time-periodic wind forcing, we chose idealized wind-stress patterns based on the analysis of observational data. For simplicity, we considered at first separately seasonal variations in the magnitude of wind-stress forcing and in the meridional position of the line of zero wind-stress curl, followed by a combination of the two.

Multiply periodic flows arise in the case of varying wind-stress magnitudes for the small basin. EOF analysis of the spatio-temporal patterns in this case clearly shows a superposition of the externally driven and internally generated modes of variability, modified by their nonlinear interaction. When seasonal migration of the wind-stress pattern is used for the forcing, the flow in the basin is affected more substantially than for seasonal changes in intensity. As the range of excursion of the atmospheric jet axis increases, the inertial recirculation cells near the basin’s western boundary are substantially weakened and reduced in size, while small-scale features fill most of the basin.

The present results on wind migration thus confirm and refine those of earlier studies. Liu (1996) showed that, in a barotropic QG model, the weakening of the inertial recirculation cells by changes in the wind pattern is due to two processes: (i) the mismatch between the fast spin-up time of the Sverdrup interior and the western boundary currents, on the one hand, and the slow spin-up of the inertial recirculation, on the other; and (ii) the chaotic mixing of potential vorticity ($PV$) anomalies between the two cells of the recirculation dipole (see also Yang and Liu, 1994; Yang, 1996). The first process was neutralized by slower baroclinic Rossby waves replacing the fast barotropic ones in a 1.5-layer version of Liu's (1996) QG model; our model also uses a reduced-gravity, 1.5-layer formulation, albeit in an SW context. The second process thus seems more important.
Our study indicates that the effectiveness of the migrating wind pattern depends quite strongly on the extent of the meridional excursions of the atmospheric jet. Within a realistic range, the effect is still moderate and probably counterbalanced by the presence of total or partial slip at the western boundary (JJG), which tends to concentrate and reinforce the recirculation dipole. For high Reynolds numbers (Berloff and McWilliams, 1999; Simonnet et al., 2003b, 2005), the role of eddy diffusion (Berloff and McWilliams, 2002a, 2002b, 2003) may be comparable to or greater than that of chaotic mixing (Yang and Liu, 1994; Yang, 1996; Coulliette and Wiggins, 2000). Thus localized sub- and superdiffusive mixing could be present in different parts of the flow domain; see also Huber et al. (2001) for a discussion of these topics in the atmosphere.

The simulations in a larger rectangular basin were performed for less viscous regimes and produce solutions with more realistic recirculation gyres and an elongated jet stream with small meridional-to-zonal aspect ratio. These solutions are dominated by periodicities of 14 and 6–7 years for the time-independent as well as the time-dependent cases explored. Moron et al. (1998) found both of these periodicities in a century of SST data for the North Atlantic, while a 7–8-yr peak is present in several other atmospheric and oceanic data sets (Wunsch, 1999; Da Costa and Colin de Verdière, 2002). This peak agrees well with a periodicity present in the meridional axis position of the Gulf Stream (Speich et al., 1995; Simonnet et al., 2005); in our model, it appears to be quite robust to realistic changes in the intensity and north–south migration of wind forcing.
References


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Figure 1. Bifurcation diagram for time-constant wind forcing $\alpha$, in the small rectangular basin (1000 km x 2000 km). The central panel shows normalized transport difference $TD$ with increasing $\alpha$; all model parameters as in Table 1a, while $\alpha_A = 1.3$ and $R_d = 80$ km. The top and bottom panels illustrate 10-yr segments of the $TD$ time series for the solutions indicated by arrows pointing to the middle panel: (top panels) positive branch, $\alpha = 1.08$ (periodic) and $\alpha = 1.20$ (quasi-periodic); (bottom panels) negative branch, $\alpha = 0.9$ (periodic) and $\alpha = 1.1$ (quasi-periodic).

Figure 2. The leading EOFs for $\alpha = 0.9$ on the lower branch, after the first Hopf bifurcation: (a, b) spatial patterns, and (c, d) power spectra of the associated PCs; same parameter settings as for bottom-left panel of Fig. 1.

