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State and Parameter Estimation for a Coupled Ocean-Atmosphere Model

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Parameter Estimation

a) Dynamical model (continuous time)

$dx/dt = M(x, \mu) + \eta(t)$

$y^o = H(x) + \varepsilon(t)$

Simple (EKF) idea – augmented state vector

$d\mu/dt = 0$, $X = (x^T, \mu^T)^T$

b) Statistical model

$L(\rho)\eta = w(t)$, L – AR(MA) model, $\rho = (\rho_1, \rho_2, \ldots. \rho_M)$

Examples: 1) Dee *et al*. (*IEEE*, 1985) – estimate a few parameters in the covariance matrix $Q = E(\eta, \eta^T)$; also the bias $<\eta> = E\eta$.

2) POPs – Hasselmann (1982, Tellus); Penland (1989, *MWR*; 1996, *Physica D*); Penland & Ghil (1993, *MWR*) – estimate L and Q entirely from data, linear.

3) $dx/dt = M(x, \mu) + \eta$: estimate both M & Q from data (Dee, 1995, *QJRMS*);

nonlinear approach – empirical mode reduction (EMR: Kravtsov *et al*., 2005; Kondrashov *et al*., 2005, 2006).

Extended Kalman Filter (EKF)

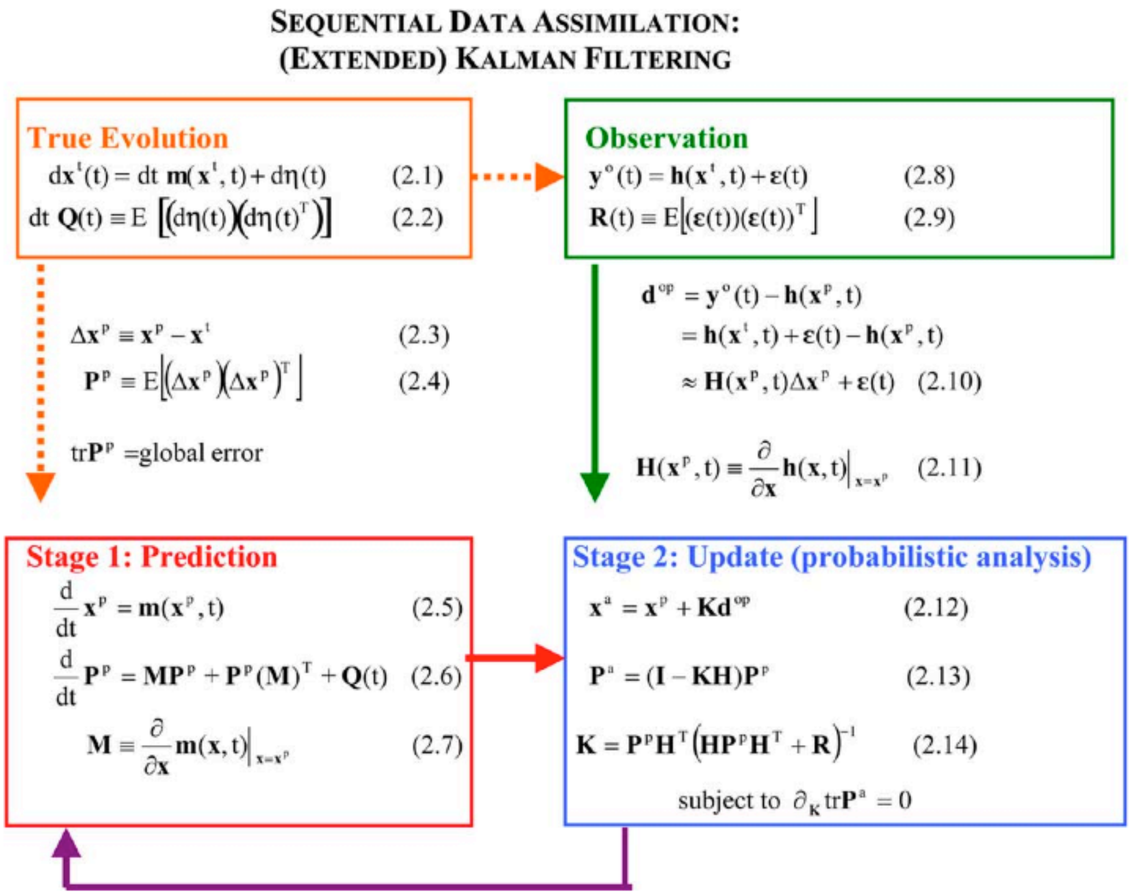


Fig. 1. A flow-chart representation of the EKF method (see Table 1 for definitions of the symbols).

Sequential parameter estimation

- "**State augmentation**" method – uncertain parameters are treated as additional state variables.
- Example – one unknown parameter μ, discrete time:

$$\bar{x}_k = \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} = \begin{pmatrix} F(x_{k-1}, \mu_{k-1}) \\ \mu_{k-1} \end{pmatrix} + \begin{pmatrix} \epsilon_k \\ \epsilon^{\mu}_{k-1} \end{pmatrix}$$

$$y^o_k = \begin{pmatrix} H & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} + \epsilon^0 = \bar{H}\bar{x}_k + \epsilon^0$$

$$\bar{x}^a_k = \bar{x}^f_k + \bar{K}(y^o_k - \bar{H}\bar{x}^f_k); \quad \bar{K} = \bar{P}^f \bar{H}^T (\bar{H}\bar{P}^f \bar{H}^T + R)^{-1}$$

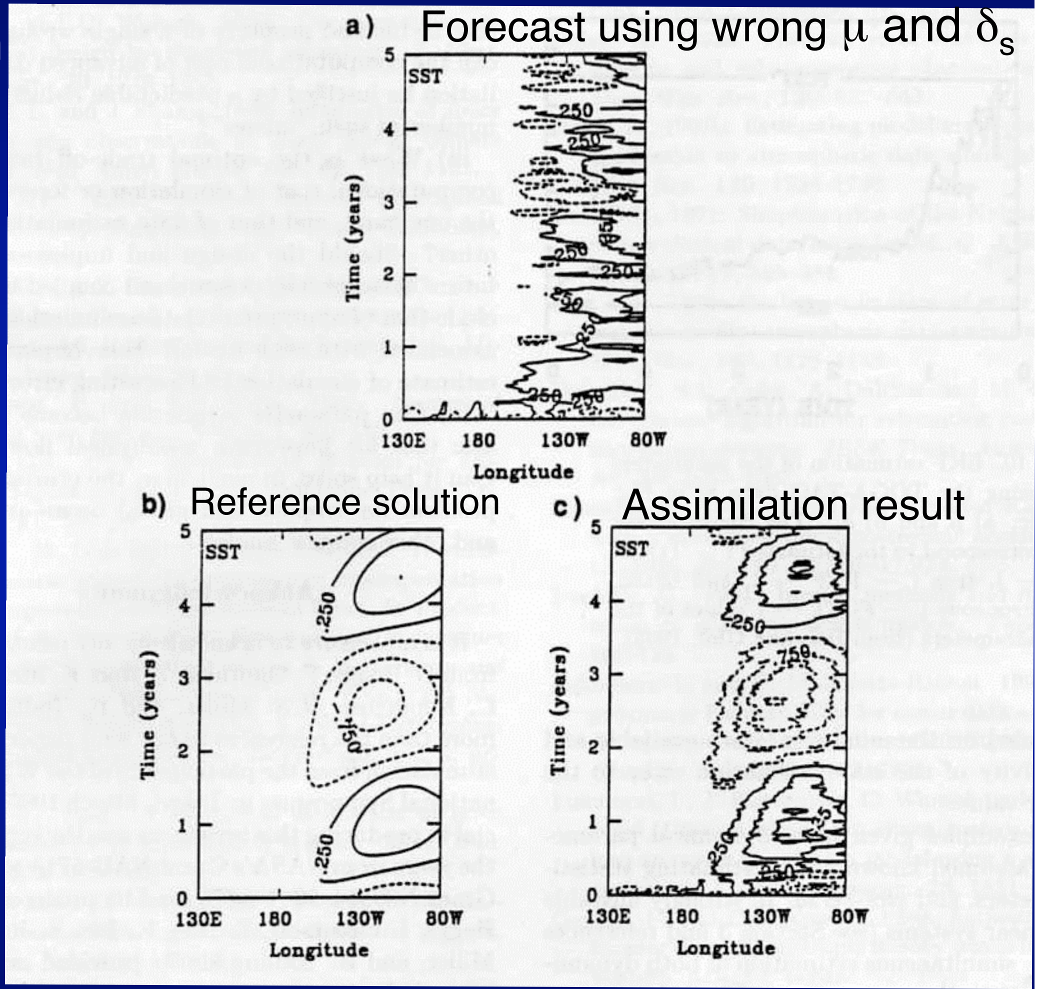
- **The parameters are not directly observable, but** the **cross-covariances** drive parameter changes from innovations of the state:

