Data Assimilation in Meteorology and Oceanography

Michael Ghil
Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

Joint work with

Dmitri Kondrashov, UCLA, and many others:

please see http://www.atmos.ucla.edu/tcd/

Outline

- Data in meteorology and oceanography
  - *in situ &* remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - transfer of information
    - between variables & regions
  - stability of the fcst.–assimilation cycle
  - filters & smoothers
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic ("classic")
  - stochastic – "dynamics" & "physics"
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

Main issues

- The solid earth stays put to be observed, the atmosphere, the oceans, & many other things, do not.
- Two types of information:
  - direct → observations, and
  - indirect → dynamics (from past observations);

  both have errors.
- Combine the two in (an) optimal way(s)
- Advanced data assimilation methods provide such ways:
  - sequential estimation → the Kalman filter(s), and
  - control theory → the adjoint method(s)
- The two types of methods are essentially equivalent for simple linear systems (the duality principle)

Main issues (continued)

- Their performance differs for large nonlinear systems in:
  - accuracy, and
  - computational efficiency
- Study optimal combination(s), as well as improvements over currently operational methods (OI, 4-D Var, PSAS).

Outline

➤ Data in meteorology and oceanography
  - *in situ &* remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - filters & smoothers
  - stability of the fcst.-assimilation cycle
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic (“classic”)
  - stochastic – “dynamics” & “physics”
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

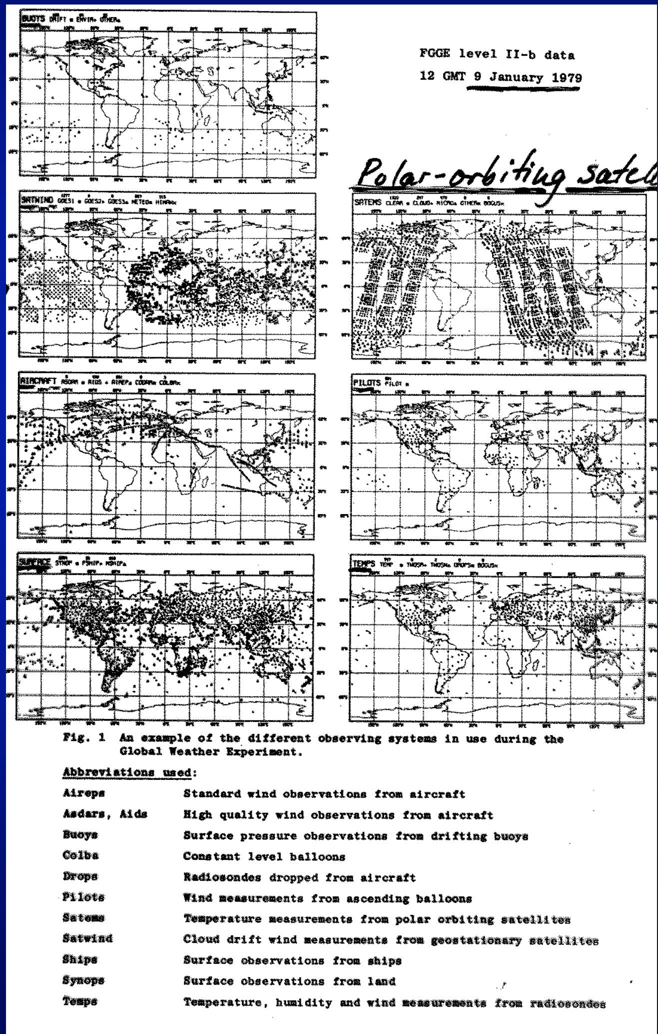
Atmospheric data

Drifting buoys: P_s – 267

Cloud-drift: V – 2x2259

Aircraft: V – 2x1100

Ship & land surface: P_s, T_s, V_s – 4x3446



Polar orbiting satellites: T – 5x2048

Balloons : V – 2x581x10

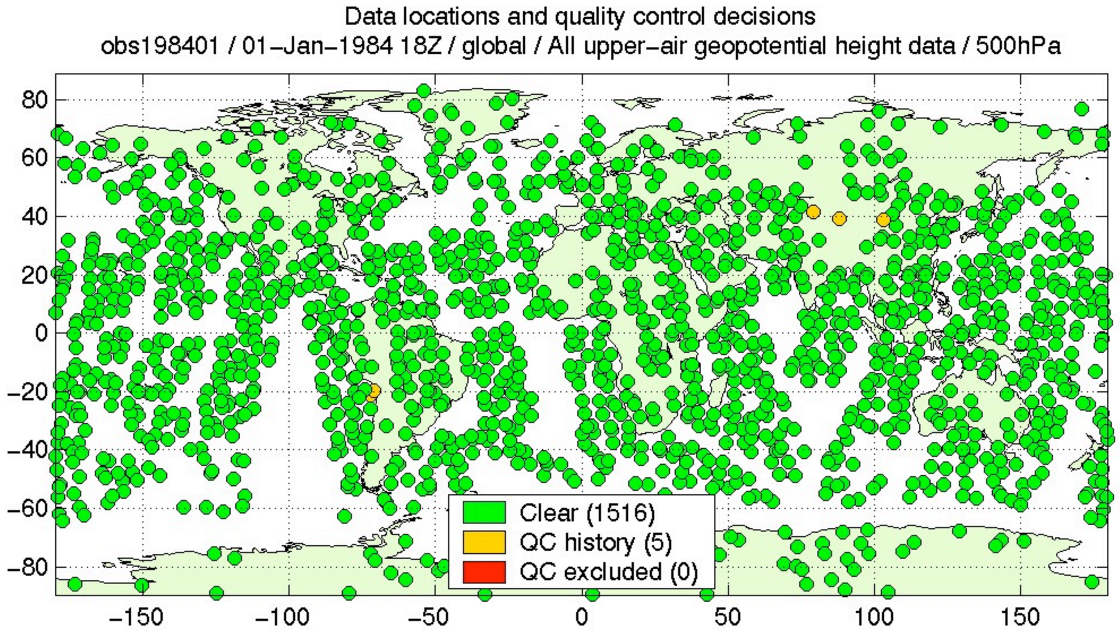
Radiosondes : T, V - 3x749x10

Total no. of observations = $0(10^5)$ scalars per 12h–24h

⁂ $0(10^2)$ observations/[(significant *d-o-f*) x (significant Δt)]

Bengtsson, Ghil & Källén (eds.): *Dynamic Meteorology, Data Assimilation Methods* (1981)

Observational network



Quality control – preliminary & as part of the assimilation cycle

Ocean data – past

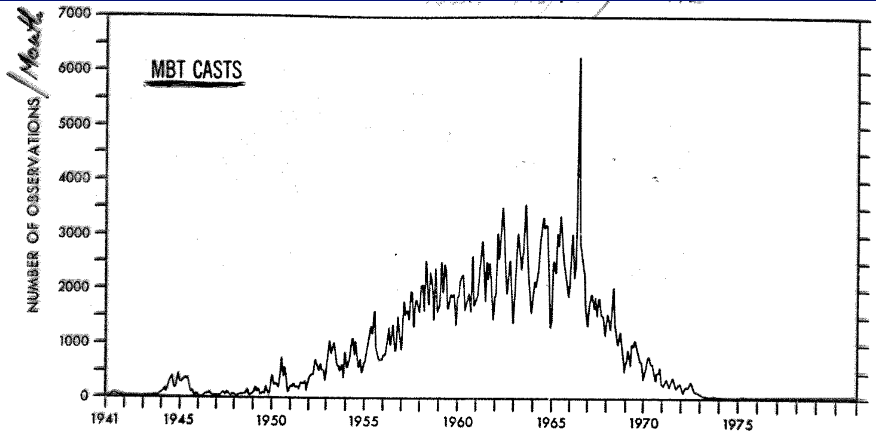


Figure 4.—Time series of MBT casts.

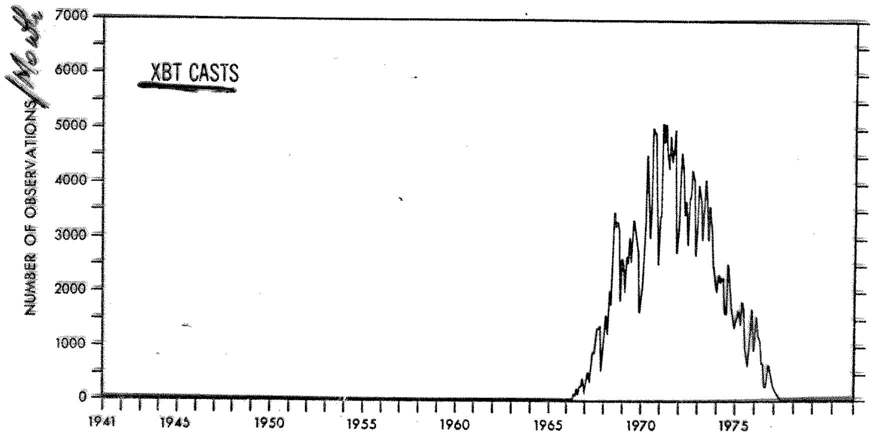
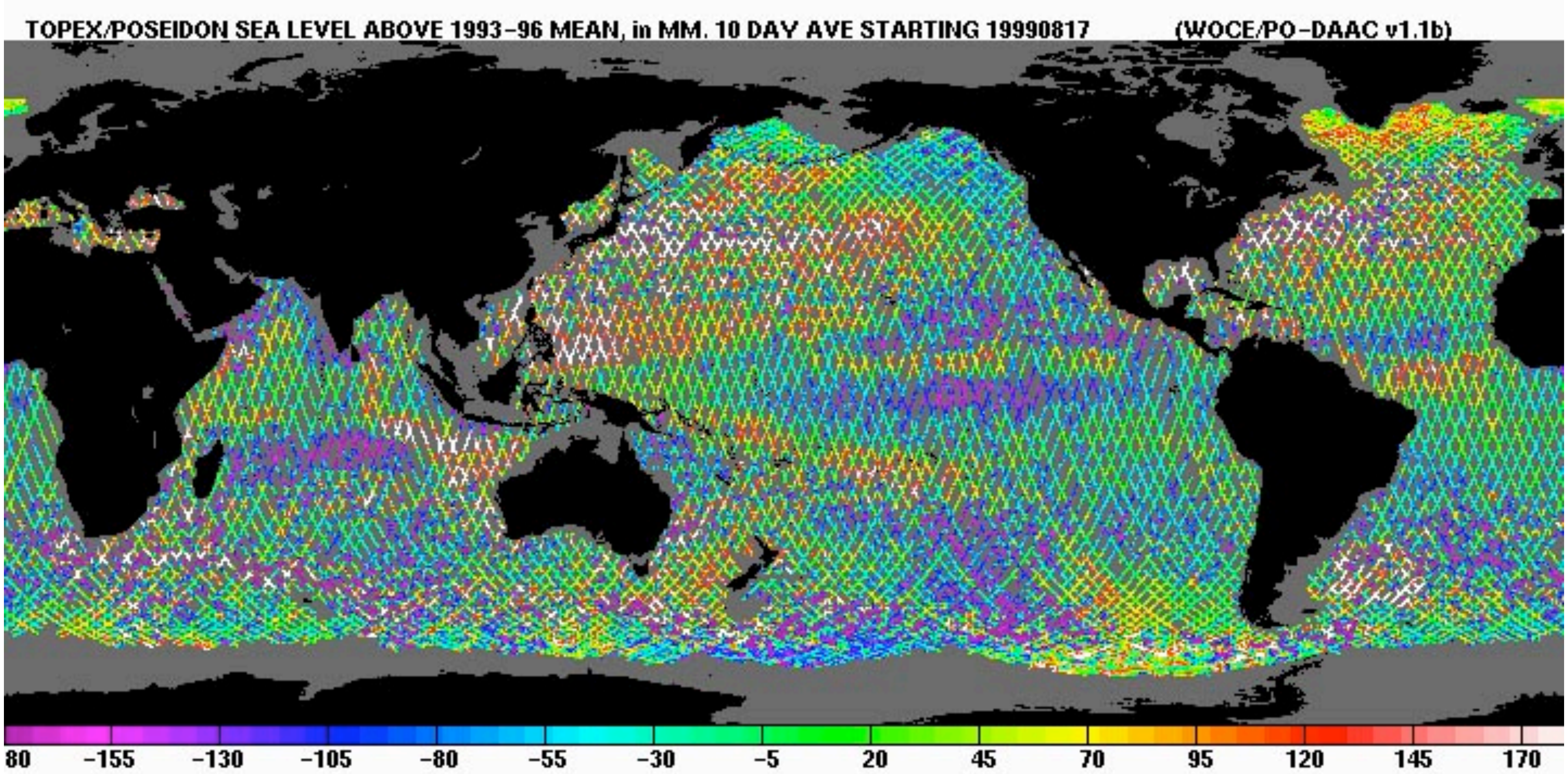


Figure 5.—Time series of XBT casts.

Total no. of oceanographic observations/met. ob'sns
= $O(10^{-4})$ for the past; &
= $O(10^{-1})$ for the future :
Syd Levitus (1982).

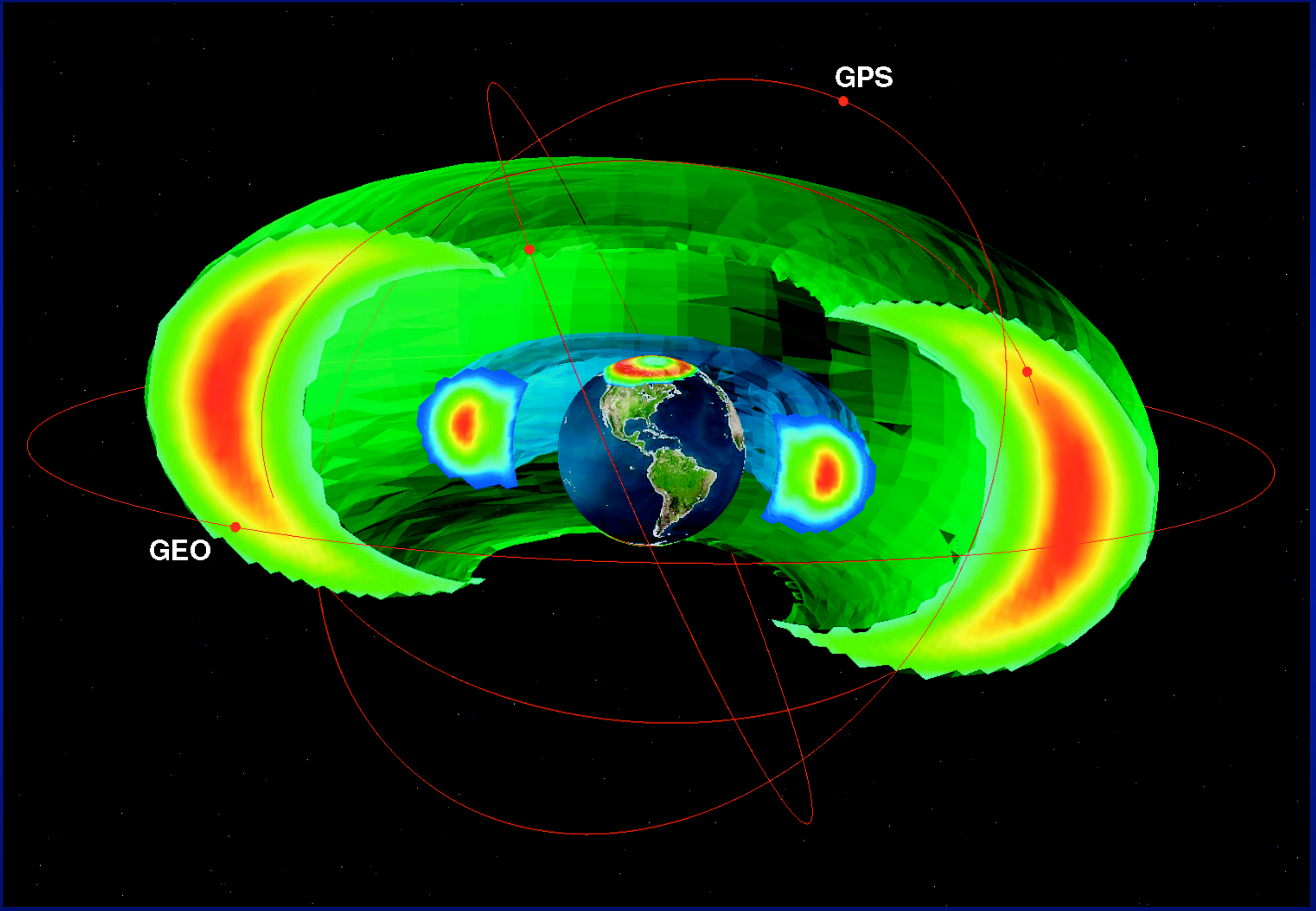
Ocean data – present & future

Altimetry ⇒ sea level; scatterometry ⇒ surface winds & sea state; acoustic tomography ⇒ temperature & density; etc.



