A Nonlinear Stochastic Model of the El Niño–Southern Oscillation: Random Dynamical Systems and Predictability

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1. Motivation

- What is the effect of stochastic (atmospheric?) perturbations on models of ENSO variability?
- Aim: Use of random dynamical system (RDS) theory to assess the dynamical effects of noise — “PDF dynamics”.

2. Random Dynamical Systems (RDSs)

- We have a model of the noise (\(\Omega, \mathcal{F}, \mathcal{P}, \theta\)) that is parametrized by time. The parametrization of realizations \(\omega\) is provided by a driving system \(\theta\) which is an ergodic one-parameter group.
- The dynamics is viewed on a “phase space × probability space,” \(X \times \Omega\), called the bundle, and the cocycle property enables one to treat trajectories as flows on this bundle.
- A path of the stochastic process corresponds to a selection of points in each fiber of the resulting bundle. Fibers are “glued together” by noise. The cocycle, also called RDS, provides a fiber-by-fiber view of the dynamics.

3. Random attractors and invariant measures

- We are looking for measures \(\mu\) on \(\Omega \times X\) that are invariant w.r.t. the dynamics. Central to our approach is the concept of disintegration \(\mu_\omega\) of \(\mu\). Mathematically, it is given by \(\mu(B) = \int \mu_\omega(B|\omega) d\omega\), with \(B\), the intersection of a measurable set \(B\) of \(X \times \Omega\) with an \(\omega\)-fiber.
- \(\mu_\omega\) is numerically computable for each fiber. It is supported by a geometric object: the random attractor (red on Fig. 3).

4. Application to an ENSO model

- Model: Low-order, coupled tropical-atmosphere-ocean model of ENSO (Timmermann & Jin, GRL 2002, hereafter T3M model). Three variables: thermocline depth anomaly \(h\), and SSTs \(T_1\) and \(T_2\) in the western and eastern basin.
- \(T_1 = \alpha(T_2 - T_1) - \frac{\bar{z}}{H}(T_2 - T_1)\),
- \(T_2 = \alpha(T_1 - T_2) - \frac{\bar{z}}{H}(T_2 - T_1)\),
- \(\dot{h} = \left(\dot{h} + \beta L(\sigma)\right)\),
- \(T_{sub} = \frac{T_1 - T_2}{2} - \frac{1}{2}\left(1 - \tanh(H + h_0 - z_0/h)\right)\).

5. Deterministic case: Shilnikov route to chaos

- Complex deterministic dynamics: Hopf bifurcation and Shilnikov horseshoes associated with multi-pulse homoclinic orbits, as \(\epsilon\) varies. Interdecadal chaotic variability. Note the presence of windows of stability with one attracting limit cycle.

6. Stochastic dynamics: Numerical results

- Below are random attractors for two different noise levels and different values \(\epsilon\) of zonal advection.
- Golden areas are most frequently visited (log-scale). The measures \(\mu_{\omega}\) are nearly singular.

7. Random attractor and predictability

- An episode in the random attractor’s life for the T3M model.

References