$$\bar{P}^f = \begin{pmatrix} P^f_{xx} & P^f_{x\mu} \\ P^f_{\mu x} & P^f_{\mu\mu} \end{pmatrix}; \quad \bar{K} = \begin{pmatrix} P^f_{xx} H^T \\ P^f_{\mu x} H^T \end{pmatrix} (H P^f_{xx} H^T + R)^{-1}$$

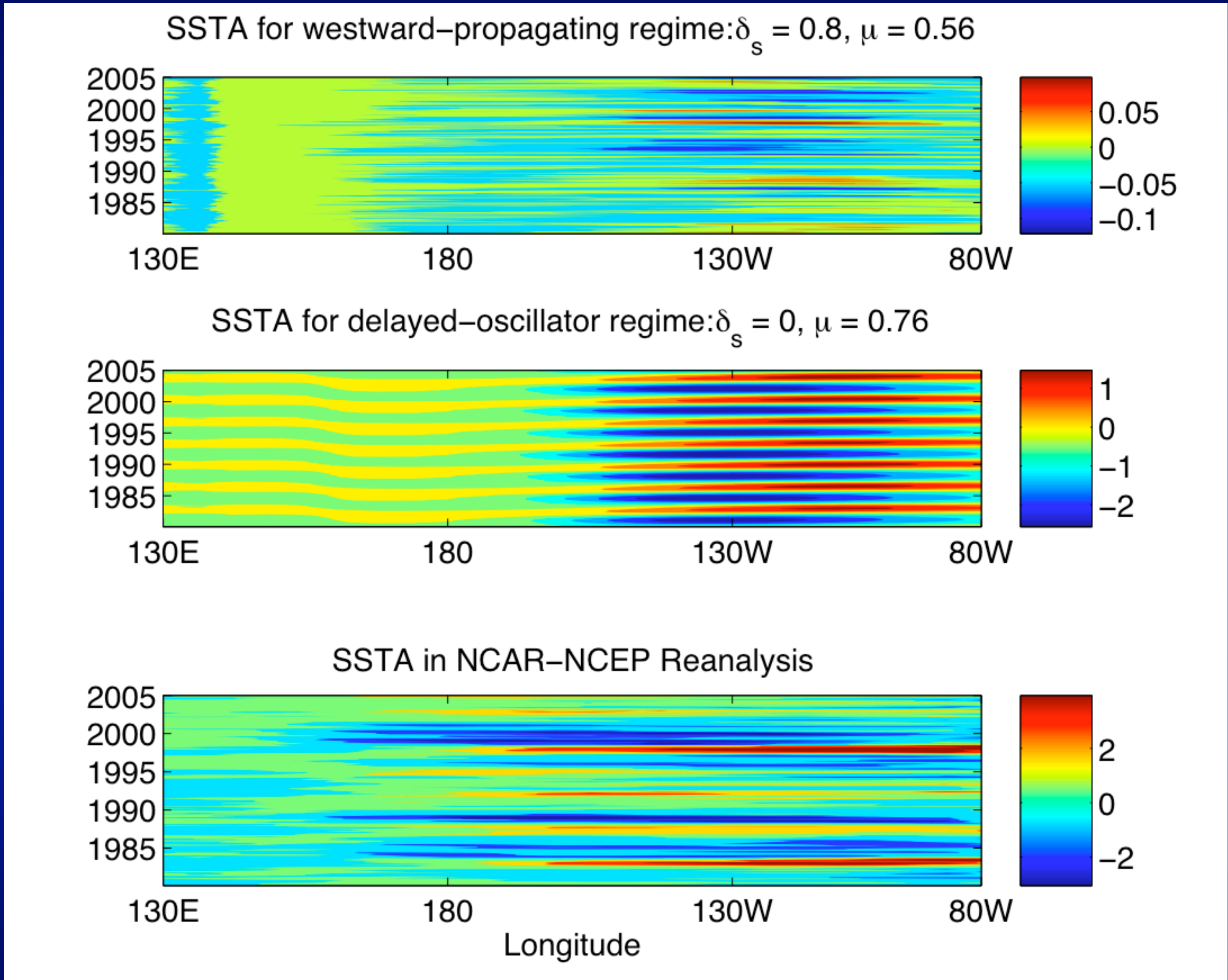
- Parameter estimation is always a **nonlinear problem,** even if the model is **linear** in terms of the model state: use **Extended Kalman Filter (EKF).**

