Nonlinear Dynamics, Poor Data, and What to Make of Them?

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Motivation & Outline

1. Data sets in the geosciences are often short and contain errors: this is both an obstacle and an incentive.

2. Phenomena in the geosciences often have both regular aspects ("cycles") and irregular ones ("noise").

3. Different spatial and temporal scales:
   one person's noise is another person's signal.

4. Need both deterministic and stochastic modeling.

5. Regularities include (quasi-)periodicity ➔ spectral analysis via "classical" methods — see SSA-MTM Toolkit.

6. Irregularities include scaling and (multi-)fractality ➔ "spectral analysis" via Hurst exponents, dimensions, etc.

7. Does some combination of the two, + deterministic and stochastic modeling, provide a pathway to prediction?

Joint work with *many people*: please visit these two Web sites for details

TCD — http://www.atmos.ucla.edu/tcd/ (key person: Dmitri Kondrashov!)

E2-C2 — http://www.ipsl.jussieu.fr/~ypsce/py_E2C2.html

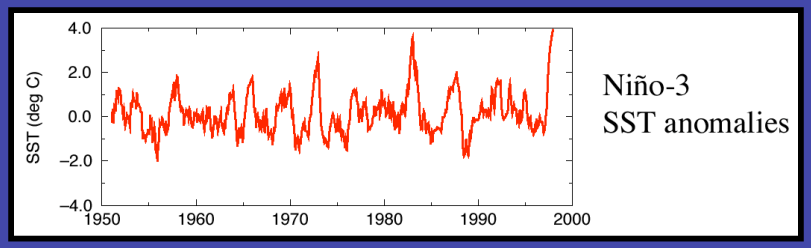
Singular Spectrum Analysis

Singular Spectrum Analysis (SSA)

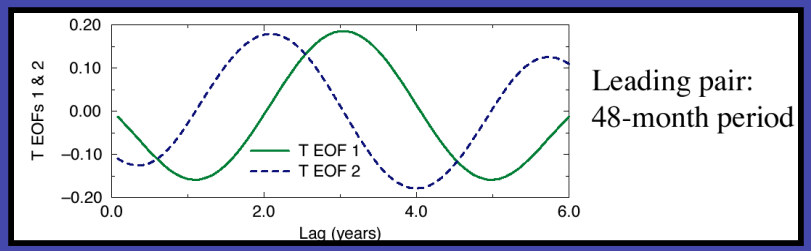
SSA decomposes geophysical time series into
Temporal EOFs (T-EOFs) and
Temporal Principal Components (T-PCs),
based on the series' lag-covariance matrix

Selected parts of the series can be
reconstructed, via
Reconstructed Components (RCs)