Courtesy of Tony Lee, JPL

Space physics data



Space platforms in Earth's magnetosphere

Basic ideas of data assimilation and sequential estimation - I

Simple illustration

Want to estimate
u - temperature of this room, based on the readings
u_1 and u_2 of the two thermometers.

Estimate $\hat{u} = \alpha_1 u_1 + \alpha_2 u_2$

Interpretation will be:
$u_1 = u_f$ - **first guess** (of numerical forecast model)
$u_2 = u_o$ - **observation** (R/S, satellite, etc.)
$\hat{u} = u_a$ - **objective analysis**

Basic ideas of data assimilation and sequential estimation - II

If u_1 and u_2 are unbiased, and $\hat{u}$ should be unbiased, then

$\alpha_1 + \alpha_2 = 1$,

so one can write $\hat{u} = u_1 + \alpha_2(u_2 - u_1)$: updating (sequential)

If u_1 and u_2 are uncorrelated, and have

$A_1 = \sigma_1^{-2}$, $A_2 = \sigma_2^{-2}$: known standard deviations,

Then the minimum variance estimator(*) is

$$\hat{u} = u_1 + A_2 /(A_2 - A_1) (u_2 - u_1)$$

and its accuracy is

$$\hat{A} = (A_1 + A_2) \geq \max \{A_1, A_2\}$$

* BLUE = Best Linear Unbiased Estimator

Kalman Filter - I

Linear unbiased data assimilation scheme

(1a) $\mathbf{x}_k^{\mathrm{f}} = \mathbf{M}_{k-1}\mathbf{x}_{k-1}^{\mathrm{a}}$, f–forecast, a–analysis

(1b) $\mathbf{x}_k^{\mathrm{a}} = \mathbf{x}_k^{\mathrm{f}} + \mathbf{K}_k(\mathbf{y}_k^{\mathrm{o}} - \mathbf{H}_k\mathbf{x}_k^{\mathrm{f}})$, $\mathbf{x}$–state vector

Remarks

1. Many operational schemes are (1a,b).
2. Want *optimal* $\mathbf{K}$.

Assumptions

(2a) $\mathbf{x}_k^{\mathrm{t}} = \mathbf{M}_{k-1}\mathbf{x}_{k-1}^{\mathrm{t}} + \eta_{k-1}^{\mathrm{t}}$, t–true state;

(2b,c) $\overline{\eta_k^{\mathrm{t}}} = 0$, $\overline{\eta_k^{\mathrm{t}}\eta_l^{\mathrm{t}}} = \mathbf{Q}_k\delta_{kl}$, system noise;

(2d) $\mathbf{y}_k^{\mathrm{o}} = \mathbf{H}_k\mathbf{x}_k^{\mathrm{t}} + \epsilon_k^{\mathrm{o}}$, o–observations;

(2e,f,g) $\overline{\epsilon_k^{\mathrm{o}}} = 0$, $\overline{\epsilon_k^{\mathrm{o}}\epsilon_l^{\mathrm{o}}} = \mathbf{R}_k\delta_{kl}$, $\overline{\epsilon_k^{\mathrm{o}}\eta_k^{\mathrm{t}}} = 0$.

Kalman Filter - II

Given (1,2), the error covariance matrices

$$\mathbf{P}_k^{\mathrm{f,a}} \equiv \overline{(\mathbf{x}_k^{\mathrm{f,a}} - \mathbf{x}_k^{\mathrm{t}})(\mathbf{x}_k^{\mathrm{f,a}} - \mathbf{x}_k^{\mathrm{t}})^{\mathrm{T}}}$$

evolve in a known way:

(3a) $\mathbf{P}_k^{\mathrm{f}} = \mathbf{M}_{k-1}\mathbf{P}_{k-1}^{\mathrm{a}}\mathbf{M}_{k-1}^{\mathrm{T}} + \mathbf{Q}_{k-1}$,

(3b) $\mathbf{P}_k^{\mathrm{a}} = (\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)\mathbf{P}_k^{\mathrm{f}}(\mathbf{I} - \mathbf{K}_k\mathbf{H}_k)^{\mathrm{T}} + \mathbf{K}_k\mathbf{R}_k\mathbf{K}_k^{\mathrm{T}}$.

Remarks

1. By advancing $\mathbf{P}_k^{\mathrm{f,a}}$ along with $\mathbf{x}_k^{\mathrm{f,a}}$, one can
   - know how well $\mathbf{x}_k^{\mathrm{t}}$ is estimated by $\mathbf{x}_k^{\mathrm{f,a}}$ for any $\mathbf{K}_k$;
   - determine the optimal $\mathbf{K}_k$.
2. Problems
   - computational — N forecasts
   - parameter estimation — find $\mathbf{M}_k$, $\mathbf{Q}_k$, $\mathbf{R}_k$ in the assimilation process
   - nonlinearities — $\mathbf{M}_k = \mathbf{M}_k(\mathbf{x}_k)$

Kalman Filter - III

Optimal gain matrix $\mathbf{K}_k$ is obtained by minimizing

$$\overline{(\mathbf{x}_k^a - \mathbf{x}_k^{\mathrm{t}})^{\mathrm{T}} \mathbf{A} (\mathbf{x}_k^a - \mathbf{x}_k^{\mathrm{t}})} \quad \text{for} \quad \mathbf{A}^{\mathrm{T}} = \mathbf{A} \geq 0.$$

N.B. The *Kalman filter* is (1,3) with this optimal K.

The result is

$$(4) \quad \mathbf{K}_k = \mathbf{K}_k^* \equiv \mathbf{P}_k^{\mathrm{f}} \mathbf{H}_k^{\mathrm{T}} (\mathbf{H}_k \mathbf{P}_k^{\mathrm{f}} \mathbf{H}_k^{\mathrm{T}} + \mathbf{R}_k)^{-1}.$$

Properties of K–filter

1) $\mathbf{K}_k^*$ is independent of $\mathbf{A}$ chosen in minimization (energy or enstrophy norm).

2) $\mathbf{K}_k^*$ depends on $\mathbf{P}_k^{\mathrm{f}}$: $\mathbf{K}_k \sim [1 + (\mathbf{P}_k^{\mathrm{f}})^{-1} \mathbf{R}_k]^{-1}$

3) Given (4),

$$\mathbf{P}_k^{\mathrm{a}} = (\mathbf{I} - \mathbf{K}_k \mathbf{H}_k) \mathbf{P}_k^{\mathrm{f}}.$$

4) The filter is sequential ("recursive"), *i.e.*, all past observations can be discarded (one–step processing).

Kalman Filter - IV

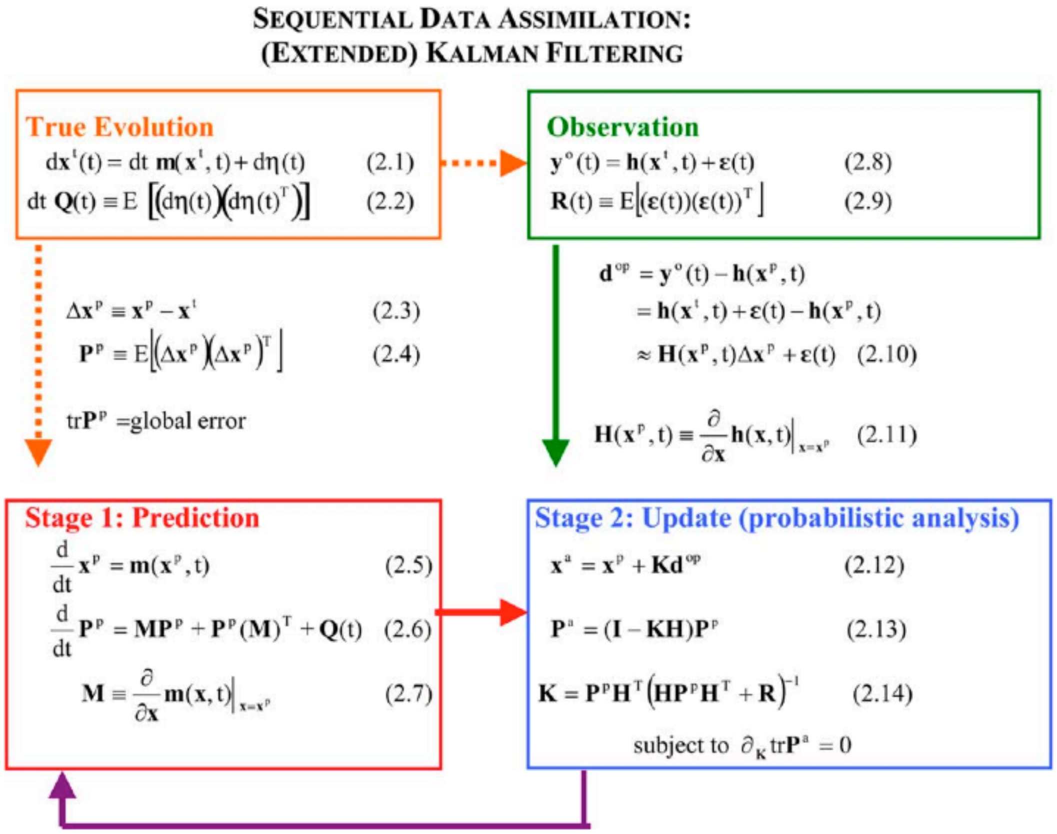


Fig. 1. A flow-chart representation of the EKF method (see Table 1 for definitions of the symbols).

Basic concepts: barotropic model

Shallow-water equations in 1-D, linearized about $(U,0,\Phi)$, $fU = -\Phi_y$

U = 20 ms^{-1}, f = 10^{-4}s^{-1}, $\Phi = gH$, $H \approx$ 3 km.

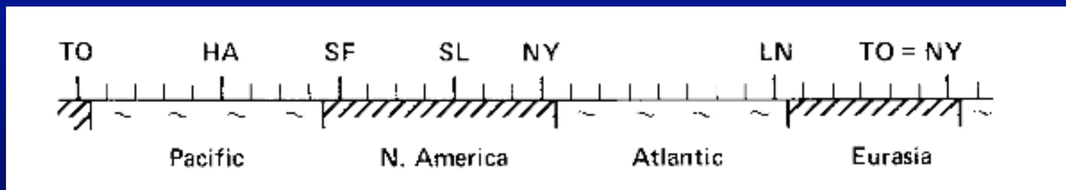
$$u_t + Uu_x + \phi_x - fv = 0$$

$$v_t + Uv_x + fu = 0$$

$$\phi_t + U\phi_x + \Phi u_x - fUv = 0$$

PDE system discretized by finite differences, periodic B. C.

$\mathbf{H}_k$: observations at synoptic times, over land only.



Ghil *et al*. (1981), Cohn & Dee (Ph.D. theses, 1982 & 1983), etc.

Trade-off between variables

•Some variables

are observed,

•others are not.

Height
obs’d

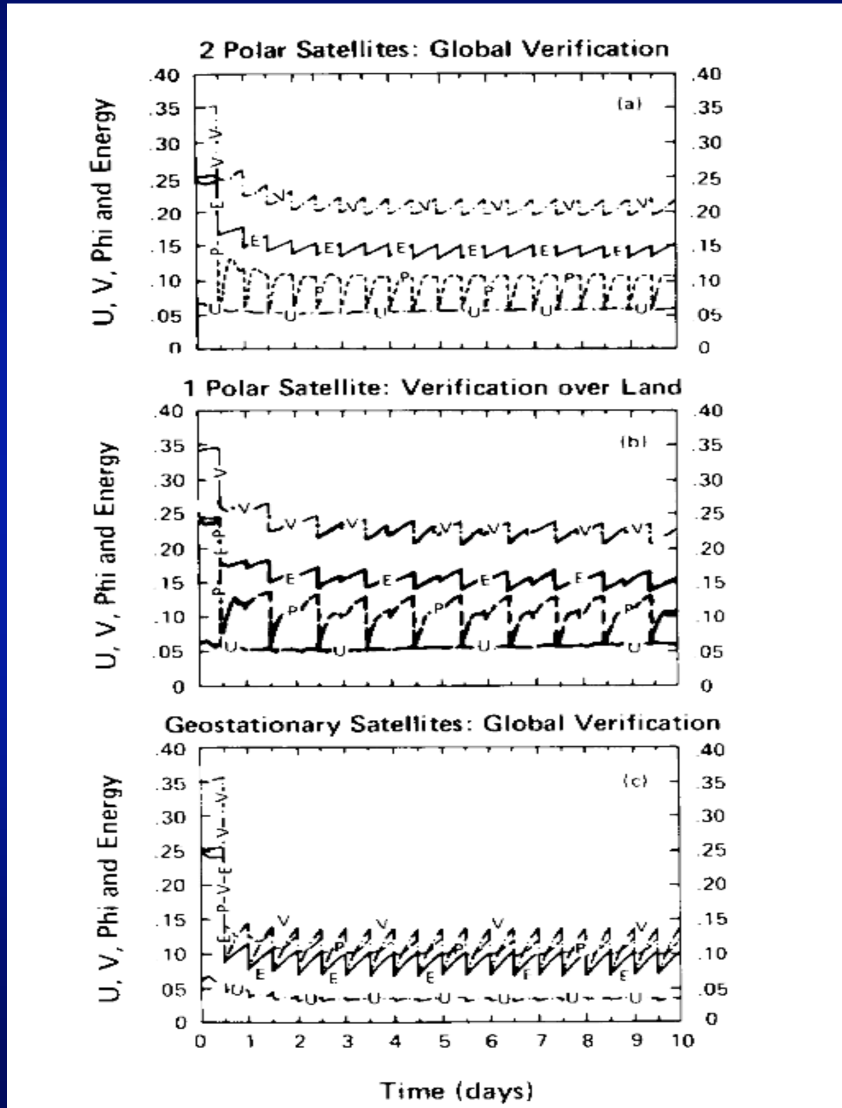
Wind info. is better (here)

than height info.