Parameter estimation for coupled O-A system

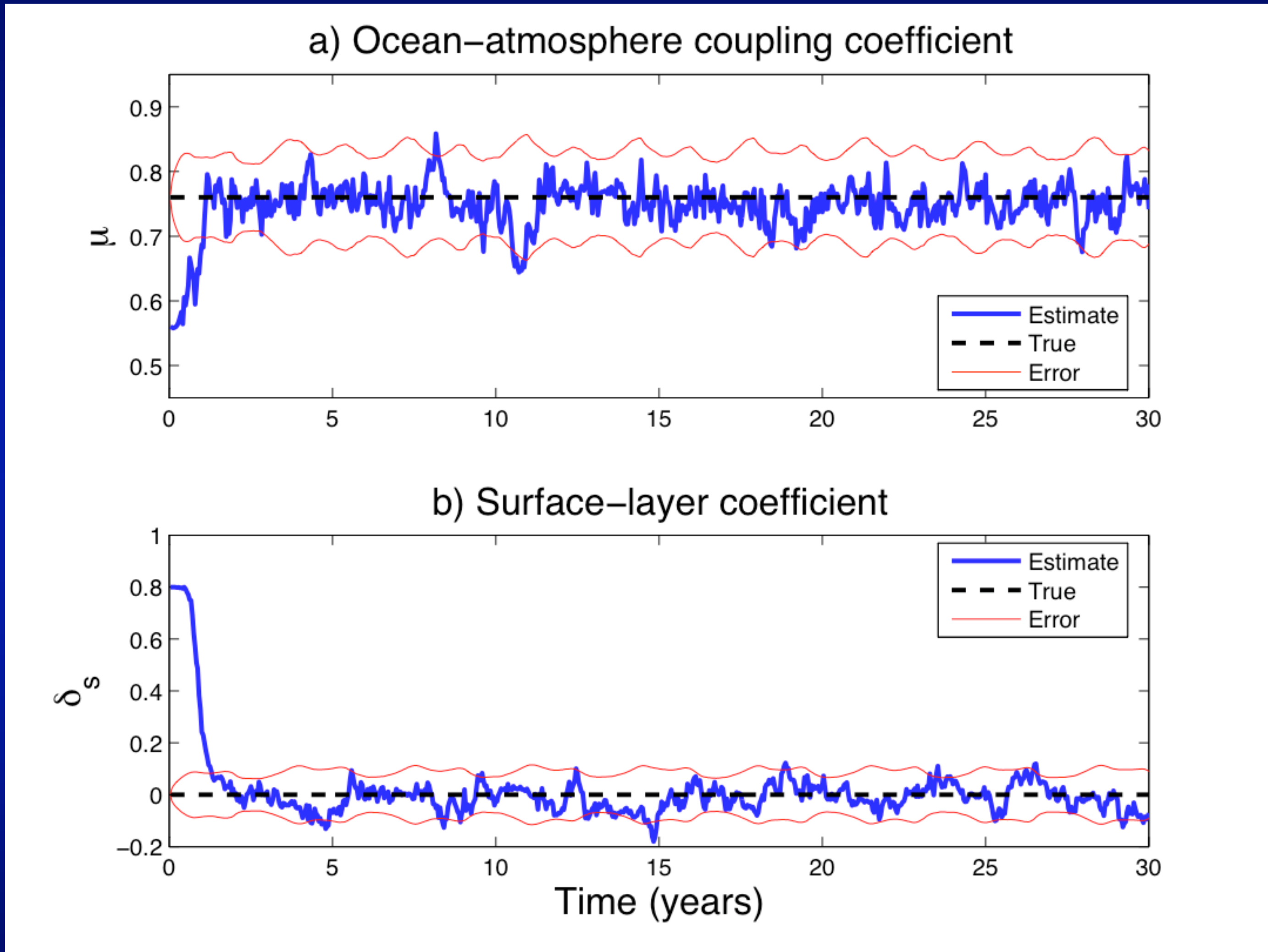
- Intermediate coupled model (ICM: Jin & Neelin, *JAS*, 1993)
- Estimate the state vector $W = (T', h, u, v)$, along with the coupling parameter μ and surface-layer coefficient δ_s by assimilating data from a single meridional section.
- The ICM model has errors in its initial state, in the wind stress forcing, & in the parameters.
- Hao & Ghil (1995, *Proc. WMO Symp. DA Tokyo*); Ghil (1997, *JMSJ*); Sun *et al*. (2002, *MWR*).
- *Current work with D. Kondrashov, J.D. Neelin (UCLA), & C.-j. Sun + I. Fukumori (JPL).*