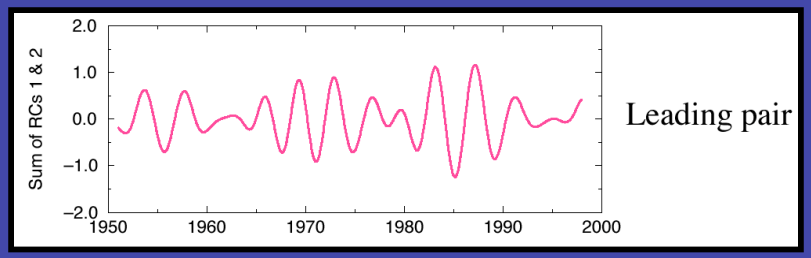
Time series



T-EOFs



RCs

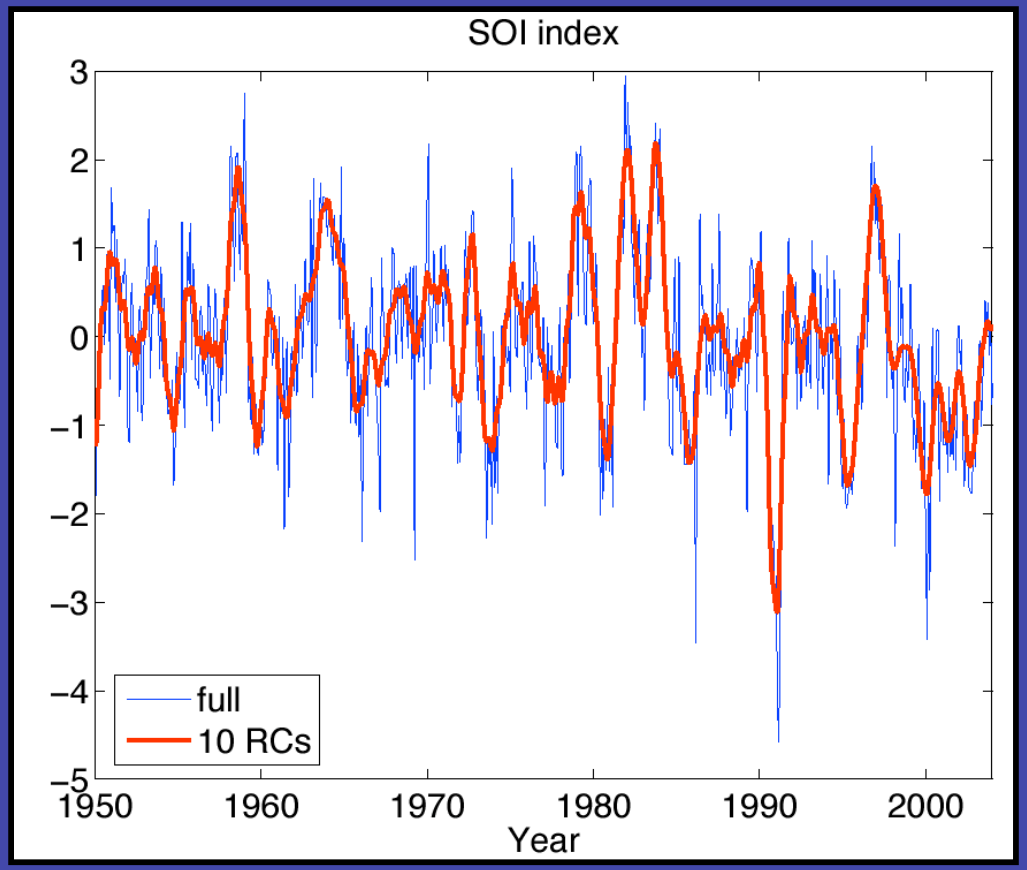


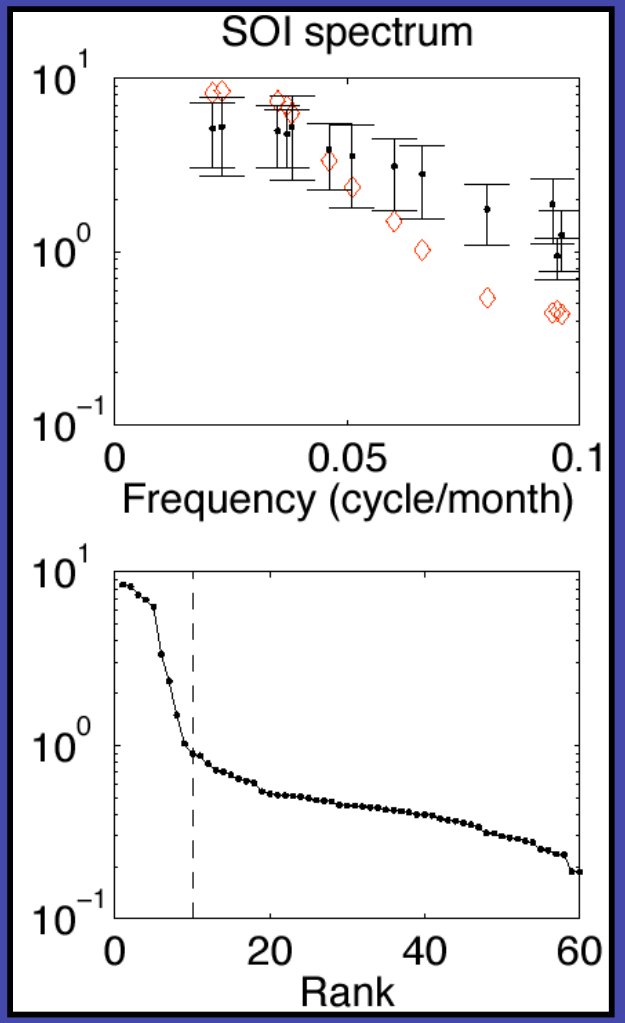
- SSA is good at isolating oscillatory behavior via paired eigenelements.
- SSA tends to lump signals that are longer-term than the window into one or two trend components.

Selected References:

Vautard & Ghil, 1989, *Physica* D;
Ghil *et al*., 2002, *Rev. Geophys.*

Singular Spectrum Analysis (SSA) and M-SSA, cont'd.





- Break in slope of SSA spectrum distinguishes *"significant"* from *"noise"* EOFs
- Formal Monte-Carlo test (Allen & Smith, 1994) identifies 4-yr and 2-yr ENSO oscillatory modes
- A window size of M = 60 is enough to "resolve" these modes in a monthly SOI time series

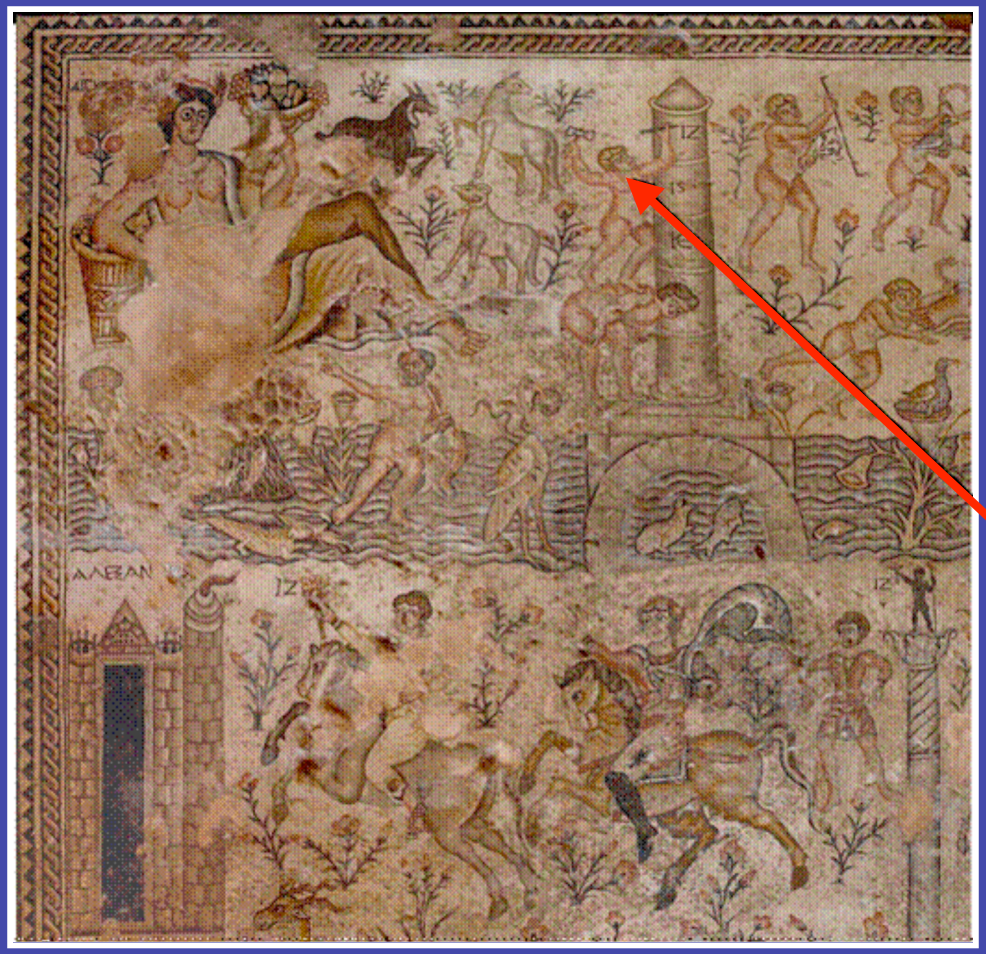
The Nile River Records Revisited: How good were Joseph's predictions?