Observing System Simulation Experiments (OSSE)

Identical twins *vs.* real observations

Wind
obs’d



Conventional network

Relative weight of observational *vs.* model errors

$P_\infty = QR/[Q + (1 - \Psi^2)R]$

(a) $Q = 0 \Rightarrow P_\infty = 0$

(b) $Q \neq 0 \Rightarrow$ (i), (ii) and (iii):

(i) "good" observations

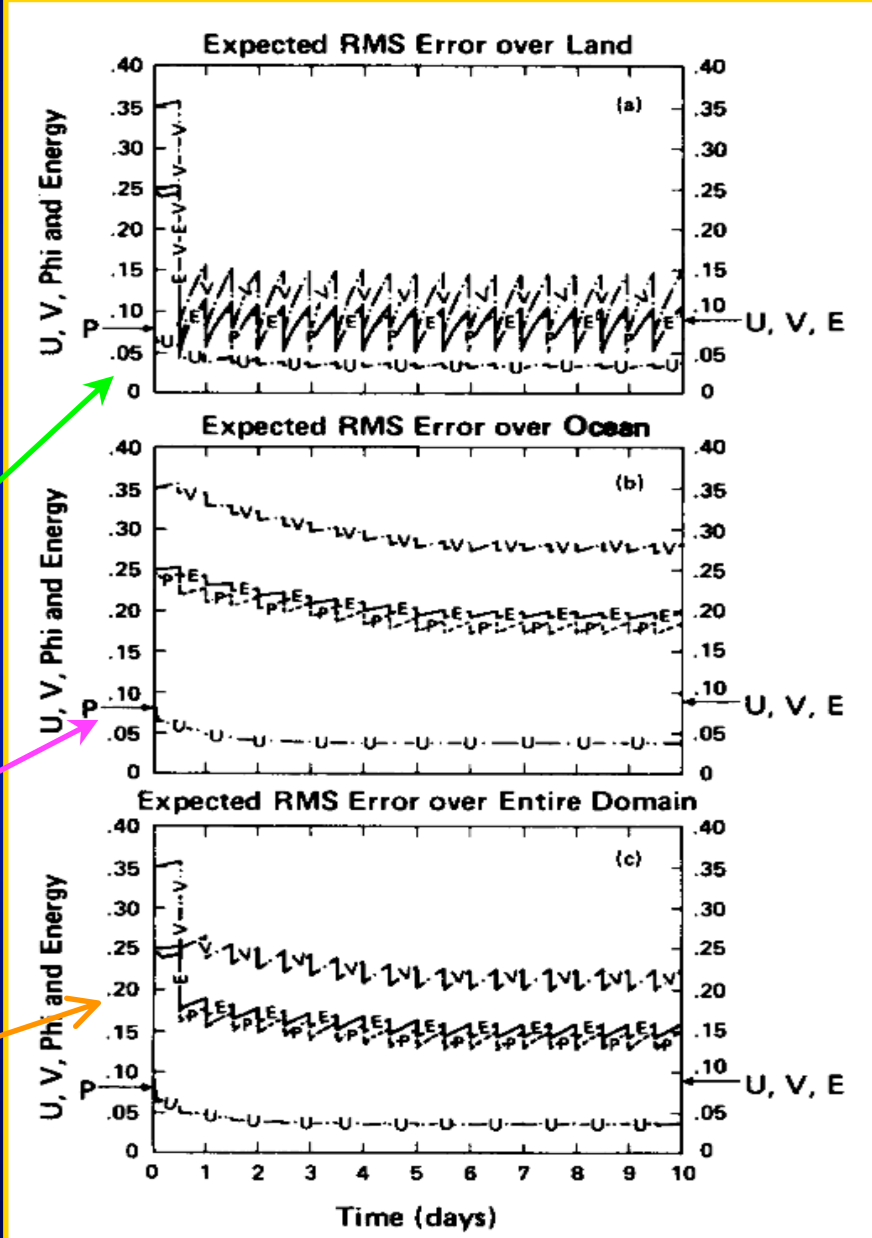
$R << Q \Rightarrow P_\infty \approx R$;

(ii) "poor" observations

$R >> Q \Rightarrow P_\infty \approx Q/(1 - \Psi^2)$;

(iii) always (provided $\Psi^2 < 1$)

$P_\infty \leq \min\{R, Q/(1 - \Psi^2)\}$.



Advection of information

Upper panel (NoSat):

Errors advected off the ocean

ϕ_{300}

Lower panel (Sat):

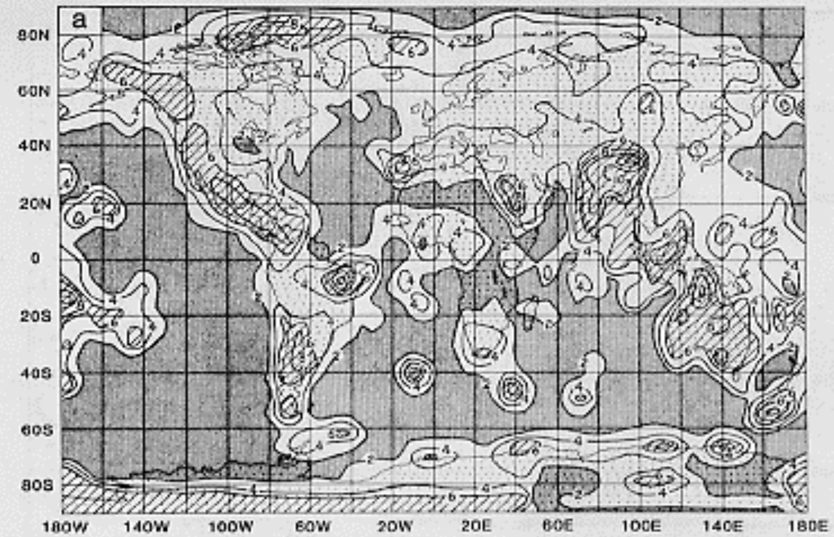
Errors drastically reduced, as info. now comes in, off the ocean

ϕ_{300}

Halem, Kalnay, Baker & Atlas

(*BAMS*, 1982)

{6h fcst} - {conventional (NoSat)}



{"first guess"} - {FGGE analysis}

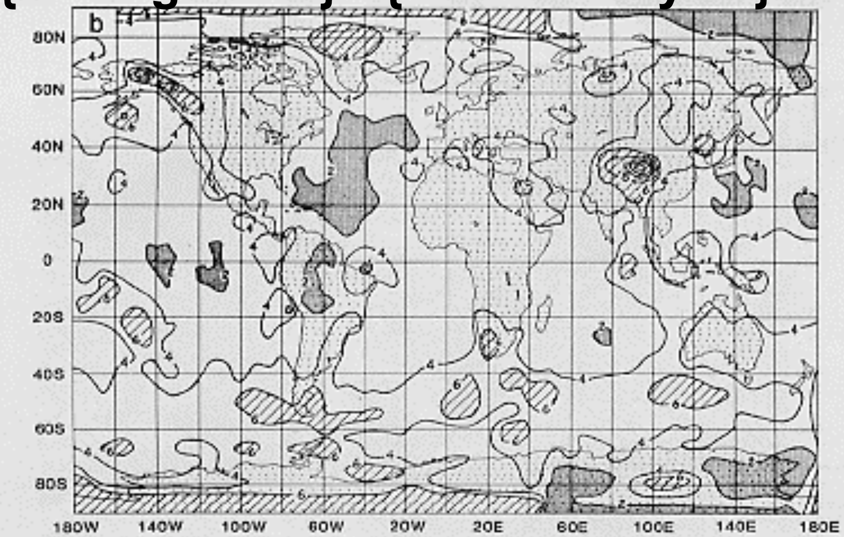


FIG. 5. The rms difference between the 6 h forecast of the 300 mb geopotential height field and the analysis for the period 5–21 January 1979. Contour interval is 20 m. a) Rms difference between the NOSAT analysis and forecast. b) Rms difference between the FGGE analysis and forecast.

Evolution of DA – I

TABLE I. CHARACTERISTICS OF DATA ASSIMILATION SCHEMES IN OPERATIONAL USE AT THE END OF THE 1970s[a]

Organization or country	Operational analysis methods	Analysis area	Analysis/forecast
Australia	Successive correction method (SCM)	SH[d]	12 hr
	Variational blending techniques	Regional	6 hr
Canada	Multivariate 3-D statistical interpolation	NH[d] Regional	6 hr (3 hr for the surface)
France	SCM; wind-field and mass-field balance through first guess	NH	6 hr
	Multivariate 3-D statistical interpolation	Regional	
F.R. Germany	SCM. Upper-air analyses were built up, level by level, from the surface	NH	12 hr (6 hr for the surface)
	Variational height/wind adjustment		Climatology only as preliminary fields
Japan	SCM	NH	12 hr
	Height-field analyses were corrected by wind analyses	Regional	
Sweden	Univariate 3-D statistical interpolation	NH	12 hr
	Variational height/wind adjustment	Regional	3 hr
United Kingdom	Hemispheric orthogonal polynomial method		
	Univariate statistical interpolation (repeated insertion of data)	Global	6 hr
U.S.A.	Spectral 3-D analysis	Global	
	Multivariate 3-D statistical interpolation	Global	6 hr
U.S.S.R.	2-D[c] statistical interpolation	NH	12 hr
ECMWF[b]	Multivariate 3-D statistical interpolation	Global	6 hr

[a] After Gustafsson (1981).
[b] European Centre for Medium Range Weather Forecasts.
[c] 2-D is in a horizontal plane.
[d] Southern Hemisphere and Northern Hemisphere, respectively.

Transition from "early" to "mature" phase of DA in NWP:

- no Kalman filter ⇨ Ghil *et al.*, 1981(*)
- no adjoint ⇨ Lewis & Derber (*Tellus*, 1985); Le Dimet & Talagrand (*Tellus*, 1986)

(*) Bengtsson, Ghil & Källén (Eds., 1981), *Dynamic Meteorology: Data Assimilation Methods.*

M. Ghil & P. M.-Rizzoli (*Adv. Geophys.*, 1991).

Evolution of DA – II

TABLE IV. DUALITY RELATIONSHIPS BETWEEN STOCHASTIC ESTIMATION AND DETERMINISTIC CONTROL[a]

A. Continuous (linear) Kalman Filter

System Model	$\dot{\mathbf{w}}^t(t) = F(t)\mathbf{w}^t(t) + G(t)\mathbf{b}^t(t), \quad \mathbf{b}^t(t) \sim \mathrm{N}[0, Q(t)]$
Measurement Model	$\mathbf{w}^o(t) = H(t)\mathbf{w}^t(t) + \mathbf{b}^o(t), \quad \mathbf{b}^o(t) \sim \mathrm{N}[0, R(t)]$
State estimation	$\dot{\mathbf{w}}^a(t) = F(t)\mathbf{w}^a(t) + K(t)[\mathbf{w}^o(t) - H(t)\mathbf{w}^a(t)], \quad \mathbf{w}^a(0) = \mathbf{w}_0^a$
Error covariance propagation (Riccati Equation)	$\dot{P}(t) = F(t)P(t) + P(t)F^T(t) + G(t)Q(t)G^T(t) - K(t)R(t)K^T(t), \quad P(0) = P_0$
Kalman Gain	$K(t) = P(t)H^T(t)R^{-1}(t)$
Initial conditions	$E[\mathbf{w}^t(0)] = \mathbf{w}_0^a, \quad E\{[\mathbf{w}^t(0) - \mathbf{w}_0^a][\mathbf{w}^t(0) - \mathbf{w}_0^a]^T\} = P_0$
Assumptions	$R^{-1}(t)$ exists; $E\{\mathbf{b}^t(t)[\mathbf{b}^o(t')]^T\} = 0$
Performance Index	$p^{f,a}(t) = E\{[\mathbf{w}^{f,a} - \mathbf{w}^t][\mathbf{w}^{f,a} - \mathbf{w}^t]^T\}$

B. Continuous (linear) Optimal Control

System Model	$\dot{\mathbf{w}}^t(t) = \tilde{F}(t)\mathbf{w}(t) + \tilde{H}(t)\mathbf{u}(t)$
Measurement Model	$\mathbf{w}^o(t) = \mathbf{w}(t)$ (all system variables are measured)
Performing control	$\mathbf{u}(t) = -\tilde{K}(t)\mathbf{w}(t)$
Performance propagation (Riccati Equation)	$\dot{\tilde{P}}(t) = -\tilde{F}^T(t)\tilde{P}(t) - \tilde{P}(t)\tilde{F}(t) - \tilde{Q}(t) + \tilde{P}(t)\tilde{H}(t)\tilde{K}(t)$
Control Gain	$\tilde{K}(t) = \tilde{R}^{-1}(t)\tilde{H}(t)\tilde{P}(t)$
Terminal conditions	$\mathbf{w}(t_f) = 0$; $\mathbf{P}(t_f) = \tilde{Q}_f$
Cost function	$J[\mathbf{w},\mathbf{u}] = \mathbf{w}_f^T\tilde{Q}_f\mathbf{w}_f + \int_0^{t_f}[\mathbf{w}^T(t)\tilde{Q}(t)\mathbf{w}(t) + \mathbf{u}^T(t)\tilde{R}(t)\mathbf{u}(t)]\,dt$

C. Estimation-Control Duality

Estimation	Control
t_0 initial time	t_f final time
$\mathbf{w}(t)$ unobservable state variable of random process	$\mathbf{w}(t)$ observable state variable to be controlled
$\mathbf{w}^o(t)$ random observations	$\mathbf{u}(t)$ deterministic control
$F(t)$ dynamic matrix	$\tilde{F}^T(t)$ dynamic matrix
$Q(t)$ covariance matrix for the model errors	$\tilde{Q}(t)$ quadratic matrix defining acceptable errors on model variables
$H(t)$ effect of observations on state variables	$\tilde{H}(t)$ effect of control on state variables
$P(t)$ covariance of estimation error under optimization	$\tilde{P}(t)$ quadratic performance under optimization
$K(t)$ weighting on observation for optimal estimation	$\tilde{K}(t)$ weighting on state for optimal control

[a] (A), Kalman filter as the optimal solution for the former problem; (B), optimal solution for the latter problem; (C), equivalences between the two (after Kalman, 1960, and Gelb, 1974, Section 9.5; courtesy of R. Todling).

Cautionary note:

"Pantheistic" view of DA:

- variational ~ KF;
- 3- & 4-D Var ~ 3- & 4-D PSAS.