Figure 3. Spatial patterns corresponding to the four leading EOFs for $\alpha = 1.1$ on the lower branch, after the second Hopf bifurcation; same parameter settings as for bottom-right panel of Fig. 1.

Figure 4. Spatial patterns corresponding to the four leading EOFs for $\alpha = 1.08$ on the upper branch, after the Hopf bifurcation; same parameter settings as for top-left panel of Fig. 1.

Figure 5. Spatial patterns of the four leading EOFs for a solution with free-slip boundary conditions, $\alpha = 1.0$, $\alpha_A = 3.5$ and $R_d = 80.0$ km, on the lower branch.

Figure 6. $PV$-vs.-$\psi$ scatter plots for different flow regimes, along the upper (top panels, a–c) and lower (bottom panels, d–f) branches of the bifurcation diagram. The values of $\alpha$ are given on each panel, while other parameter values are as in Fig. 1; no-slip boundary conditions were used. (a, d) steady; (b, e) periodic; and (c, f) aperiodic. Points that
represent the dipole region are in red and lie very close to a smooth curve. The blue points correspond to grid points along the western boundary (x = 0). Each point in the scatter plots is based on an average over 20 years of PV and $\psi$ for a given grid point.

Figure 7. Effect of the Rossby radius of deformation $R_d$ on PV-$\psi$ scatter plots: (a–d) $R_d = 80, 100, 120$ and 140 km, for $\alpha = 1.0$ and $\alpha_A = 1.3$ on the lower branch. Points representing the dipole region are in red; long-term averages over 20 years are used, as in Fig. 6.

Figure 8. Solution behaviour for the large rectangular basin (3000 km x 2010 km), for $\alpha = 0.3$ and free-slip boundary conditions: (right- and left-top panels) simulated long-term average streamfunction field and PV-$\psi$ scatter plot; (middle and bottom panels) time series of the asymmetric energy $\Delta E$ and its maximum entropy spectrum.

Figure 9. The 7-yr oscillation reconstructed using the leading pair of modes and plotted at eight equidistant phases (a)–(h). The plots show the anomalous upper-layer thickness, i.e. the difference between the field in the corresponding phase and the mean over the oscillation.

Figure 10. As in Figure 9, but for the 14-yr oscillation.

Figure 11. Bifurcation diagram for intensity-modulated wind-stress forcing, in the small basin; this type of forcing is also referred to as “standing oscillation”. The central panel shows TD against the wind-stress intensity $\alpha$. The TD time series over 20 yr = 240 months in the periodic, quasi-periodic and aperiodic regimes are shown for the positive branch in the three top panels and for the negative branch in the bottom panels. Same symbols and parameter values as in Fig. 1.

Figure 12. The aperiodic TD and KE time series off the lower branch, for 200 years of simulation, for $\alpha = 2.0$ are shown in the top and middle panels. The instantaneous PV-$\psi$ scatter plots for the points A and B marked on the TD series, corresponding to negative and
positive TD values, are shown in the bottom left and right panels respectively.

Figure 13. Quasi-periodicity route to chaos as the nonlinearity $\alpha_\tau$ is increased; free-slip boundary conditions in the large basin, along the lower branch, at $\alpha_\lambda = 3.5$. The $KE$–$PE$ phase-space plots are shown in the left panels, for six selected values of $\alpha_\tau$, while the corresponding time series of TD appear in the right panels; these series are keyed to the phase-space plots and colour-coded accordingly.

Figure 14. Spatio-temporal variability of a solution for the standing oscillation in the wind-stress forcing, upper branch; this solution corresponds to the top-left panel in Fig. 11 ($\alpha_\tau = 0.8$, purely periodic solution). Top panel: two leading EOFs of the streamfunction field; middle panel: TD and KE time series for the last 10 yr of a 40-yr run; bottom panel: time series of the two extrema of the streamfunction, $|\psi_{po}|$ and $|\psi_{tr}|$ for the same time interval.