Michael Ghil, *ENS & UCLA,*

Yizhak Feliks, *IIBR & UCLA,*

Dmitri Kondrashov, *UCLA*

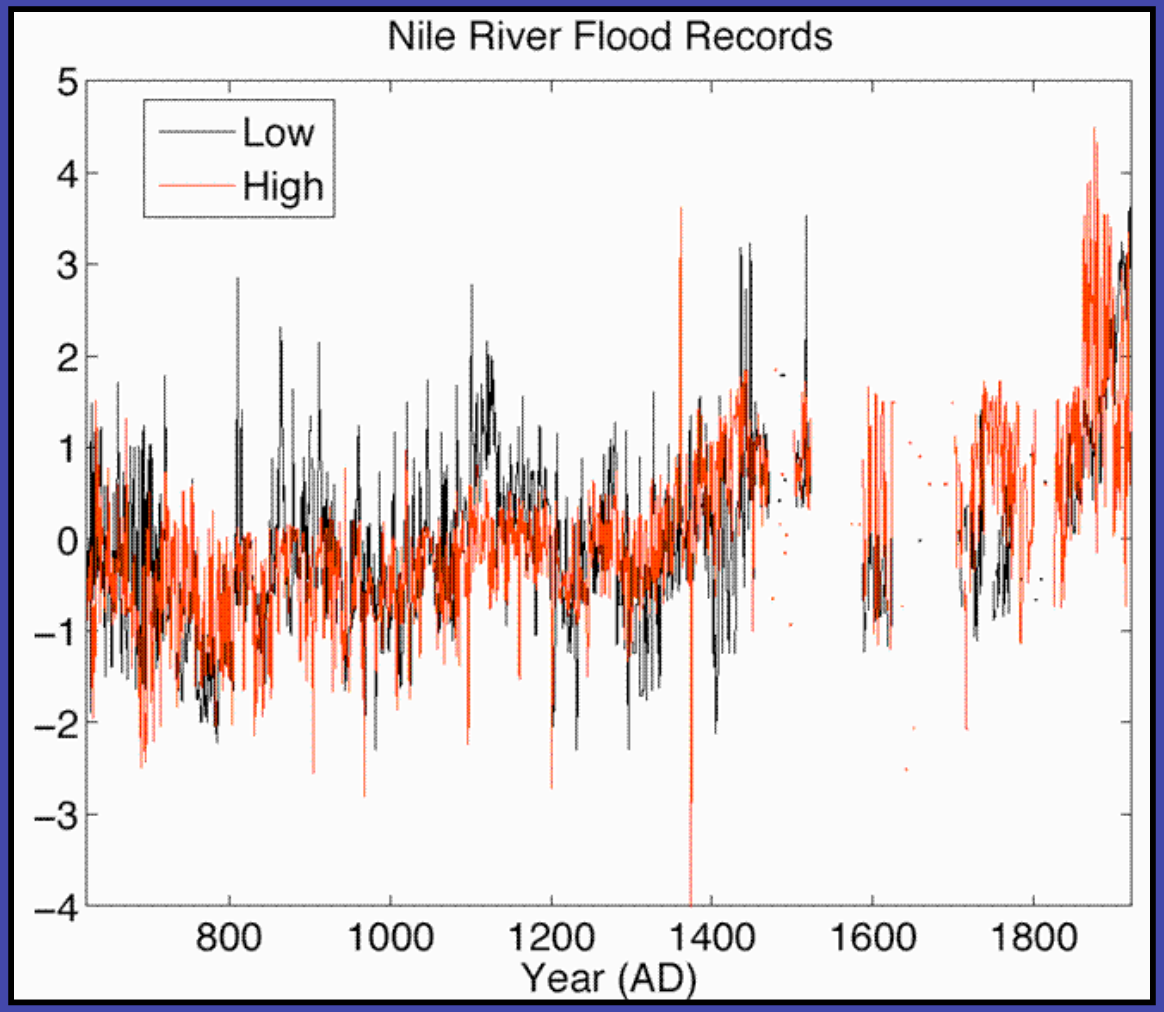
Why are there data missing?



Hard Work

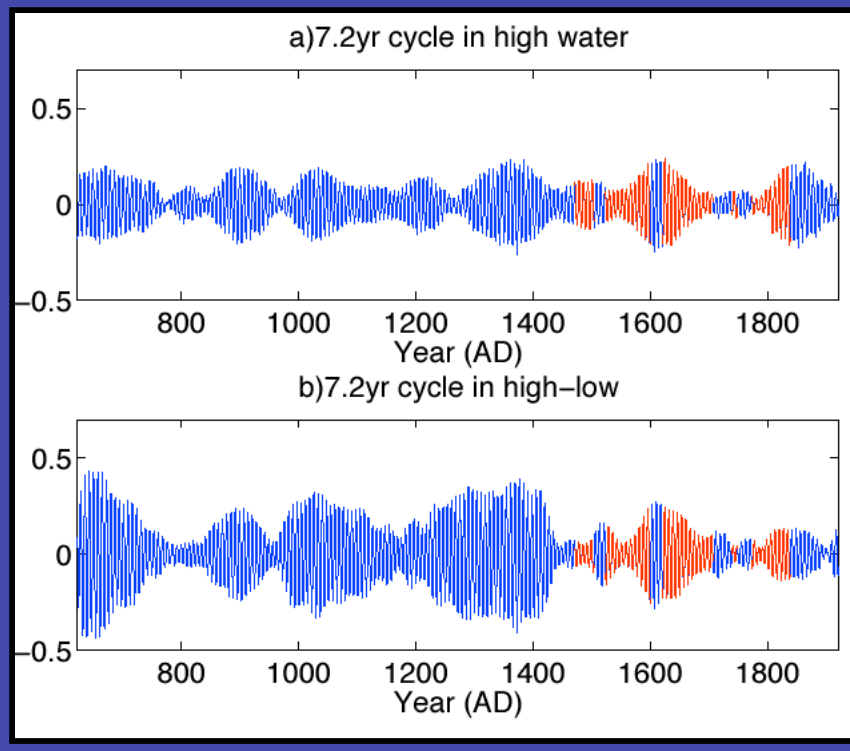
- Byzantine-period mosaic from ***Zippori***, the capital of ***Galilee*** (1st century B.C. to 4th century A.D.); photo by ***Yigal Feliks***, with *permission from the Israel Nature and Parks Protection Authority*)

Historical records are full of "gaps"....