Fashionable to claim it's all the same but it's not:

- **God** is in **everything**,
- **but the devil is in the details.**

M. Ghil & P. M.-Rizzoli (1991, *Adv. Geophys.*)

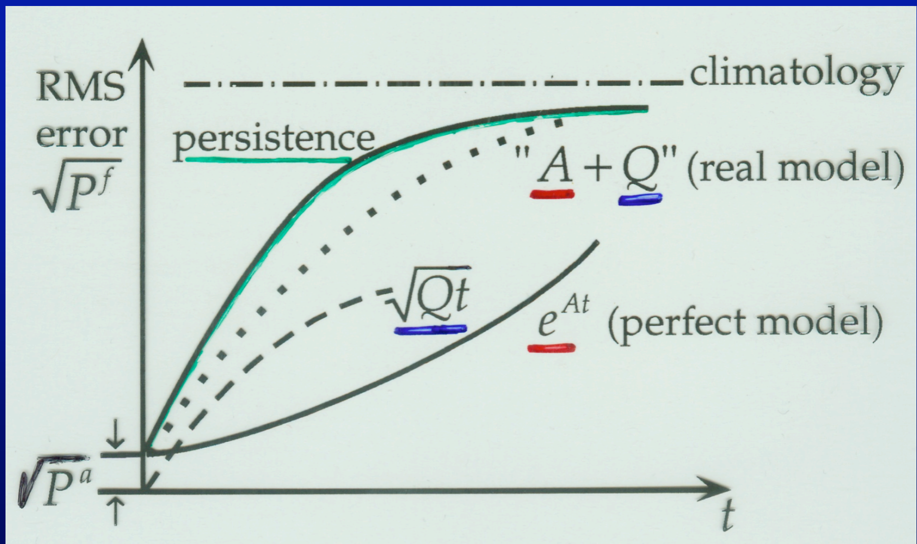
Outline

- Data in meteorology and oceanography
  - *in situ &* remotely sensed
- ➤ Basic ideas, data types, & issues
  - how to combine data with models
  - stability of the fcst.–assimilation cycle
  - filters & smoothers
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic ("classic")
  - stochastic – "dynamics" & "physics"
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

Error components in forecast–analysis cycle

$$\underbrace{P^f}_{\text{first-guess error}} \cong \underbrace{P^a}_{\text{analysis error}} + \Delta t(\underbrace{2AP^a}_{\text{id. twins error growth}} + \underbrace{Q}_{\text{modeling error}})$$

$$(\Psi = e^{A\Delta t} \cong 1 + A\Delta t)$$



The relative contributions to error growth of

- **analysis error**
- **intrinsic error growth**
- **modeling error (stochastic?)**

Assimilation of observations: Stability considerations

Free-System Dynamics (sequential-discrete formulation): ***Standard breeding***

forecast state; model integration from a previous analysis

$$\mathbf{x}_{n+1}^{f} = M(\mathbf{x}_{n}^{a})$$

Corresponding perturbative (tangent linear) equation

$$\delta\mathbf{x}_{n+1}^{f} = \mathbf{M}\delta\mathbf{x}_{n}^{a}$$

Observationally Forced System Dynamics (sequential-discrete formulation): ***BDAS***

If observations are available and we assimilate them:

Evolutive equation of the system, subject to forcing by the assimilated data

$$\mathbf{x}_{n+1}^{a} = \left[\mathbf{I} - \mathbf{K}H \circ\right] M(\mathbf{x}_{n}^{a}) + \mathbf{K}\mathbf{y}_{n+1}^{o}$$

Corresponding perturbative (tangent linear) equation, if the same observations are assimilated in the perturbed trajectories as in the control solution

$$\delta\mathbf{x}_{n+1}^{a} = \left[\mathbf{I} - \mathbf{K}\mathbf{H}\right]\mathbf{M}\delta\mathbf{x}_{n}^{a}$$

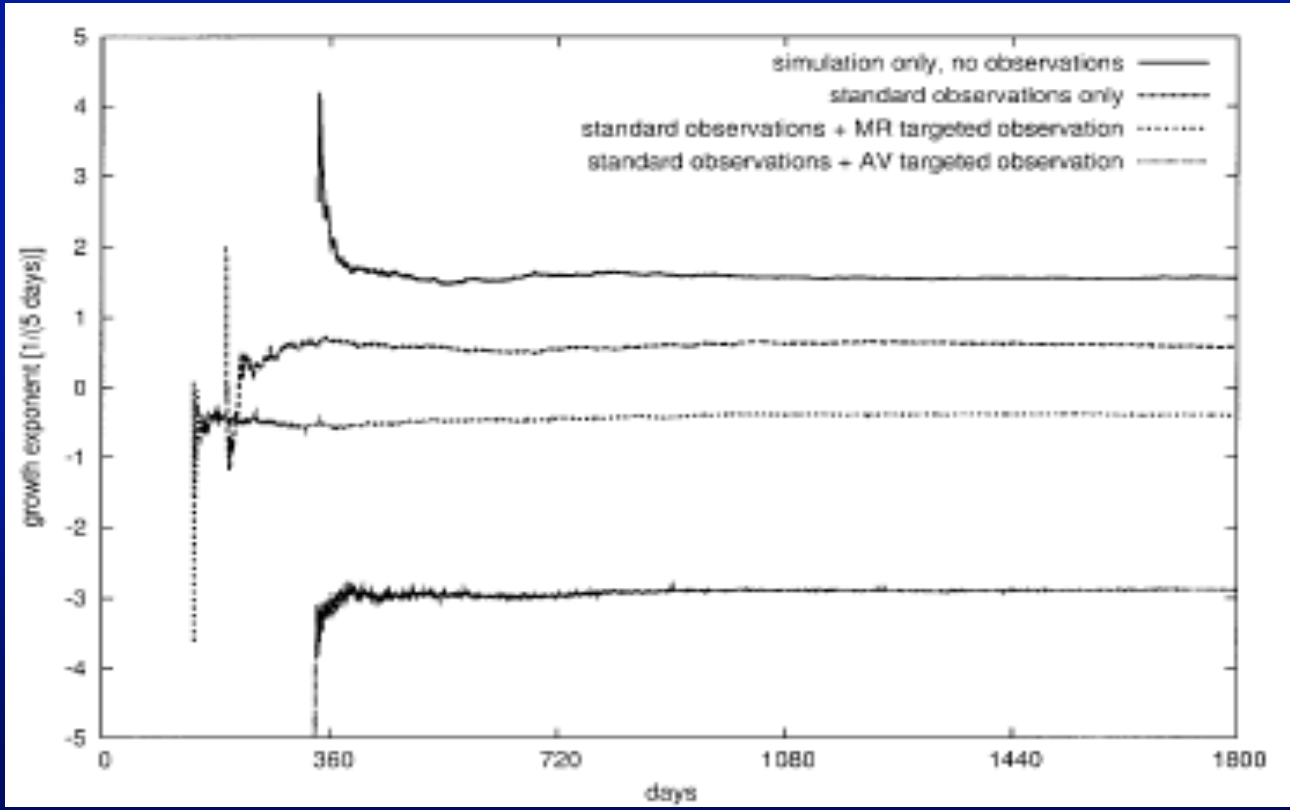
- The matrix (**I – KH**) is expected, in general, to have a stabilizing effect;
- the free-system instabilities, which dominate the forecast step error growth, can be reduced during the analysis step.

Joint work with A. Carrassi, A. Trevisan & F. Uboldi

Stabilization of the forecast–assimilation system – I

Assimilation experiment with a low-order chaotic model

- Periodic 40-variable Lorenz (1996) model;
- Assimilation algorithms: replacement (Trevisan and Uboldi, 2004), replacement + one adaptive obs'n located by multiple replication (Lorenz, 1996), replacement + one adaptive obs'n located by BDAS and assimilated by AUS (Trevisan & Uboldi, *JAS*, 2004).



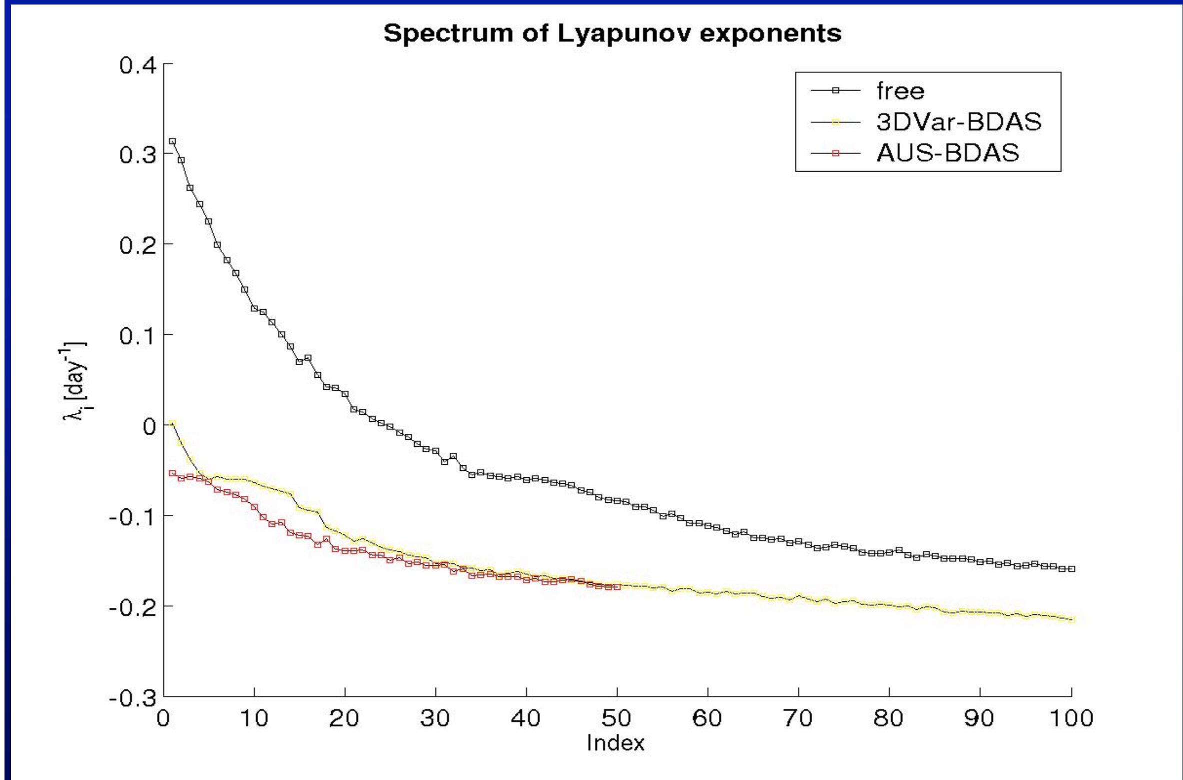
Trevisan & Uboldi (*JAS*, 2004)

Stabilization of the forecast–assimilation system – II

Assimilation experiment with an intermediate atmospheric circulation model

- 64-longitudinal x 32-latitudinal x 5 levels periodic channel QG-model (Rotunno & Bao, 1996)
- Perfect-model assumption
- Assimilation algorithms: 3-DVar (Morss, 2001); AUS (Uboldi *et al.,* 2005; Carrassi *et al*., 2006)

Observational forcing ⇒ Unstable subspace reduction



➤ ***Free System***

Leading exponent:

$\lambda_{max} \approx 0.31$ days^{-1};

Doubling time ≈ 2.2 days;

Number of positive exponents:

$N^{+} = 24$;

Kaplan-Yorke dimension ≈ 65.02.

➤ ***3-DVar–BDAS***

Leading exponent:

$\lambda_{max} \approx 6\text{x}10^{-3}$ days^{-1};

➤ ***AUS–BDAS***

Leading exponent:

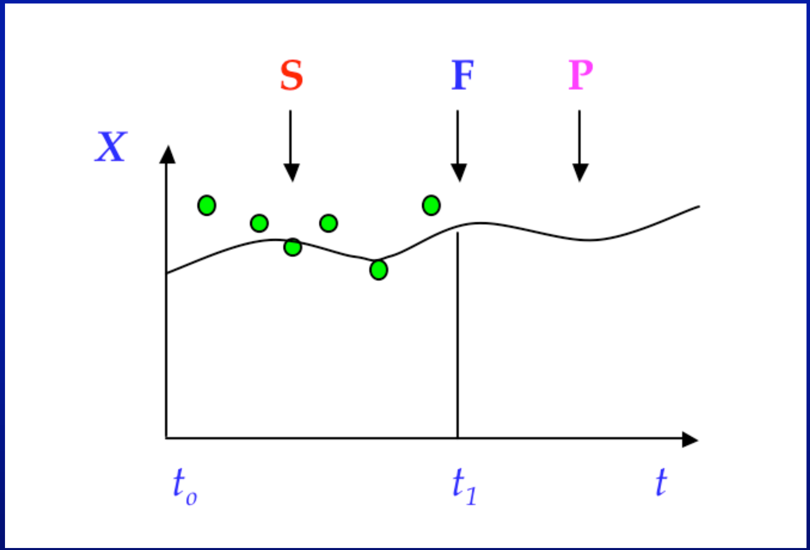
$\lambda_{max} \approx -0.52\text{x}10^{-3}$ days^{-1}

Outline

- Data in meteorology and oceanography
  - *in situ &* remotely sensed
- ➤ Basic ideas, data types, & issues
  - how to combine data with models
  - stability of the fcst.–assimilation cycle
  - filters & smoothers
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic ("classic")
  - stochastic – "dynamics" & "physics"
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

The main products of estimation[*]

- Filtering (F) – "video loops"
- Smoothing (S) – full-length feature "movies"
- Prediction (P) – NWP, ENSO



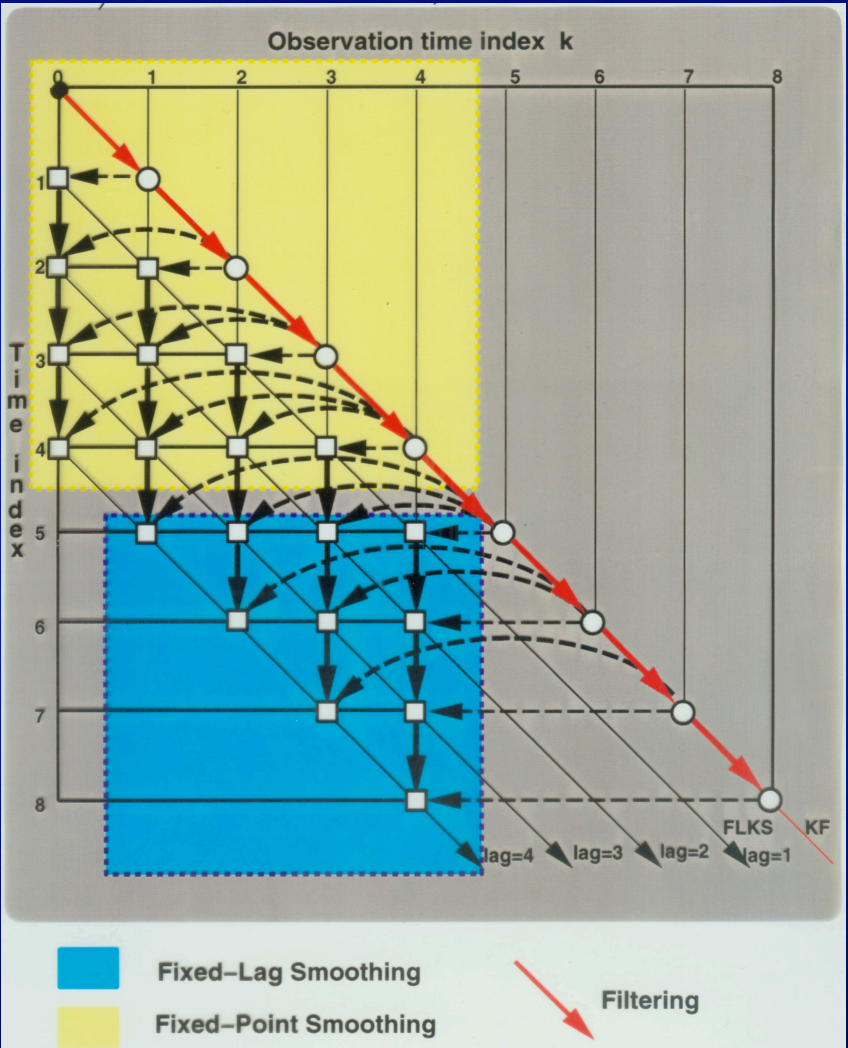
Distribute all of this over the Web to

- scientists, and the
- "person in the street" (or on the information superhighway).

In a general way: Have fun!!!

[*] N. Wiener (1949, MIT Press)

Kalman smoother

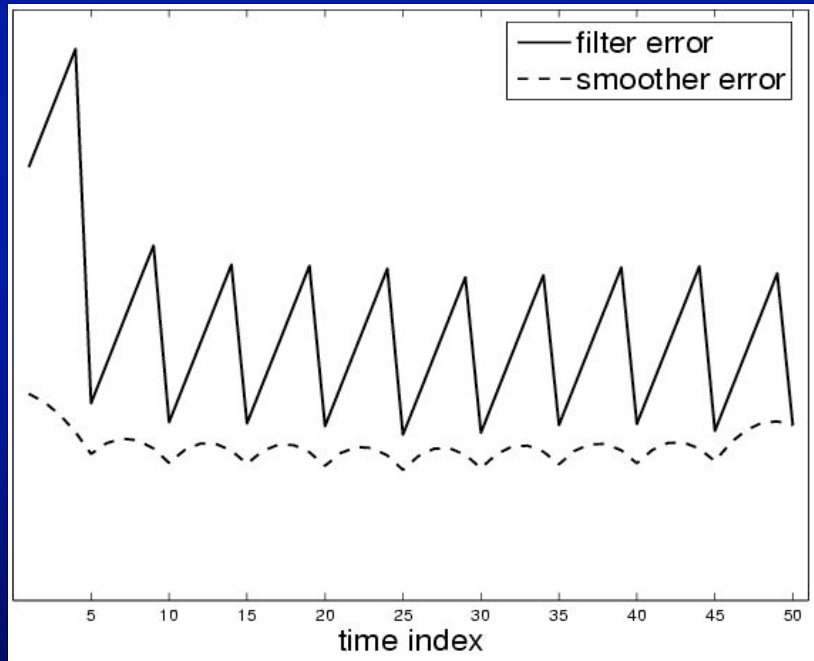


For a fixed interval, weak constrained 4-D Var is equivalent to the sequential ("Kalman") smoother.