Coupled O-A Model (ICM) vs. Observations

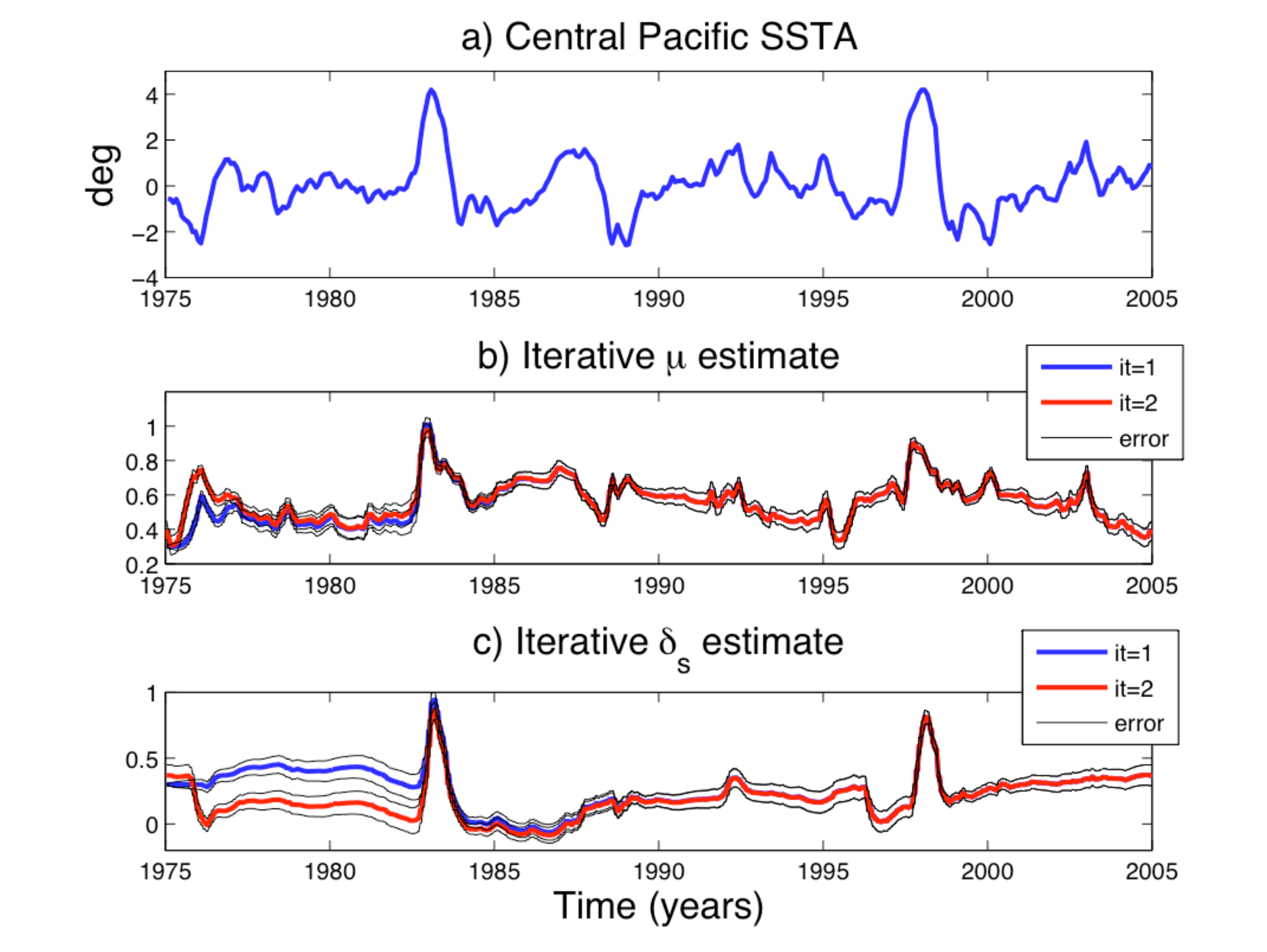


Convergence of Parameter Values – I



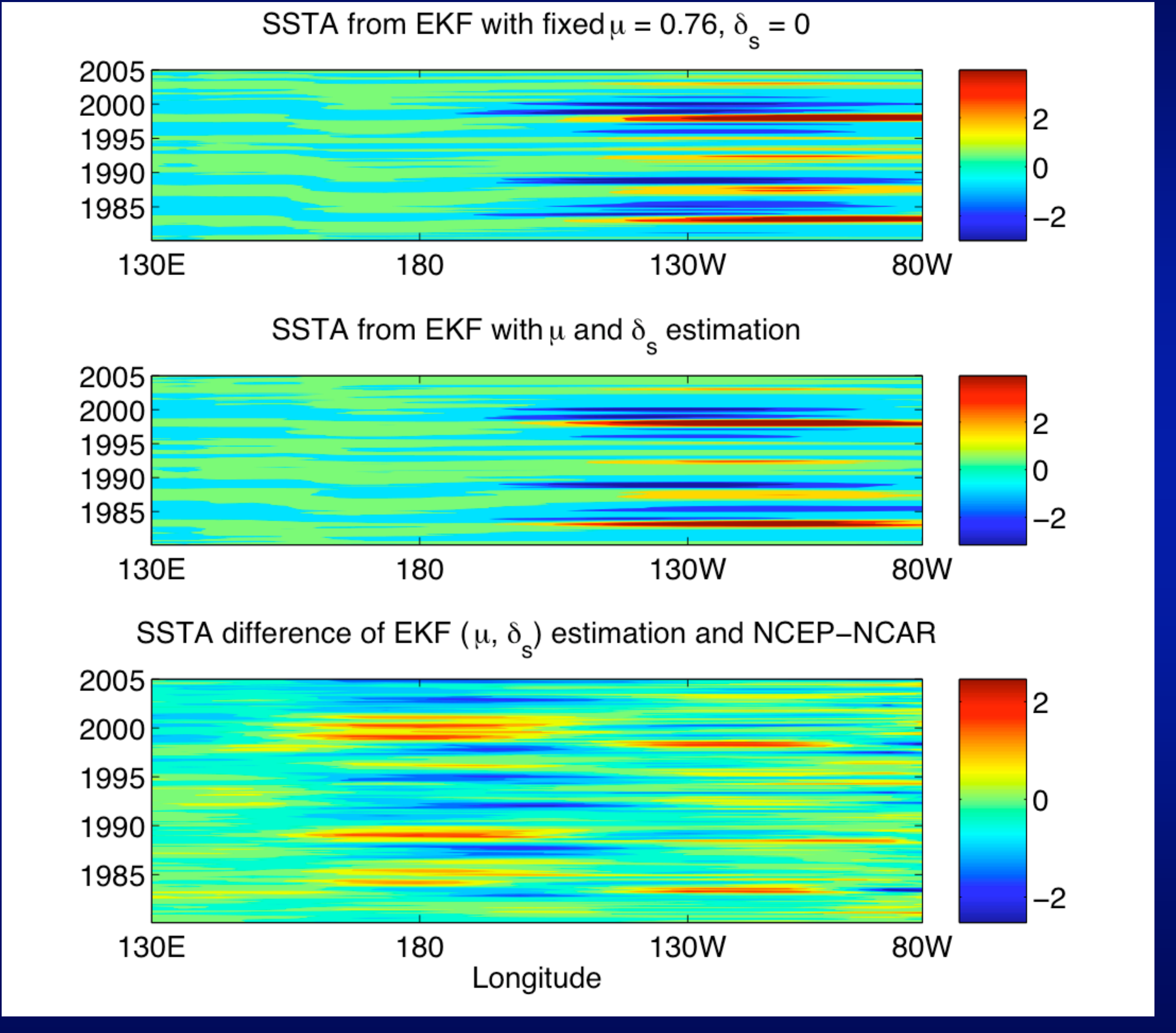
Identical-twin experiments

Convergence of Parameter Values – II



SST anomaly data from reanalysis

EKF results with and w/o parameter estimation



Summary & future work

- Sequential estimation (EKF, etc.) of model (& noise) parameters is possible.
- The state augmentation method is conceptually simple: it uses state observations only to estimate model parameters, too.
- Including parameter estimation improves the state estimation, given the same observing system.
- Identical-twin experiments in an ICM of tropical ocean-atmosphere variability exhibit convergence of the parameters within a few years; this convergence depends on the initial error in the parameters, but not on the model or observational errors.