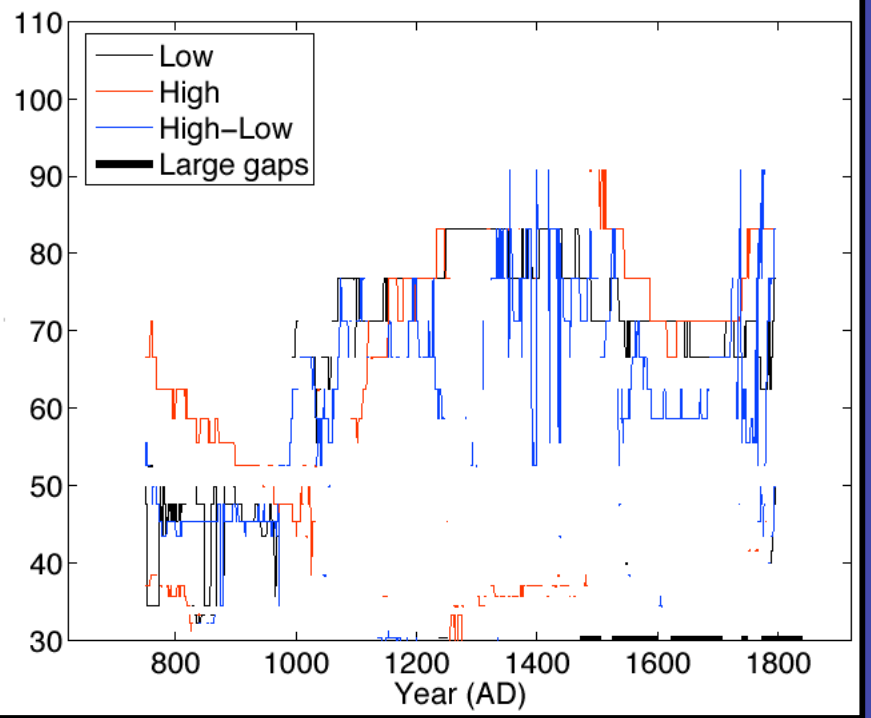


Annual maxima and minima of the water level at the nilometer on Rodah Island, Cairo.

Significant Oscillatory Modes

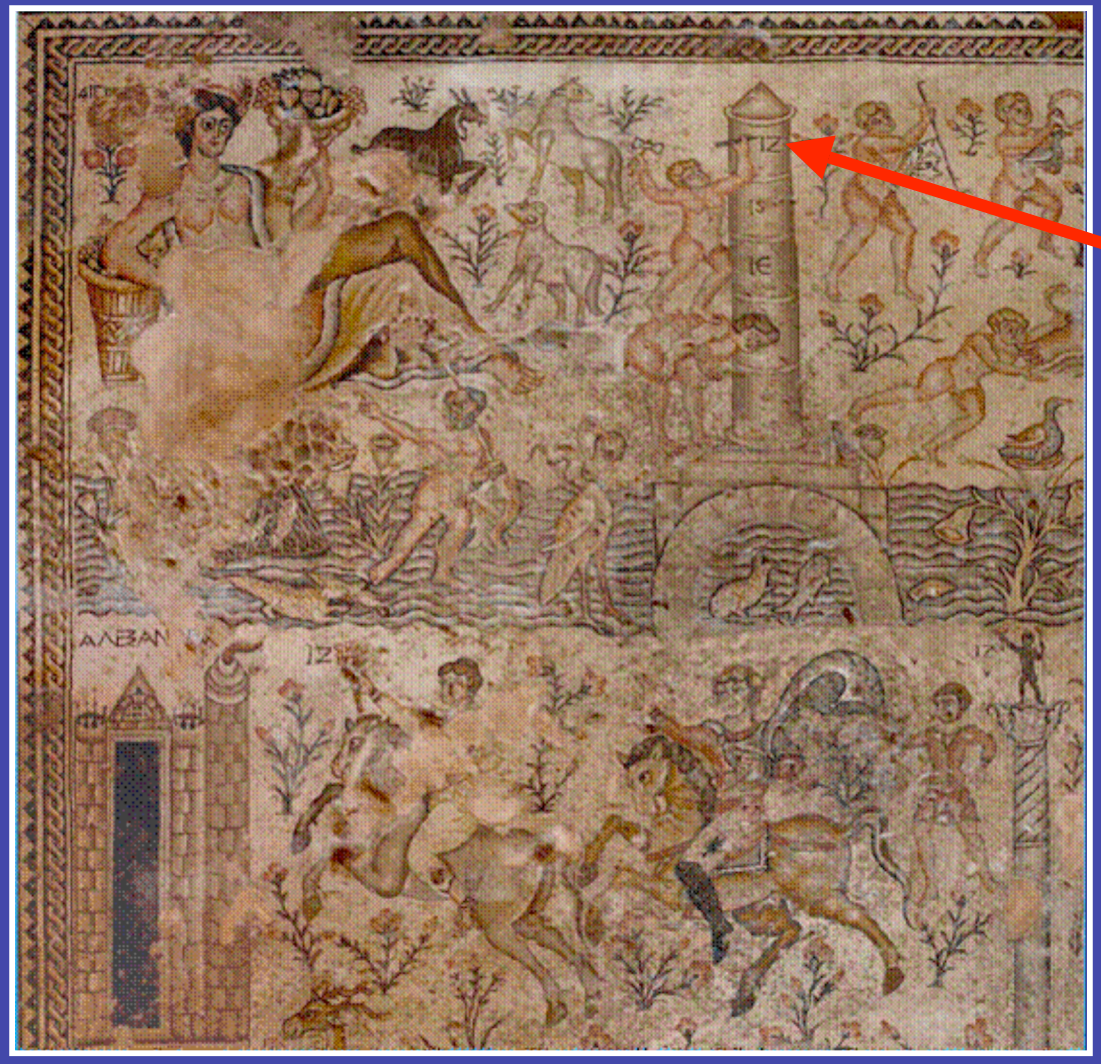


SSA reconstruction of the 7.2-yr mode in the extended Nile River records: (a) high-water, and (b) difference. Normalized amplitude; reconstruction in the large gaps in red.



Instantaneous frequencies of the oscillatory pairs in the low-frequency range (40–100 yr). The plots are based on multi-scale SSA [Yiou *et al*., 2000]; local SSA performed in each window of width $W = 3M$, with $M = 85$ yr.

How good were Joseph's predictions?