Cohn, Sivakumaran & Todling (*MWR*, 1994)

Smoothing *vs.* Filtering: The Backward Sequential Smoother (BSS)

- A “smoother” **is** smoother than a filter.
- But which smoother is
  - smoothest
  - cheapest
  - easiest to implement?



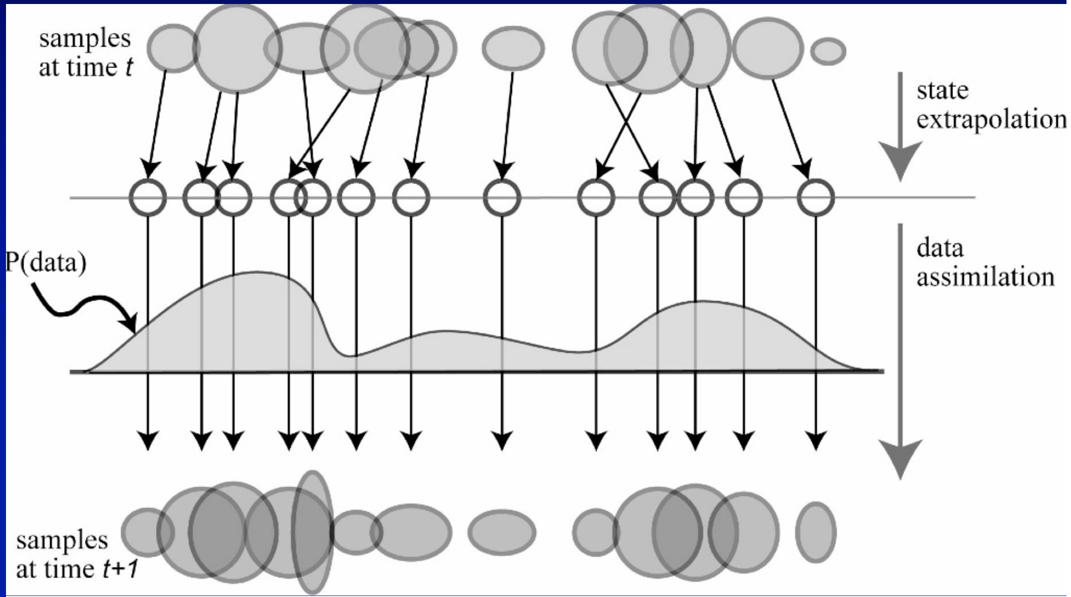
Joint work with T. M. Chin, J. B. Jewell, & M. J. Turmom, JPL

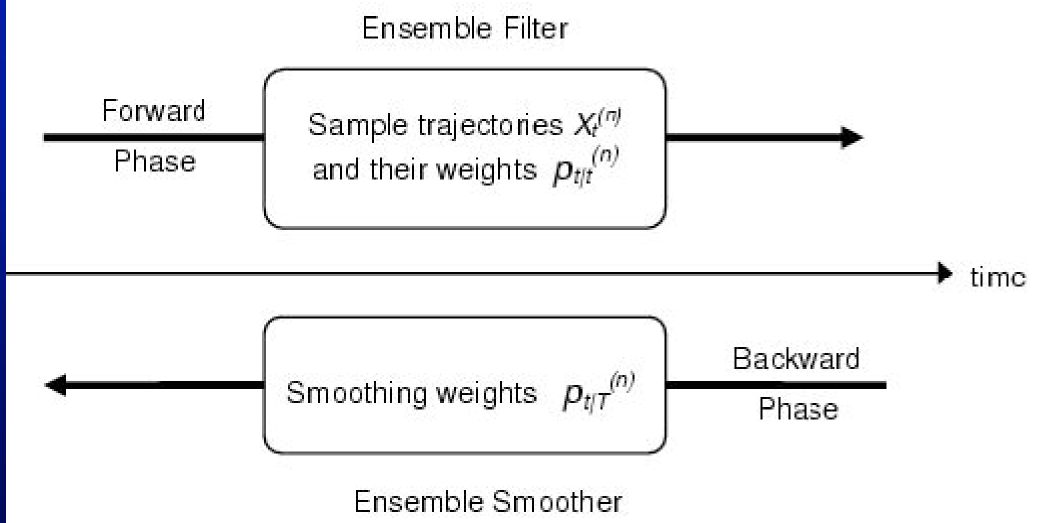
EnKF, RPF, MCMC and the BSS

The BSS retrospectively updates a set of weights for ensemble members.

It can

- work with either EnKF- or RPF-generated ensembles;
- is relatively inexpensive; and
- works well for highly nonlinear, illustrative examples:
  - the double-well potential, &
  - the Lorenz (1963) model.



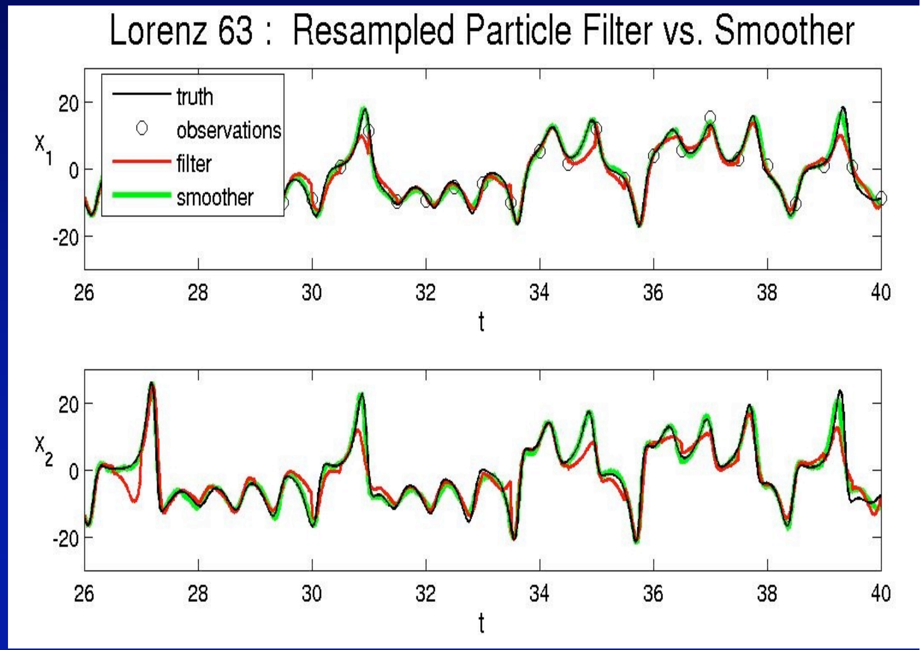


BSS Performance for the Lorenz (1963) System

Data: x_1 and x_3, every $\Delta t = 0.5$

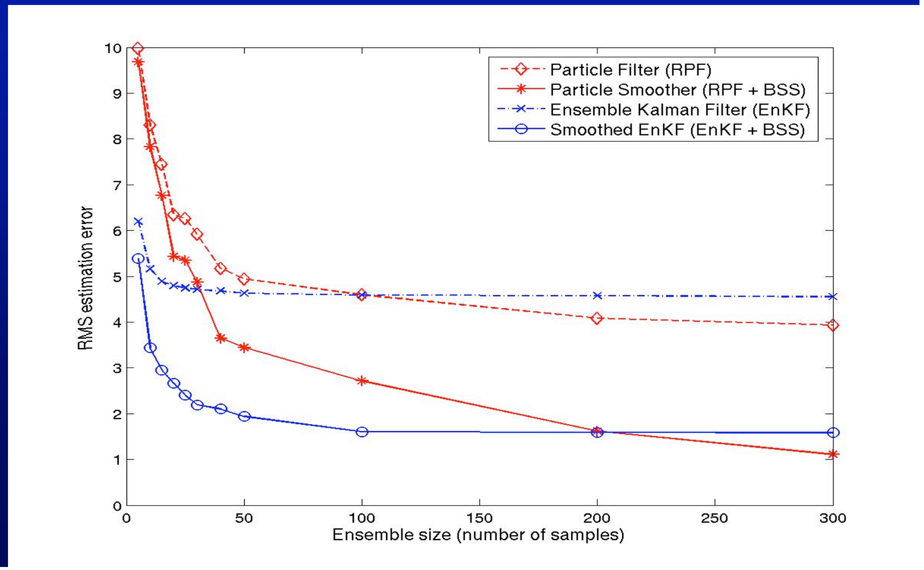
Upper panel: RPF vs. smoother

Smoother follows ob'ns (O) better in x_1 and is more realistic in x_2.



Lower panel: RPF *vs.* EnKF, & filter (- - -) *vs.* smoother (----)

Smoother better than filter, & EnKF better than RPF for very small ensemble size *N*, but RPF takes over as *N* increases.



Outline

- Data in meteorology and oceanography
  - *in situ &* remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - filters & smoothers
  - stability of the fcst.-assimilation cycle
- ➤ Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic ("classic")
  - stochastic – "dynamics" & "physics"
- Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

Parameter Estimation

a) Dynamical model

$dx/dt = M(x, \mu) + \eta(t)$

$y^o = H(x) + \varepsilon(t)$

Simple (EKF) idea – augmented state vector

$d\mu/dt = 0$, $X = (x^T, \mu^T)^T$

b) Statistical model

$L(\rho)\eta = w(t)$, L – AR(MA) model, $\rho = (\rho_1, \rho_2, \ldots. \rho_M)$

Examples: 1) Dee *et al*. (*IEEE*, 1985) – estimate a few parameters in the covariance matrix $Q = E(\eta, \eta^T)$; also the bias $<\eta> = E\eta$;

2) POPs - Hasselmann (1982, Tellus); Penland (1989, *MWR*; 1996, *Physica D*); Penland & Ghil (1993, *MWR*)

3) $dx/dt = M(x, \mu) + \eta$: Estimate both M & Q from data (Dee, 1995, *QJ*), Nonlinear approach: Empirical mode reduction (Kravtsov *et al*., 2005, Kondrashov *et al*., 2005)

Estimating noise – I

$Q_1 = Q_{slow}$, $Q_2 = Q_{fast}$, $Q_3 = 0$;

$R_1 = 0$, $R_2 = 0$, $R_3 = R$;

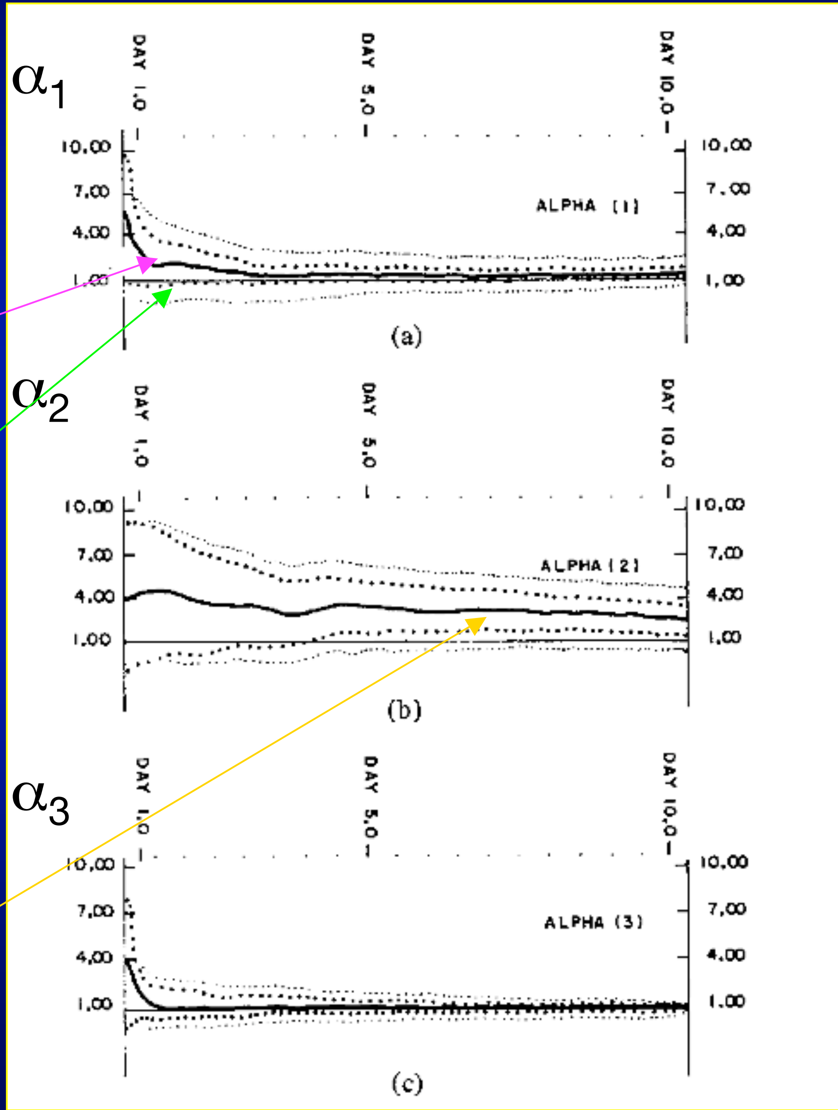
$Q = \sum \alpha_i Q_i$; $R = \sum \alpha_i R_i$;

$\alpha(0) = (6.0, 4.0, 4.5)^T$;

$Q(0) = 25 * I$.

estimated

true ($\alpha = 1$)



Dee *et al.* (1985, *IEEE Trans. Autom. Control*, **AC-30**)

Poor convergence for Q_{fast}?

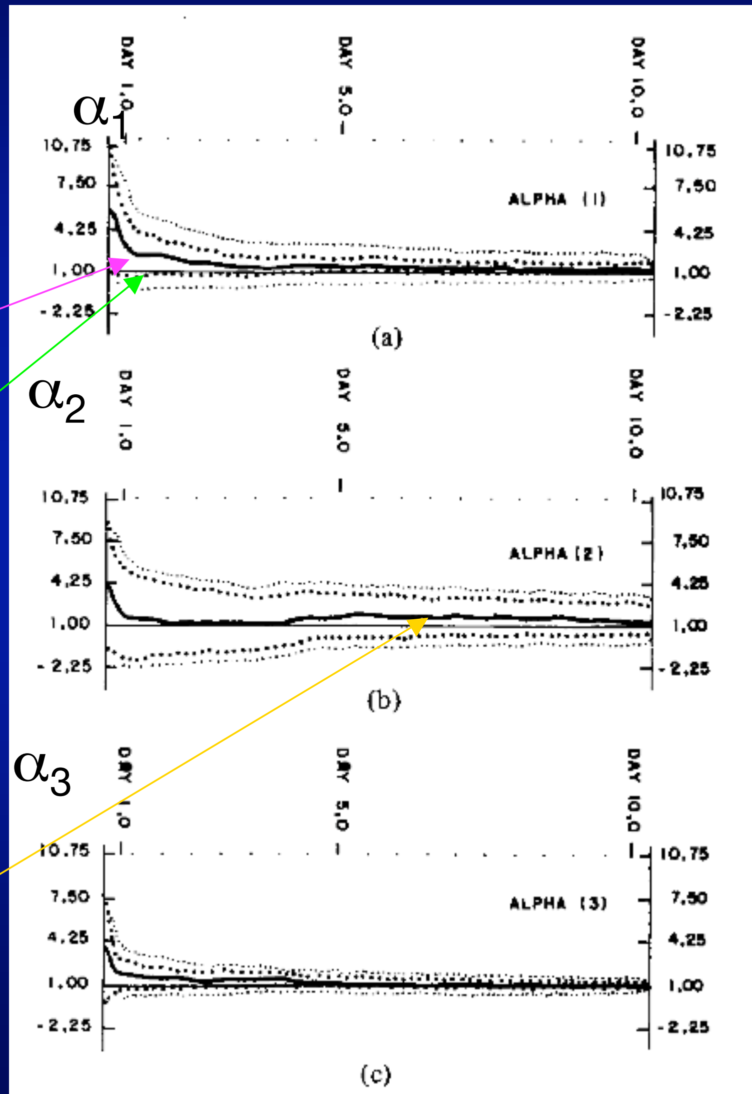
Estimating noise – II

Same choice of $\alpha(0)$, Q_i, and R_i but

$$\Theta(0) = 25 * \begin{bmatrix} 1 & 0.8 & 0 \\ 0.8 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

estimated

true ($\alpha = 1$)



Dee *et al.* (1985, *IEEE Trans. Autom. Control*, **AC-30**)

Good convergence for Q_{fast}!