Figure 15. Spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing, and a quasi-periodic solution on the upper branch ($\alpha_\tau = 1.0$, top-middle panel in Fig. 11).

Figure 16. Spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing, and a purely periodic solution on the lower branch, ($\alpha_\tau = 0.7$, bottom-left panel of Fig. 11).

Figure 17. Spatial patterns of the leading four EOFs for the quasi-periodic regime on the lower branch of Fig. 11; $\alpha_\tau = 0.9$, bottom-middle panel there.

Figure 18. Solution behaviour for a meridionally modulated wind-stress pattern, for $\alpha_\tau = 0.85$ on the upper branch; all other parameter settings as in Fig. 1. The range of excursions in the line of zero wind-stress curl is (a) $b = 400$ km; (b) $b = 800$ km; and (c) $b = 1600$ km. Top
panels: long-term average fields of $\psi$ (in red) and $PV$ (in blue); bottom panels: $PV$-vs.-$\psi$ scatter plots, with points stemming from the recirculation region, when present, in red, and those along the western boundary ($x = 0$) in blue.

Figure 19. Amplitude-modulated wind-stress forcing in the large rectangular basin, for $\alpha_t = 0.3$. (a) The maximum-entropy spectrum of the time series of asymmetric energy $\Delta E$, for $a = 0.1$ (solid line) and 1.0 (dashed line); SSA window width is 48 yr, number of reconstructed components retained is 12, and the MEM order is 30. (b, c) The root-mean-square (rms) of the streamfunction for these two values of the amplitude modulation $a$.

Figure 20. North–south migrating wind-stress forcing in the large rectangular basin, for $\alpha_t = 0.3$: (left panels) long-term averaged upper-layer thickness $H$; and (right panels) the rms of $H$. The range of excursions in the line of zero wind-stress curl is: (top panels) $b = 200$ km; (middle panels) $b = 400$ km; and (bottom panels) $b = 800$ km.

Figure 21. Time evolution of basin kinetic energy $KE$ for the solutions shown in Fig. 20: (a) $b = 200$ km, (b) $b = 400$ km, and (c) $b = 800$ km.

Figure 22. Seasonal forcing with a wind-stress changing in both intensity and meridional profile. Shown are the MEM spectra of three time series of asymmetric energy $\Delta E$, for the north-south migration factor $b = 200$ km, while $a = 1/5$ (blue), $2/3$ (red) and 1 (black), respectively; see text for details. The SSA window width is 20 yr, number of reconstructed components retained is 12, and the MEM order is 30.
## Table 1: Parameter values

### a) Fixed model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
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<tr>
<td>$f_0$, basic Coriolis parameter</td>
<td>$5 \times 10^{-5}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\beta$, rate of change of Coriolis parameter</td>
<td>$2 \times 10^{-11}$ m$^{-1}$ s$^{-1}$</td>
</tr>
<tr>
<td>$\tau_0$, wind-stress amplitude</td>
<td>$0.1$ N/m$^2$</td>
</tr>
<tr>
<td>$A$, eddy viscosity coefficient</td>
<td>$300$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td>$R$, Rayleigh friction coefficient</td>
<td>$5 \times 10^{-8}$ s$^{-1}$</td>
</tr>
<tr>
<td>$H_0$, upper-layer thickness</td>
<td>$500$ m</td>
</tr>
<tr>
<td>$\rho$, upper-layer density</td>
<td>$1022$ kg m$^{-3}$</td>
</tr>
</tbody>
</table>

### b) Control parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_T$, wind stress intensity</td>
<td>$0.4$ – $1.3$                : $0.3$</td>
</tr>
<tr>
<td>$\alpha_A$, lateral-diffusion coefficient</td>
<td>$1.3$ (no-slip)       : $1.0$ (free-slip)</td>
</tr>
<tr>
<td></td>
<td>$2.5$ – $3.5$ (free-slip)</td>
</tr>
<tr>
<td>$g'$, reduced gravity</td>
<td>$0.03$ m$^2$ s$^{-1}$ (most cases) : $0.03$ m$^2$ s$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$0.008$–$0.098$ m$^2$ s$^{-1}$   (sensitivity study)</td>
</tr>
<tr>
<td>$R_d$, Rossby radius of deformation</td>
<td>$80$ km (most cases)       : $80$ km</td>
</tr>
<tr>
<td></td>
<td>$80,100, 120$ and $140$ km (sensitivity study)</td>
</tr>
</tbody>
</table>