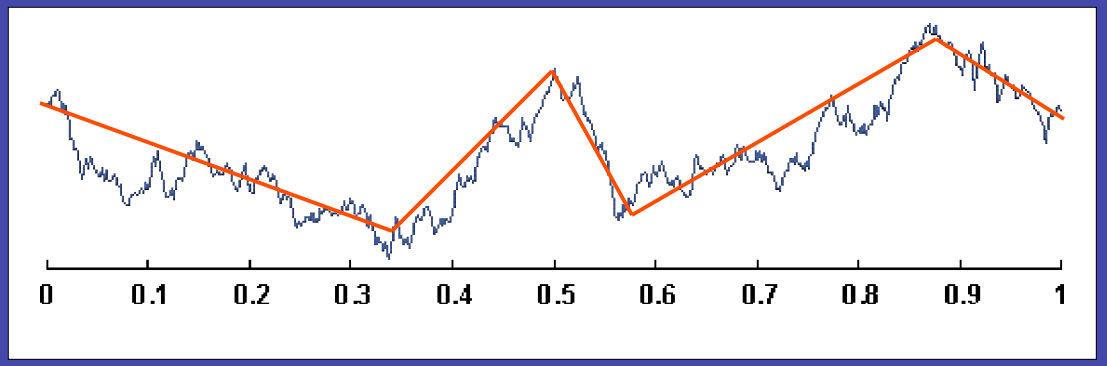
Pretty good!

Multiscale Trend Analysis

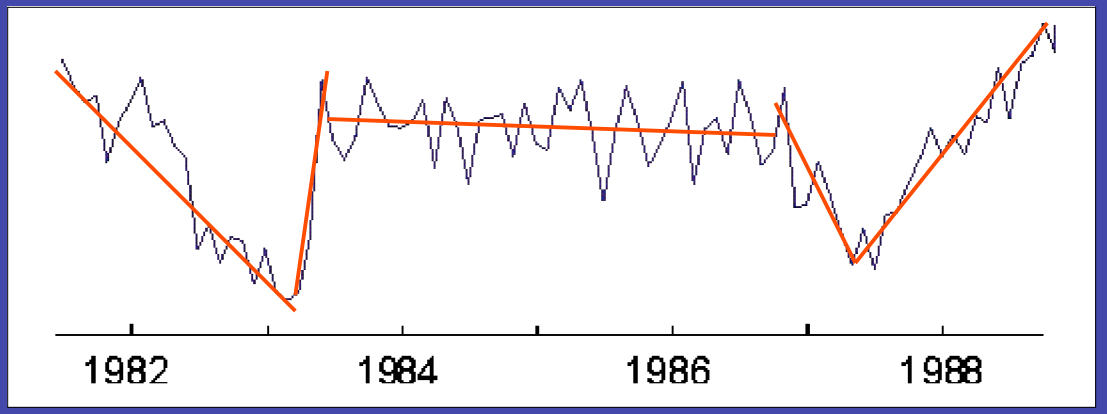
Motivation

Trends are the most intuitive features of time series and it is natural to use these features for their description and analysis