Sequential parameter estimation

- "**State augmentation**" method – uncertain parameters are treated as additional state variables.
- Example: one unknown parameter μ

$$\bar{x}_k = \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} = \begin{pmatrix} F(x_{k-1}, \mu_{k-1}) \\ \mu_{k-1} \end{pmatrix} + \begin{pmatrix} \epsilon_k \\ \epsilon^{\mu}_{k-1} \end{pmatrix}$$

$$y^o_k = \begin{pmatrix} H & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} + \epsilon^0 = \bar{H}\bar{x}_k + \epsilon^0$$

$$\bar{x}^a_k = \bar{x}^f_k + \bar{K}(y^o_k - \bar{H}\bar{x}^f_k); \quad \bar{K} = \bar{P}^f \bar{H}^T (\bar{H}\bar{P}^f \bar{H}^T + R)^{-1}$$

- **The parameters are not directly observable, but** the **cross-covariances** drive parameter changes from innovations of the state:

$$\bar{P}^f = \begin{pmatrix} P^f_{xx} & P^f_{x\mu} \\ P^f_{\mu x} & P^f_{\mu\mu} \end{pmatrix}; \quad \bar{K} = \begin{pmatrix} P^f_{xx} H^T \\ P^f_{\mu x} H^T \end{pmatrix} (H P^f_{xx} H^T + R)^{-1}$$

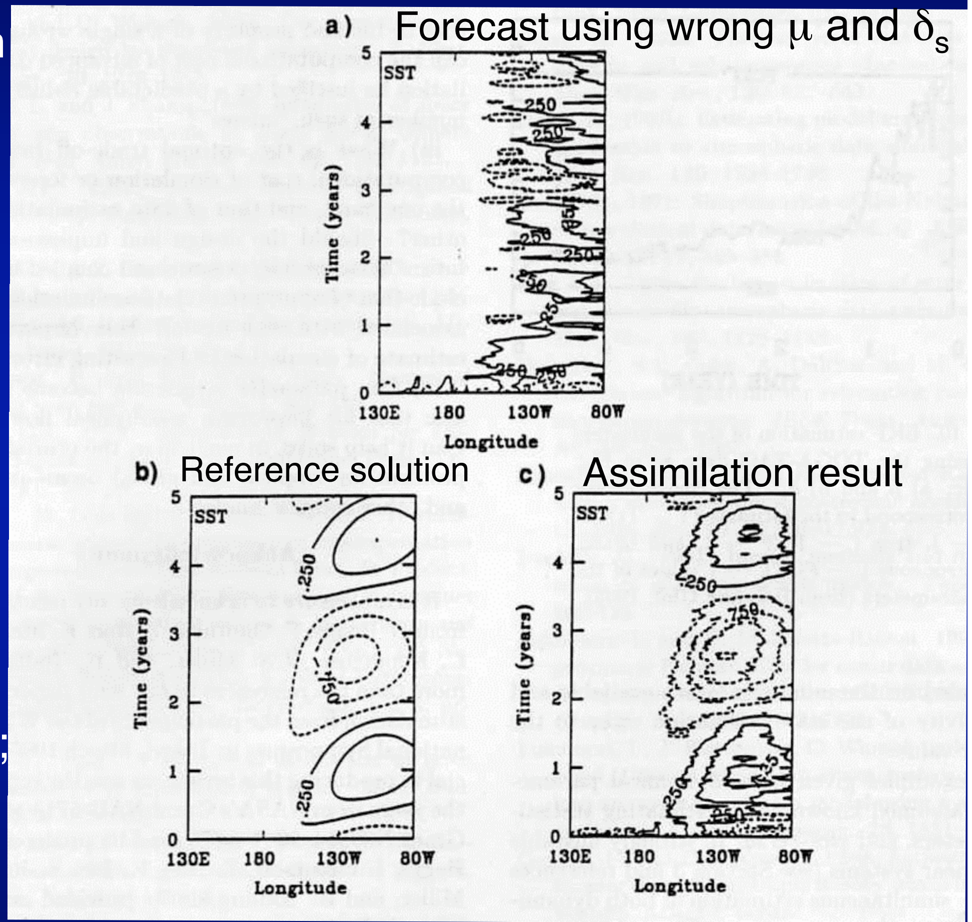
- Parameter estimation is always a **nonlinear problem,** even if the model is **linear** in terms of the model state: use **Extended Kalman Filter (EKF).**

Parameter estimation for coupled O-A system

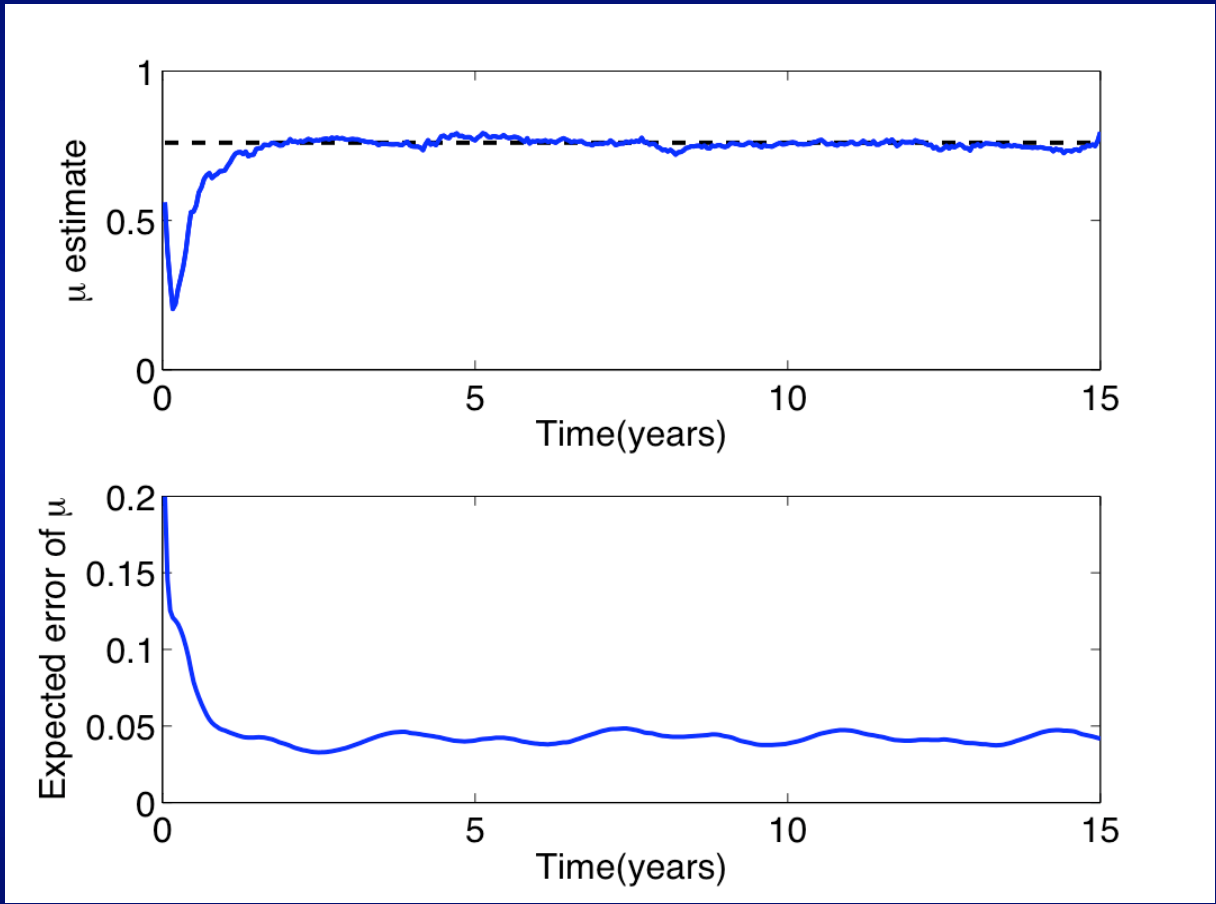
- Intermediate coupled model (ICM: Jin & Neelin, *JAS*, 1993)
- Estimate the state vector $W = (T', h, u, v)$,

  along with the coupling parameter μ and surface-layer coefficient δ_s

  by assimilating data from a single meridional section.
- The ICM model has errors in its initial state, in the wind stress forcing & in the parameters.
- M. Ghil (1997, *JMSJ*); Hao & Ghil (1995, *Proc. WMO Symp. DA Tokyo*); Sun *et al*. (2002, *MWR*).
- *Current work with D. Kondrashov, J.D. Neelin, & C.-j. Sun.*



Convergence of parameter value



Reference value: μ = 0.76;

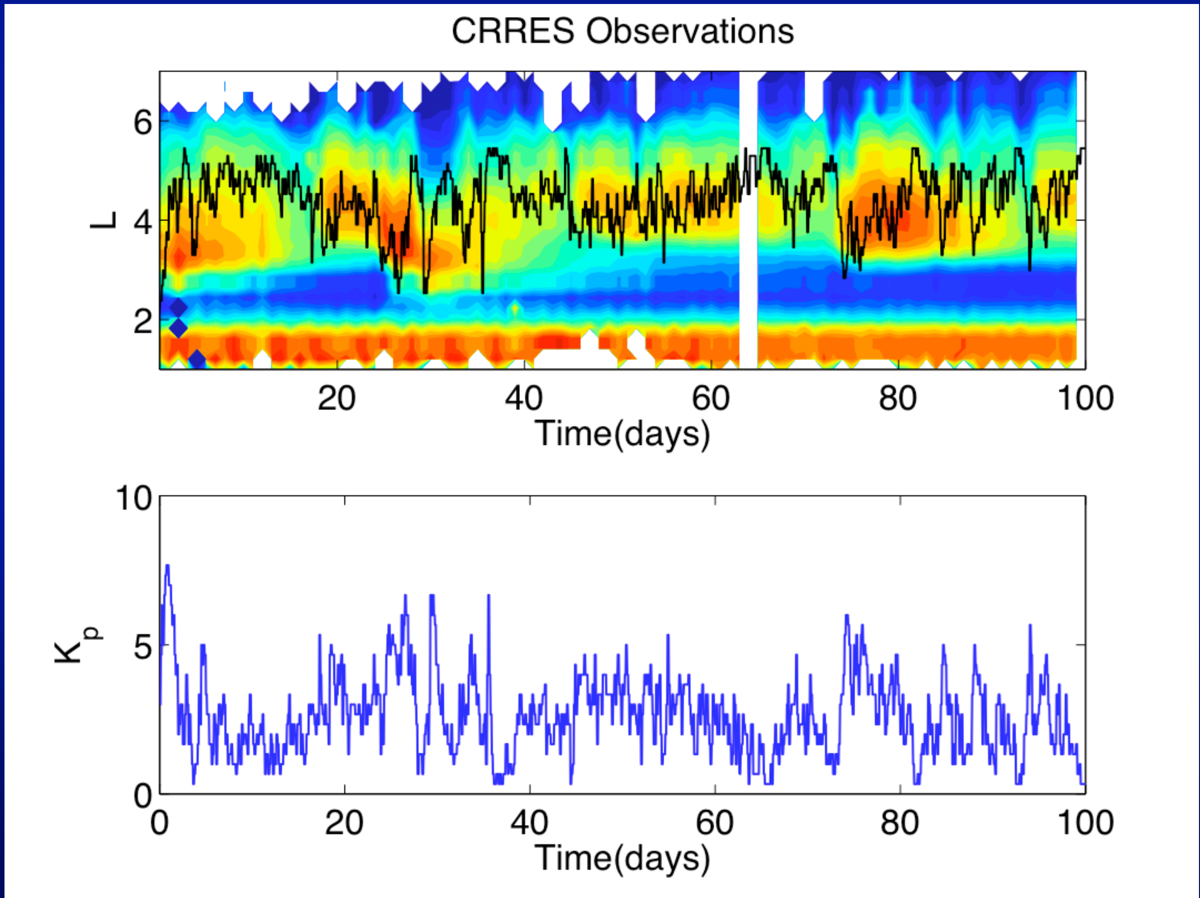
Initial model (“wrong”) value: μ = 0.6

Outline

- Data in meteorology and oceanography
  - *in situ &* remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - stability of the fcst.–assimilation cycle
  - filters & smoothers
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic ("classic")
  - stochastic – "dynamics" & "physics"
- ➤ Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

Parameter Estimation for Space Physics – I

Daily fluxes of 1MeV relativistic electrons in Earth's outer radiation belt
(CRRES observations from 28 August 1990)
***K_p* - index of solar activity (external forcing)**



*Joint work with
D. Kondrashov, Y. Shprits,
& R. Thorne, UCLA;
R. Friedel & G. Reeves,
LANL*

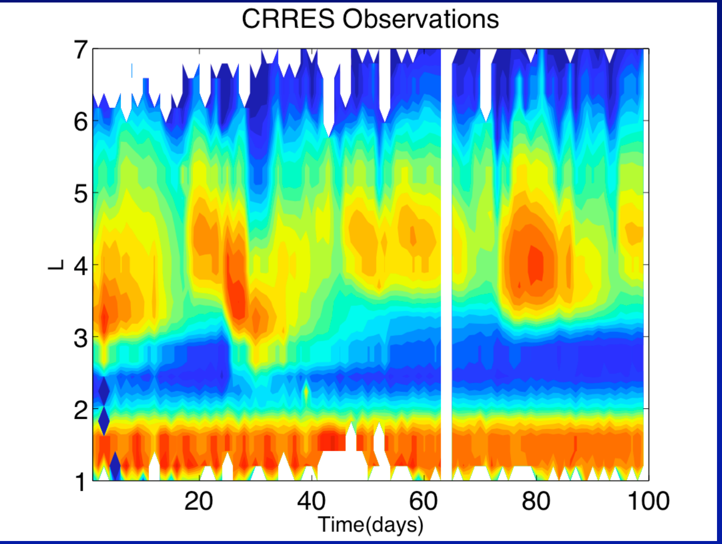
Parameter estimation for space physics – II