Table 1: Parameter values
Figure 1: Bifurcation diagram for time-constant wind forcing $\alpha_t$, in the small rectangular basin (1000 km x 2000 km). The central panel shows normalized transport difference $TD$ with increasing $\alpha_t$; all model parameters as in Table 1a, while $\alpha_A = 1.3$ and $R_d = 80$ km. The top and bottom panels illustrate 10-yr segments of the $TD$ time series for the solutions indicated by arrows pointing to the middle panel: (top panels) positive branch, $\alpha_t = 1.08$ (periodic) and $\alpha_t = 1.20$ (quasi-periodic); (bottom panels) negative branch, $\alpha_t = 0.9$ (periodic) and $\alpha_t = 1.1$ (quasi-periodic).
Figure 2: The leading EOFs for $\zeta = 0.9$ on the lower branch, after the first Hopf bifurcation: (a, b) spatial patterns, and (c, d) power spectra of the associated PCs; same parameter settings as for bottom-left panel of Fig. 1.
Figure 3: Spatial patterns corresponding to the four leading EOFs for $\alpha_1=1.1$ on the lower branch, after the second Hopf bifurcation; same parameter settings as for bottom-right panel of Fig. 1.
Figure 4: Spatial patterns corresponding to the four leading EOFs for $\alpha = 1.08$ on the upper branch, after the Hopf bifurcation; same parameter settings as for top-left panel of Fig. 1.
Figure 5: Spatial patterns of the four leading EOFs for a solution with free-slip boundary conditions, $\alpha_r = 1.0$, $\alpha_a = 3.5$ and $R_d = 80.0$ km, on the lower branch.
Figure 6: $PV$-vs-$\psi$ scatter plots for different flow regimes, along the upper (top panels, a–c) and lower (bottom panels, d–f) branches of the bifurcation diagram. The values of $\alpha_\tau$ are given on each panel, while other parameter values are as in Fig. 1; no-slip boundary conditions were used. (a, d) steady; (b, e) periodic; and (c, f) aperiodic. Points that represent the dipole region are in red and lie very close to a smooth curve. The blue points correspond to grid points along the western boundary ($x = 0$). Each point in the scatter plots is based on an average over 20 years of $PV$ and $\psi$ for a given grid point.
Figure 7: Effect of the Rossby radius of deformation $R_d$ on $PV \times \psi$ scatter plots: (a–d) $R_d = 80, 100, 120$ and $140$ km, for $\alpha_t = 1.0$ and $\alpha_A = 1.3$ on the lower branch. Points representing the dipole region are in red; long-term averages over 20 years are used, as in Fig. 6.
Figure 8: Solution behaviour for the large rectangular basin (3000 km x 2010 km), for $\alpha = 0.3$ and free-slip boundary conditions: (right- and left-top panels) simulated long-term average streamfunction field and $PV$-vs.-$\psi$ scatter plot; (middle and bottom panels) time series of the asymmetric energy $\Delta E$ and its maximum entropy spectrum.
Figure 9: The 7-yr oscillation reconstructed using the leading pair of modes and plotted at eight equidistant phases (a)–(h). The plots show the anomalous upper-layer thickness, i.e. the difference between the field in the corresponding phase and the mean over the oscillation.
Figure 10: As in Figure 9, but for the 14-yr oscillation.
Figure 11: Bifurcation diagram for intensity-modulated wind-stress forcing, in the small basin; this type of forcing is also referred to as “standing oscillation”. The central panel shows $TD$ against the wind-stress intensity $\alpha_t$. The $TD$ time series over 20 yr = 240 months in the periodic, quasi-periodic and aperiodic regimes are shown for the positive branch in the three top panels and for the negative branch in the bottom panels. Same symbols and parameter values as in Fig. 1.