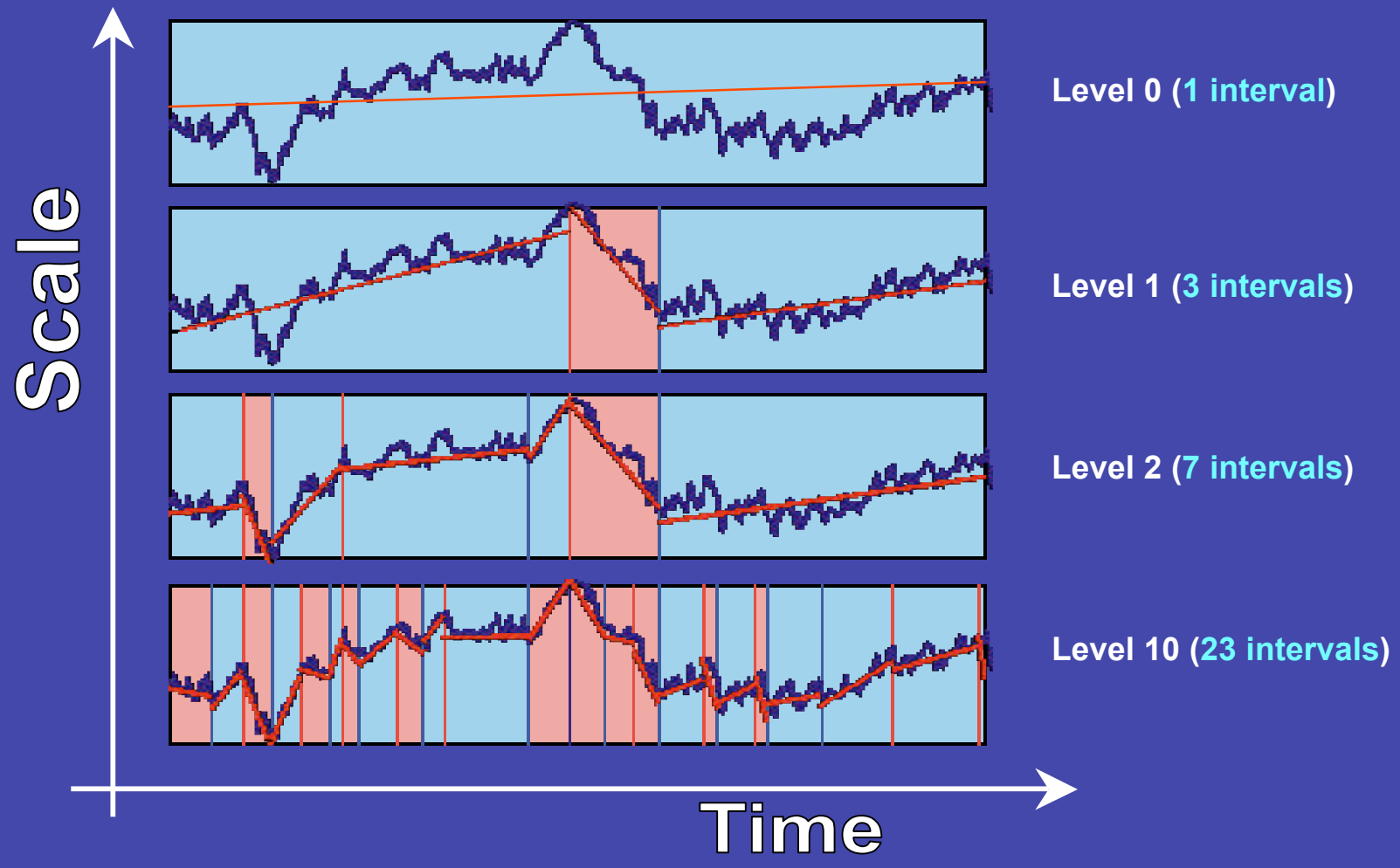
Brownian walk



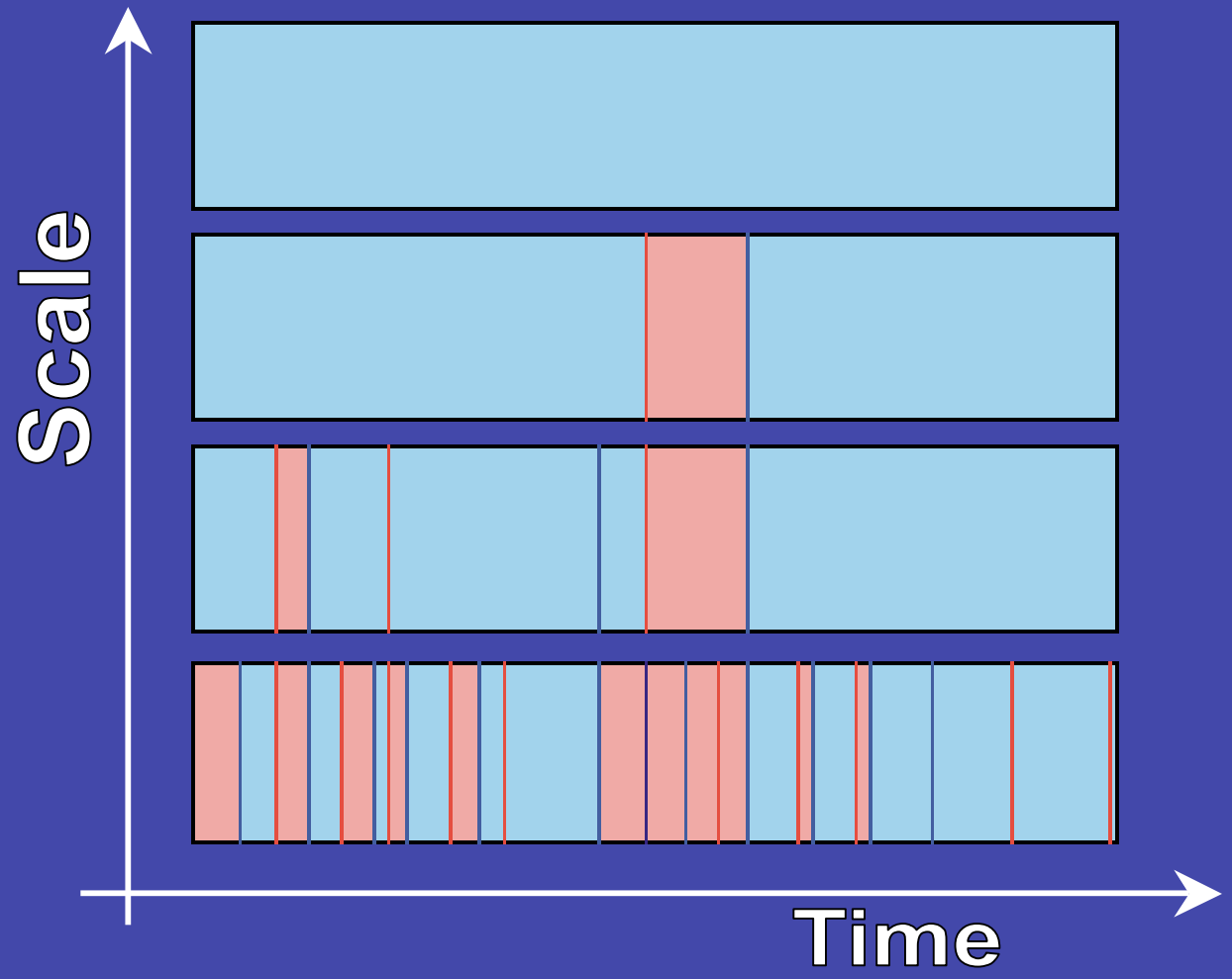
Southern Oscillation Index



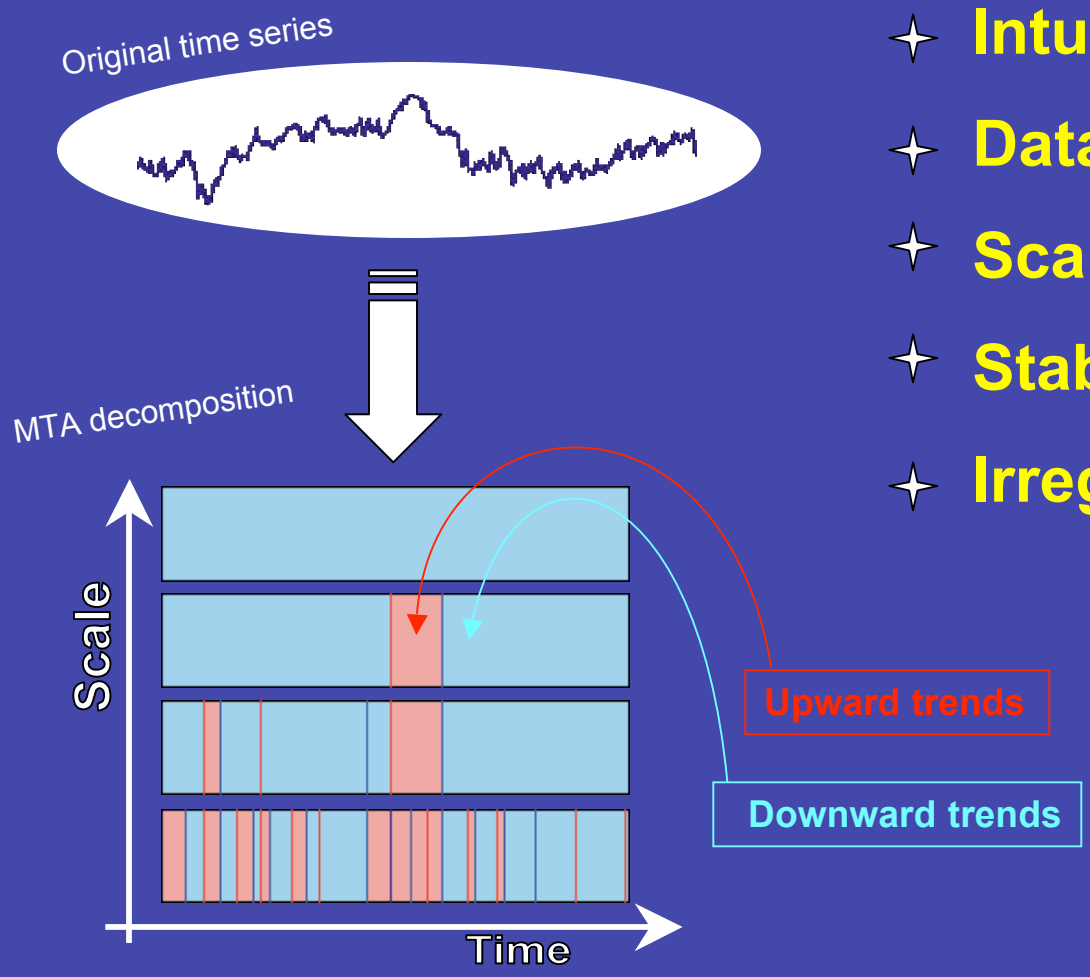
Multiscale Trend Analysis (MTA): outline