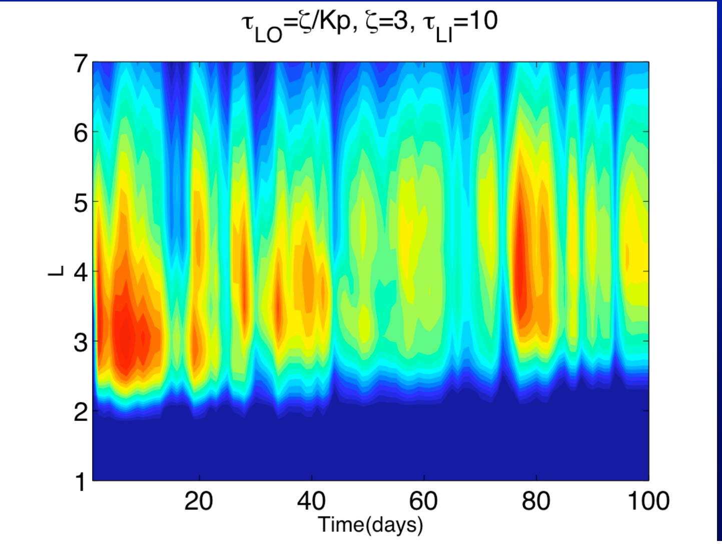
HERRB-1D code (Y. Shprits) –
estimating phase space density *f* and electron lifetime τ_L:

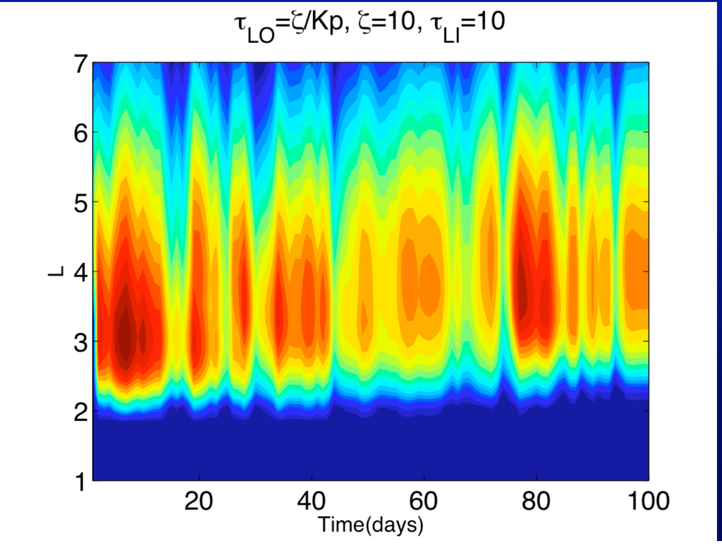
$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) - \frac{f}{\tau_L}$$

Different lifetime parameterizations for plasmasphere – out/in: $\tau_{Lo} = \zeta / K_p(t)$; τ_{Li}=const.

What are the **optimal** lifetimes to match the observations best?







Parameter estimation for space physics – III

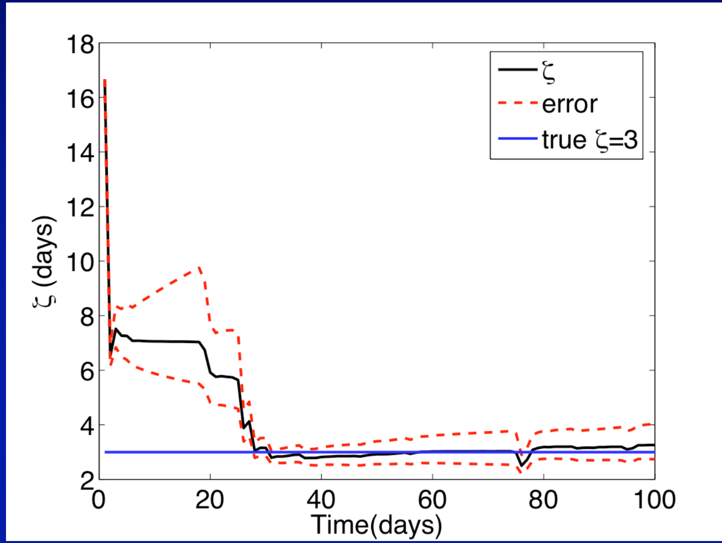
Daily observations from the "truth" —

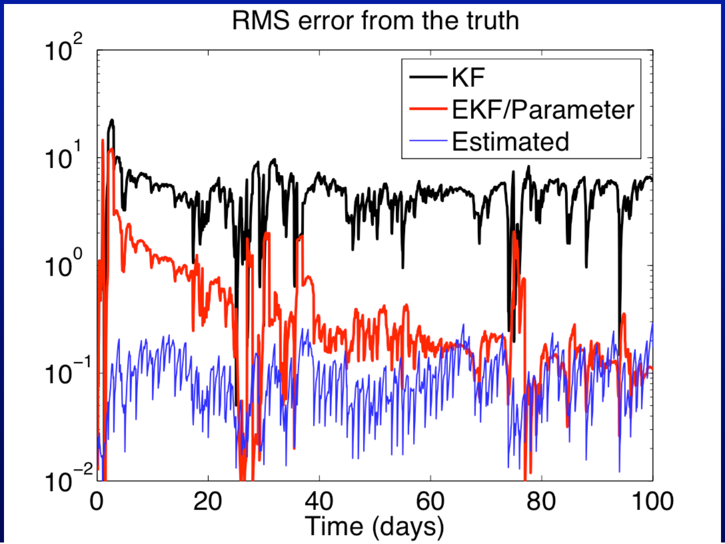
$\tau_{Lo} = \zeta/K_p$, $\zeta = 3$, and $\tau_{LI} = 20$ —

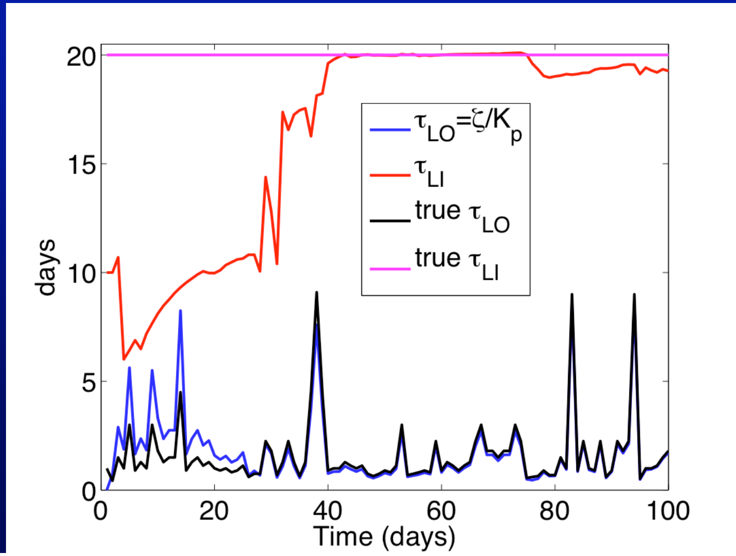
are used to correct the model's "wrong" parameters, $\zeta = 10$ and $\tau_{LI} = 10$.

The estimated error $tr(P_f) \approx$ actual

When the parameters' assumed uncertainty is large enough, their EKF estimates converge rapidly to the "truth".







Deterministic parametrization - I

1. Hierarchy of climate model: from 0-D EBMs to 3-D GCMs
2. None resolves all relevant processes on all the scales of motion;
3. "Parametrization"= representation of unresolved processes in terms of resolved variables.
4. GCMs, as well as many intermediate, 2-D models, have
   - "dynamics" ⊕ "physics," *i.e.*,
   - fluid dynamics (adiabatic, conservative, Hamiltonian) + radiation, clouds, surface processes, precipitation, etc. (forcing & dissipation);
   - both are only resolved up to a given trancation (finite differences, finite elements, spectral, etc.);
   - much of the "physics" has characteristic scales that are not resolved.

Deterministic parametrization - II

Slow and fast variables

- $\mathbf{w} = (\mathbf{u}^T, \mathbf{v}^T)^T$
- $\mathbf{u}$ ~ large-scale, slow; $\mathbf{v}$ ~ small-scale, fast
- $\mathbf{y} = \varepsilon\mathbf{x}$, $s = \varepsilon t$
- (S) $d\mathbf{u}/dt = f(\mathbf{u}, \mathbf{v})$, $\quad \mathbf{u} = \mathbf{u}(\mathbf{y}, s)$
- (F) $d\mathbf{v}/dt = \mathbf{g}(\mathbf{u}, \mathbf{v})$, $\quad \mathbf{v} = \mathbf{v}(\mathbf{x}, t)$

- Equations like (S) $\oplus$ (F) appear in slow manifold theory, "initialization," and "parametrization" of small scales.

Deterministic parametrization - III

- *General idea*: (i) Solve (F) at fixed
  $\mathbf{u}(\mathbf{y}, s_o) = \mathbf{u}_o(\mathbf{y})$ to get
  (O) $\mathbf{v} = \mathbf{v}(\mathbf{x}, t; \mathbf{u}_o)$
- then extend to an "adiabatic" solution of (ii)
  (A) $\mathbf{v} = \mathbf{v}(\mathbf{x}, t; \mathbf{u}(\mathbf{y}, s))$. Substitute (A) into (S), to yield
  (S_A) = du/dt= $f(\mathbf{u}(\mathbf{y}, s), \mathbf{v}(\mathbf{x}, t; \mathbf{u}(\mathbf{y}, s))$
- Use nonlinear averaging, on the fast & small scales,

$$\overline{(\bullet)}^F \equiv E_F(\bullet) \equiv \int dx \int dt(\bullet)$$

- applied to (S_A) to get the **net effect** of these scales on the evolution of the *slow & large ones*:
- (P) du/dt = $f(\mathbf{u}(\mathbf{y}, s); \mathbf{v}^F(\mathbf{y}, s))$.

Stochastic parametrization

- Let's keep to additive, white, Gaussian noise:

(S') $du/dt = f(u,v) + \boldsymbol{\xi}$; $\boldsymbol{\xi} = \boldsymbol{\xi}(\mathbf{y}, s) \rightarrow \mathbf{Q_S} = \mathrm{cov}(\xi, \xi^{`})$

(F') $\mathbf{dv/dt = g(u,v)}$; $\boldsymbol{\eta} = \boldsymbol{\eta}(\mathbf{x}, t) \rightarrow \mathbf{Q_F} = \mathrm{cov}(\boldsymbol{\eta}, \boldsymbol{\eta}^{`})$

- We proceed in an analogous fashion to get

(Q') $\mathbf{v} = \mathbf{v}\ (\mathbf{x}, t; \mathbf{u}_o, \omega_\eta), \omega_\eta = \{\boldsymbol{\eta}(\mathbf{x}, \tau): -\infty < \tau < t\}$

and then the "adiabatic" evolution

(A') $\mathbf{v} = \mathbf{v}(\mathbf{x}, t; \mathbf{u}(\mathbf{y}, s; \omega_\xi), \omega_\eta), \omega_\xi = \{\xi(\mathbf{y}, \sigma): -\infty < \sigma < s\}$.

- Substituting now (A') into (S') will yield, if we're lucky:

(P')

For luck, we need

- Nonlinear averaging procedure: Lie series?
- Parameter estimation procedure: EKF?
- Explicit noise generation – yes/no?

MTV

Majda, Timofeyev & Vanden Eijnden (1999, *PNAS*; 2001, *CPAM*)

Basic Strategy for Stochastic Climate Modeling

We illustrate the ideas for stochastic climate modeling on an abstract basic model involving quadratically nonlinear dynamics, which is very appropriate for modeling many aspects of atmospheric dynamics. In the abstract model, the unknown variable $\vec{z}$ evolves in time in response to a linear operator, $L\vec{z}$, and a quadratic or bilinear operator, $B(\vec{z}, \vec{z})$, and satisfies

$$\frac{d\vec{z}}{dt} = L\vec{z} + B(\vec{z}, \vec{z}). \qquad [1]$$

In stochastic climate modeling, the variable $\vec{z}$ is decomposed into an orthogonal decomposition through the variables $\vec{x}$, $\vec{y}$ by $\vec{z} = (\vec{x}, \vec{y})$. The variable $\vec{x}$ denotes the climate state of the system; the climate state necessarily evolves slowly in time compared to the $\vec{y}$ variables, which evolve more rapidly in time and are not resolved in detail in the stochastic climate model. Decomposing the dynamic equation in **1** by projecting on the $\vec{x}$ and $\vec{y}$ variables yields the equations

$$\frac{d\vec{x}}{dt} = L_{11}\vec{x} + L_{12}\vec{y} + B^1_{11}(\vec{x}, \vec{x}) + B^1_{12}(\vec{x}, \vec{y}) + B^1_{22}(\vec{y}, \vec{y}), \qquad [2]$$

$$\frac{d\vec{y}}{dt} = L_{21}\vec{x} + L_{22}\vec{y} + B^2_{11}(\vec{x}, \vec{x}) + B^2_{12}(\vec{x}, \vec{y}) + B^2_{22}(\vec{y}, \vec{y}). \qquad [3]$$

In stochastic climate modeling, the explicit nonlinear self-interaction through $B^2_{22}(\vec{y}, \vec{y})$ of the variables $\vec{y}$, which are not resolved in detail, is represented by a linear stochastic operator

$$B^2_{22}(\vec{y}, \vec{y}) \approx -\frac{\Gamma}{\varepsilon}\vec{y} + \frac{\sigma}{\sqrt{\varepsilon}}\dot{\vec{W}}(t), \qquad [4]$$

where Γ, σ are diagonal matrices (for simplicity in exposition) with positive coefficients and $\dot{\vec{W}}(t)$ is a vector-valued white-noise. We note from **4** that ε measures the ratio of the correlation time of the under-resolved $\vec{y}$-variables to the climate variables $\vec{x}$ and the requirement $\varepsilon \ll 1$ is very natural for stochastic climate models where the climate variables should change more slowly. In fact if we coarse-grain the equations in **2** and **3** with the approximation from **4** on a longer time scale, $t \to \varepsilon t$, to measure the slowly evolving climate variables, we derive the stochastic climate model

$$\begin{aligned}\frac{d\vec{x}}{dt} &= \frac{1}{\varepsilon}(L_{11}\vec{x} + L_{12}\vec{y} + B^1_{11}(\vec{x}, \vec{x}) + B^1_{12}(\vec{x}, \vec{y}) + B^1_{22}(\vec{y}, \vec{y})),\\ \frac{d\vec{y}}{dt} &= \frac{1}{\varepsilon}(L_{21}\vec{x} + L_{22}\vec{y} + B^2_{11}(\vec{x}, \vec{x}) + B^2_{12}(\vec{x}, \vec{y})) \qquad [5]\\ &\quad - \frac{\Gamma}{\varepsilon^2}\vec{y} + \frac{\sigma}{\varepsilon}\dot{\vec{W}}(t).\end{aligned}$$

Outline

- Data in meteorology and oceanography
  - *in situ* & remotely sensed
- Basic ideas, data types, & issues
  - how to combine data with models
  - stability of the fcst.–assimilation cycle
  - filters & smoothers
- Parameter estimation
  - model parameters
  - noise parameters – at & below grid scale
- Subgrid-scale parameterizations
  - deterministic ("classic")
  - stochastic – "dynamics" & "physics"
- ➤ Novel areas of application
  - space physics
  - shock waves in solids
  - macroeconomics
- Concluding remarks

Kalman filtering in a macroeconomic model

Joint work with P. Dumas [(*)], S. Hallegatte, & J.-Ch. Hourcade, CIRED

The NEDyM model

- represents a closed economy with one producer; one consumer; one type of goods;
- reproduces economic growth and business cycles;
- dynamical system with 12 state variables; and
- 15 parameters

The data

- U.S. macroeconomic data for nearly 60 years, 1947–2004;
- data from the National Bureau of Economic Research (NBER; www.nber.org);
- 5 variables: production, consumption, investment, wages, and inflation.