- When using NCEP-NCAR reanalysis data of SST anomalies, parameter estimation is successful, too.
- In this case, the estimated parameter values undergo large shifts at the two major ENSO events of 1982-83 and 1987-88: these correspond to shifts in the ICM's regimes (westward-propagating vs. delayed-oscillator).
- Collaboration with the JPL (I. Fukumori) and GFDL-Princeton (G. Philander & A. Rosati) teams aims at implementing these parameter-estimation ideas in a fully coupled O-A GCM.

Computational advances

a) Hardware

- more computing power (CPU throughput)
- larger & faster memory (3-tier)

b) Software

- better numerical implementations of algorithms
- automatic adjoints
- block-banded, reduced-rank, & other sparse-matrix algorithms
- better ensemble filters
- efficient parallelization, ….

How much DA *vs*. forecast?

- Design integrated observing–forecast–assimilation systems!

Observing system design

- **Need no more (independent) observations than *d-o-f* to be tracked:**
  - "features" (Ide & Ghil, 1997a, b, *DAO*);
  - instabilities (Todling & Ghil, 1994 + Ghil & Todling, 1996, *MWR*);
  - trade-off between mass & velocity field (Jiang & Ghil, *JPO*, 1993).
- The cost of **advanced** DA is **much** less than that of instruments & platforms:
  - at best use DA **instead** of instruments & platforms.
  - at worst use DA to determine **which** instruments & platforms (**advanced** OSSE)
- **Use any** observations, if forward modeling is possible (observing operator H)
  - satellite images, 4-D observations;
  - pattern recognition in observations and in phase-space statistics.

General references

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