MTA: scheme



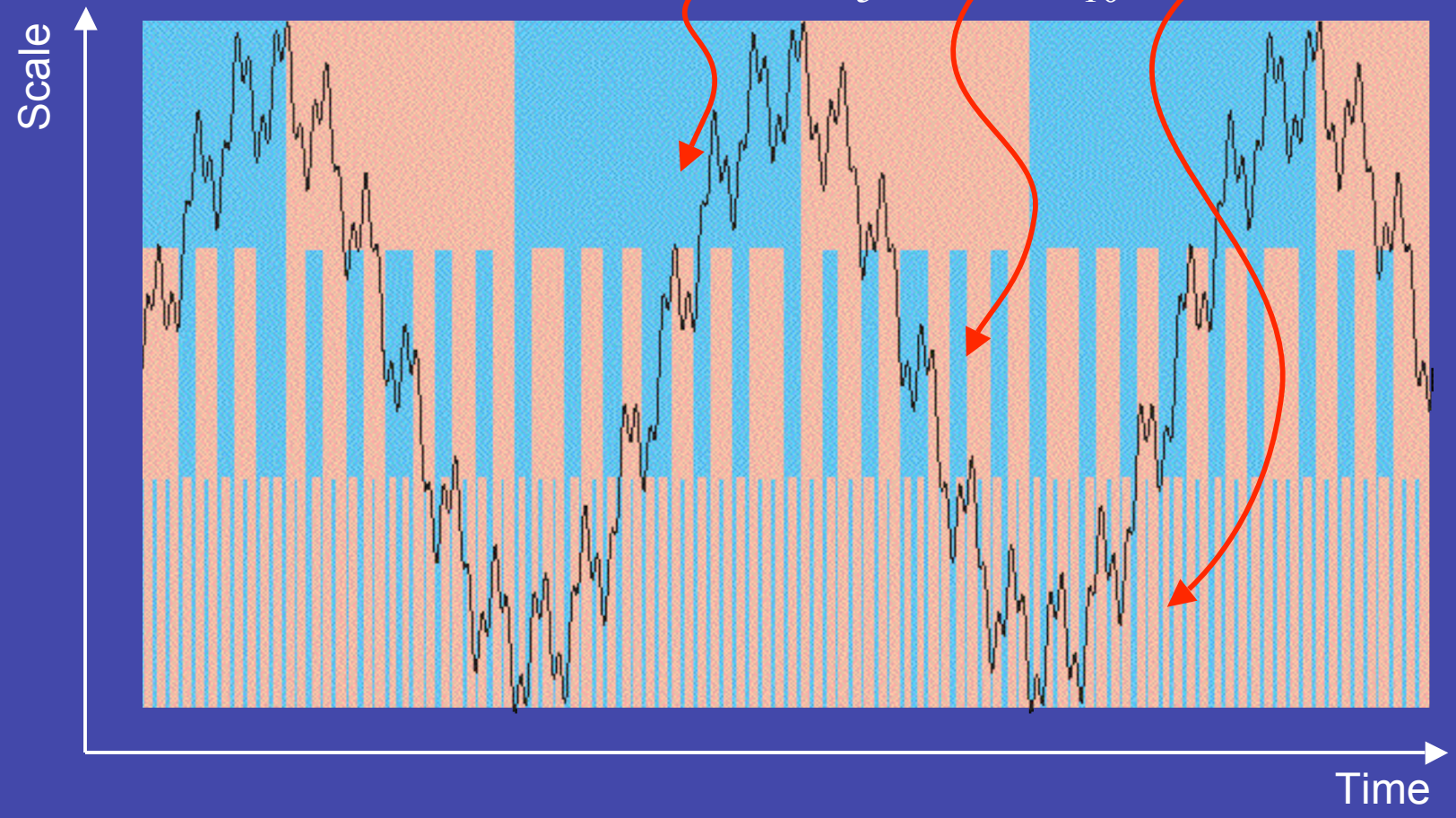
MTA: properties



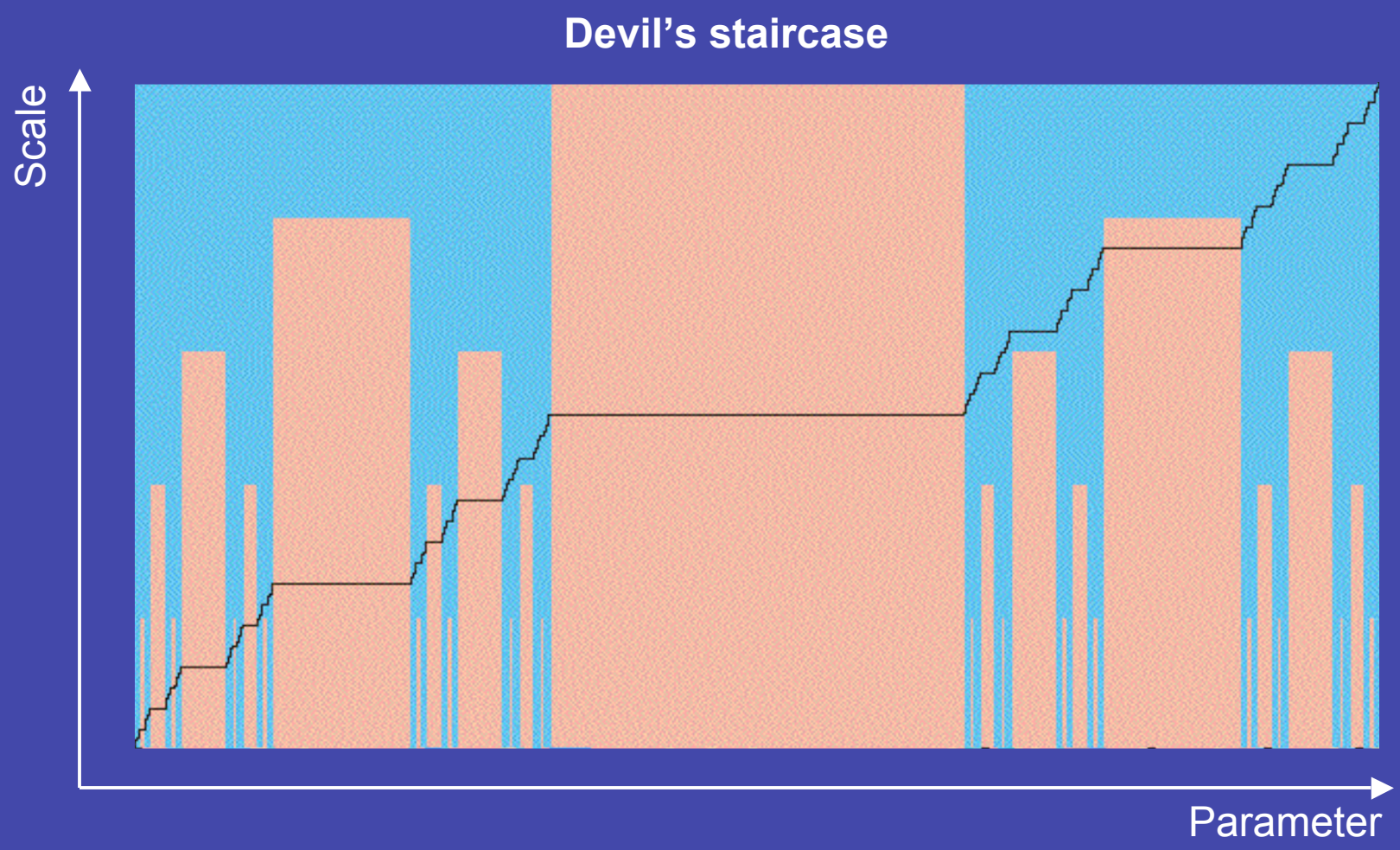
- Intuitive
- Data-adaptive
- Scale-dependent
- Stable
- Irregular time grids

MTA: examples

Three sinusoids: $f(t) = \sin(5\pi t) + \frac{1}{5}\sin(60\pi t) + \frac{1}{10}\sin(200\pi t)$



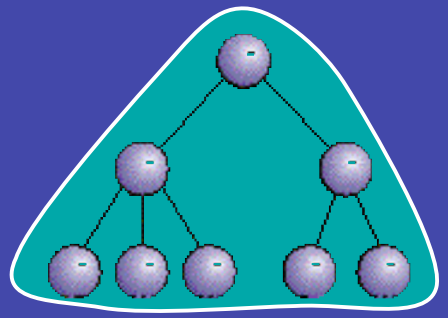
MTA: examples



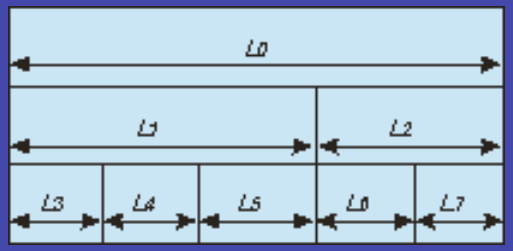
MTA-analyses

MTA allows to study time series in terms of …

Topological tree



Metric tree of interval partitions



Metric tree of fitting errors

$$L_{element} = \sum L_{children}$$

conservative system

$$E_{element} \geq \sum E_{children}$$

dissipative system

MTA: Applications

Geodynamics

Studying interactions between brittle and ductile layers of the Earth lithosphere

Collaborators: *V. Keilis-Borok, K. Aki, A. Jin, J. Liu*

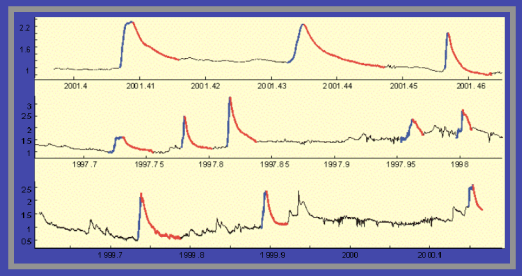
Association for Development of Earthquake Prediction, Tokyo

Hydrology

Automated detection of hydrographs, and studying scaling in hydrologic response

Collaborators: *E. Foufoula-Georgiou, B. Dodov, R. Sherestha*

St. Anthony Falls Laboratory, U. of Minnesota



Finance

Nonlinear filtering of volatility in Black-Scholes type models

$$X_t = \log S_t = \int_0^t \left(r_s - \frac{1}{2} v_s^2 \right) ds - \int_0^t v_s \, dB_s$$