Figure 12: The aperiodic $TD$ and $KE$ time series off the lower branch, for 200 years of simulation, for $\alpha_t = 2.0$ are shown in the top and middle panels. The instantaneous $PV$-$\psi$ scatter plots for the points A and B marked on the $TD$ series, corresponding to negative and positive $TD$ values, are shown in the bottom left and right panels respectively.
Figure 13: Quasi-periodicity route to chaos as the nonlinearity $\alpha_\tau$ is increased; free-slip boundary conditions in the large basin, along the lower branch, at $\alpha_A = 3.5$. The $KE–PE$ phase-space plots are shown in the left panels, for six selected values of $\alpha_\tau$, while the corresponding time series of $TD$ appear in the right panels; these series are keyed to the phase-space plots and colour-coded accordingly.
Figure 14: Spatio-temporal variability of a solution for the standing oscillation in the wind-stress forcing, upper branch; this solution corresponds to the top-left panel in Fig. 11 ($\alpha = 0.8$, purely periodic solution). Top panel: two leading EOFs of the streamfunction field; middle panel: $TD$ and $KE$ time series for the last 10 yr of a 40-yr run; bottom panel: time series of the two extrema of the streamfunction, $|\psi_{po}|$ and $|\psi_{tr}|$ for the same time interval.
Figure 15: Spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing, and a quasi-periodic solution on the upper branch ($\alpha_t = 1.0$, top-middle panel in Fig. 11).
Figure 16: Spatial patterns of the two leading EOFs of the streamfunction field, for the standing oscillation in the wind-stress forcing, and a purely periodic solution on the lower branch, ($\alpha = 0.7$, bottom-left panel of Fig. 11).
Figure 17: Spatial patterns of the leading four EOFs for the quasi-periodic regime on the lower branch of Fig. 11; $\alpha = 0.9$, bottom-middle panel there.
Figure 18: Solution behaviour for a meridionally modulated wind-stress pattern, for $\alpha = 0.85$ on the upper branch; all other parameter settings as in Fig. 1. The range of excursions in the line of zero wind-stress curl is (a) $b = 400$ km; (b) $b = 800$ km; and (c) $b = 1600$ km. Top panels: long-term average fields of $\psi$ (in red) and $PV$ (in blue); bottom panels: $PV$ vs. $\psi$ scatter plots, with points stemming from the recirculation region, when present, in red, and those along the western boundary ($x = 0$) in blue.
Figure 19: Amplitude-modulated wind-stress forcing in the large rectangular basin, for $\alpha = 0.3$. (a) The maximum-entropy spectrum of the time series of asymmetric energy $\Delta E$, for $a = 0.1$ (solid line) and 1.0 (dashed line); SSA window width is 48 yr, number of reconstructed components retained is 12, and the MEM order is 30. (b, c) The root-mean-square (rms) of the streamfunction for these two values of the amplitude modulation $a$. 
Figure 20: North–south migrating wind-stress forcing in the large rectangular basin, for \( \alpha_t = 0.3 \):
(left panels) long-term averaged upper-layer thickness \( H \); and (right panels) the rms of \( H \). The range of excursions in the line of zero wind-stress curl is: (top panels) \( b = 200 \text{ km} \); (middle panels) \( b = 400 \text{ km} \); and (bottom panels) \( b = 800 \text{ km} \).
Figure 21: Time evolution of basin kinetic energy $KE$ for the solutions shown in Fig. 20: (a) $b = 200$ km, (b) $b = 400$ km, and (c) $b = 800$ km.
Figure 22: Seasonal forcing with a wind-stress changing in both intensity and meridional profile. Shown are the MEM spectra of three time series of asymmetric energy $\Delta E$, for the north-south migration factor $b = 200$ km, while $a = 1/5$ (blue), $2/3$ (red) and 1 (black), respectively; see text for details. The SSA window width is 20 yr, number of reconstructed components retained is 12, and the MEM order is 30.