Collaborators: *B. Rozovskii, J. Cvitanic*

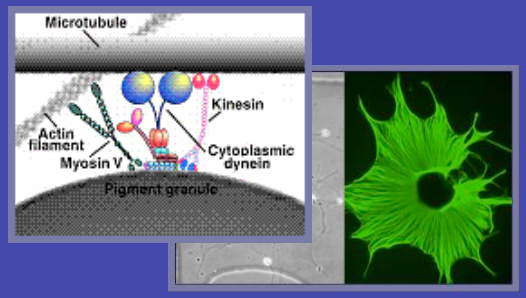
Center for Applied Math. Sciences, USC; CalTech

Cell biology

Part of a tracking procedure for studying intracellular protein motors

Collaborators: *V. Rodionov, A. Kashina*

Center for Biomedical Imaging Technology, U. of Connecticut; U. Penn.



Conclusions

1. Phenomena in the geosciences often have both regular ("cycles") and irregular ("noise") aspects.

2. We used the SSA-MTM Toolkit to analyze the periodic and quasi-periodic features ("cycles") of the SOI and Nile River records.

3. This helped us fill data gaps and study regime transitions in the amplitude and frequency modulation of climatic cycles.

4. We used Multi-Trend Analysis (MTA) to analyze irregularities, such as scaling and (multi-)fractality in a variety of applications.

5. This helped us establish a super-universality class of fractal Brownian motion, based on their hierarchical scaling properties (not shown).

6. Need both deterministic and stochastic modeling.

7. Does some combination of the two, + deterministic and stochastic modeling, provide a pathway to prediction?

TCD — http://www.atmos.ucla.edu/tcd/

E2-C2 — http://www.ipsl.jussieu.fr/~ypsce/py_E2C2.html