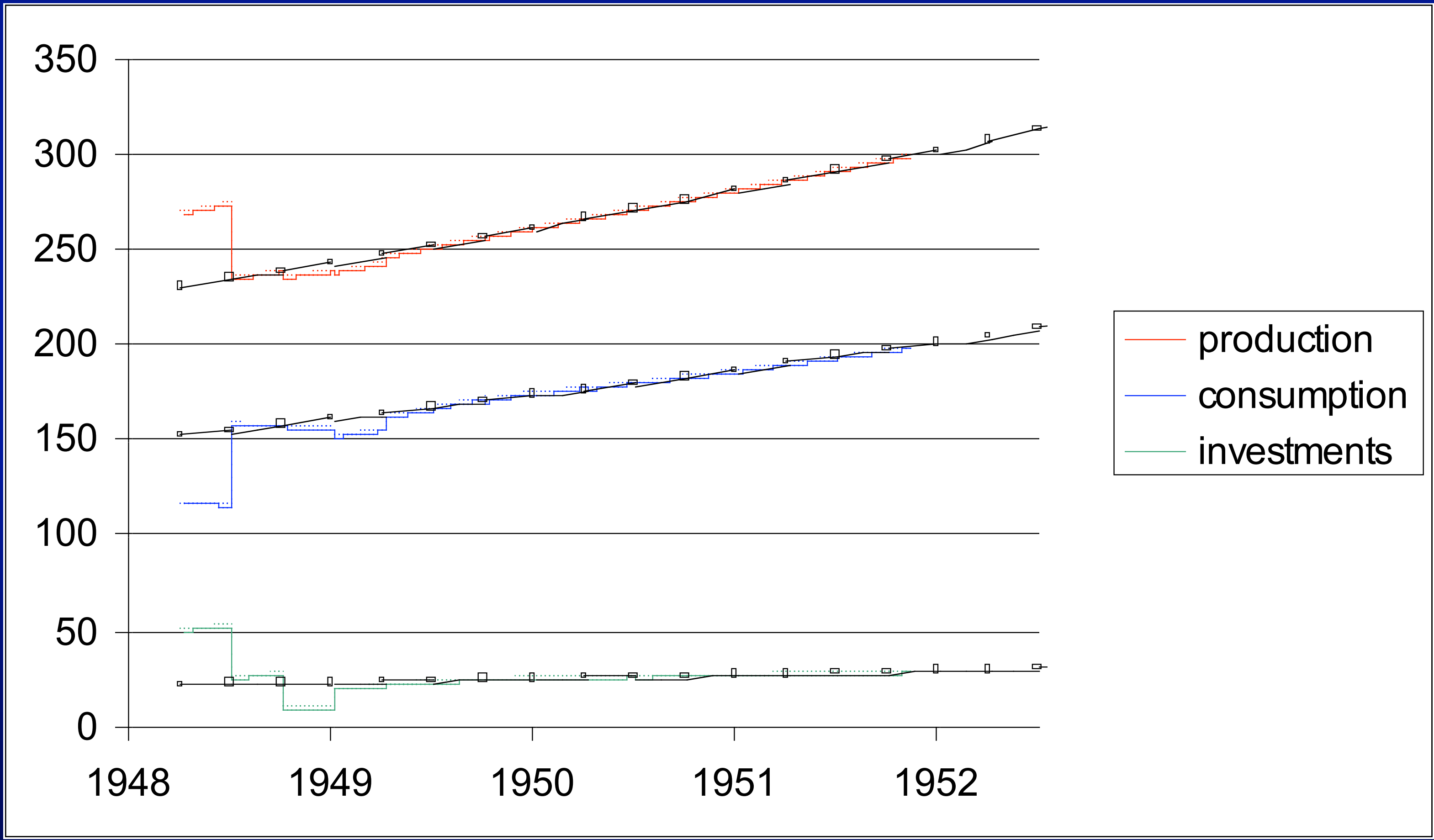
First experiments

- Calibration of the model trend only (no business cycle yet)
- "Only" 8 parameters

[(*)] also Environmental Research and Teaching Institute (ERTI), ENS

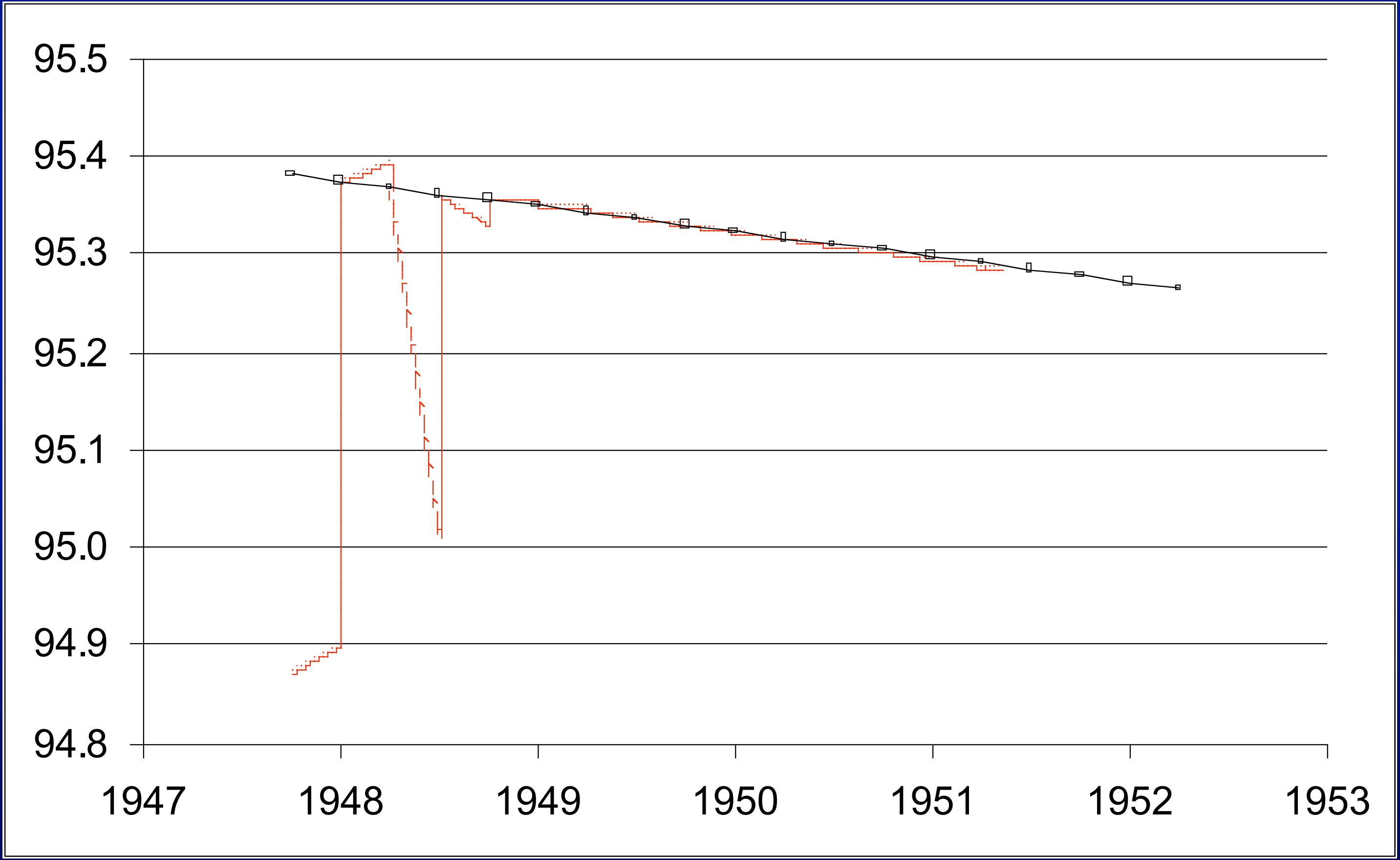
Calibration of a macroeconomic model using a Kalman filter

Model variables: production; consumption; investment



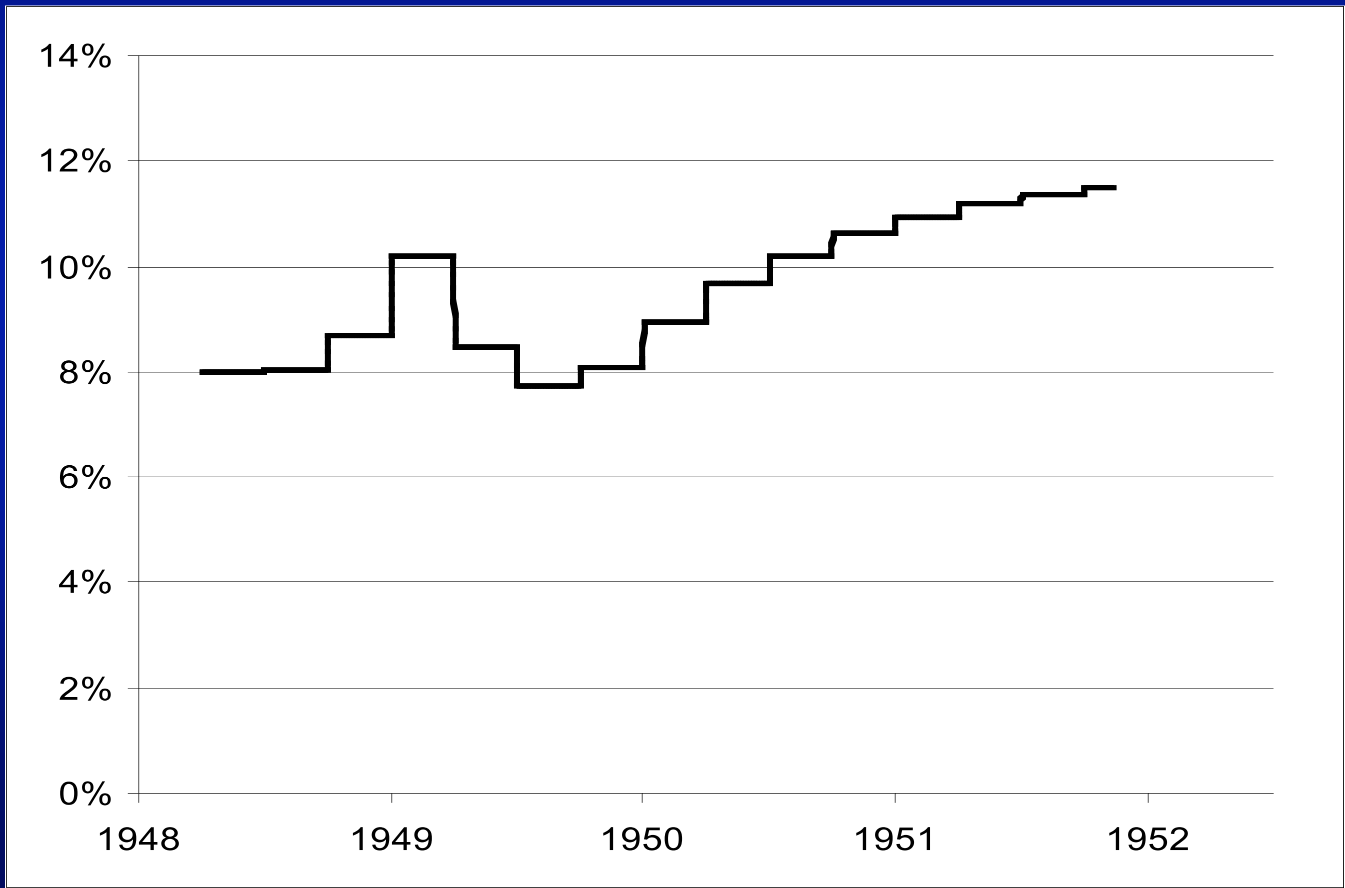
State estimation

Model variables – employment rate



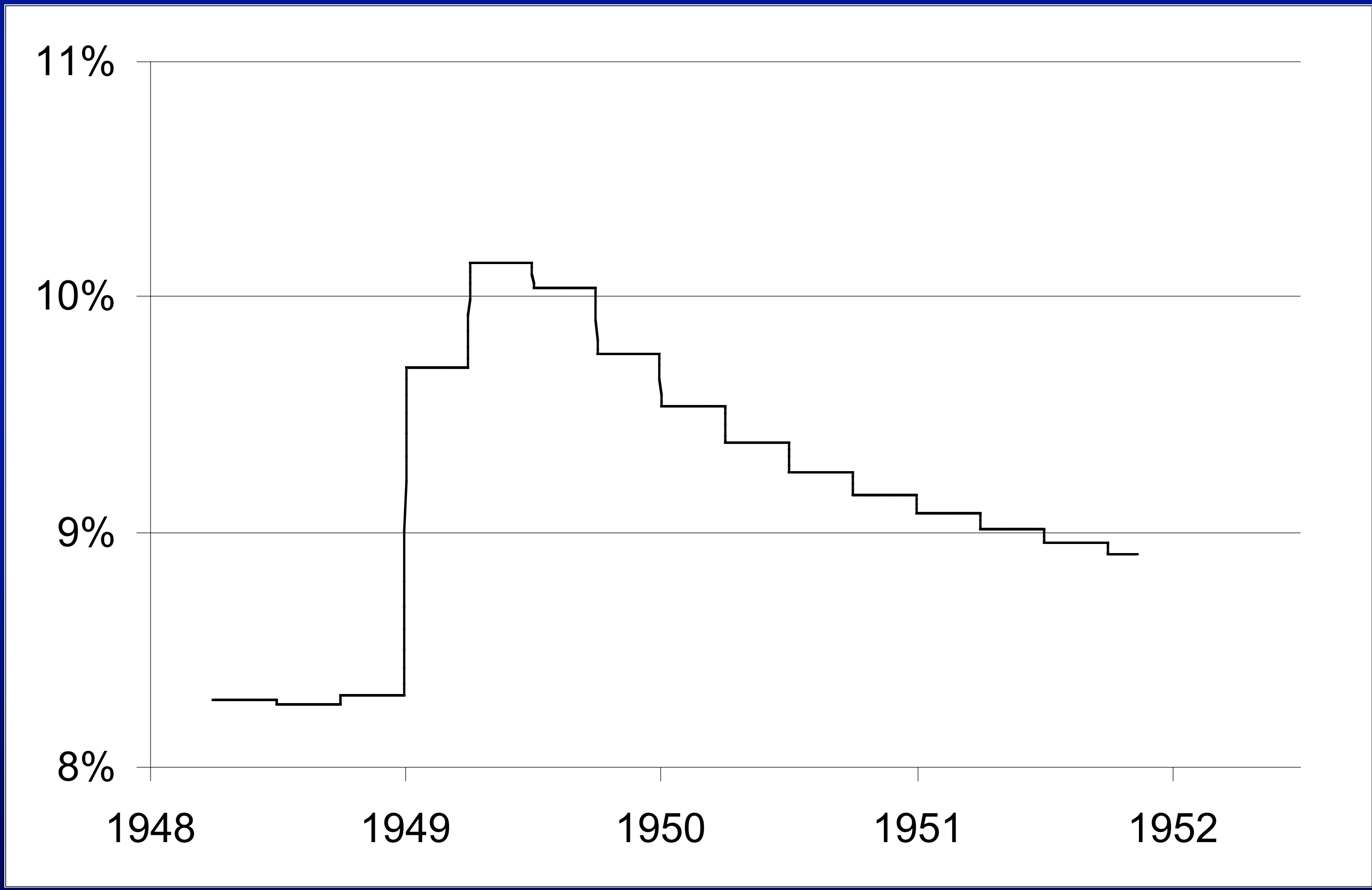
Parameter estimation

Model parameters – interest rate



Parameter estimation

Model parameters – annual rate of productivity growth



Computational advances

a) Hardware

- more computing power (CPU throughput)
- larger & faster memory (3-tier)

b) Software

- better algorithms
- automatic adjoints
- block-banded algorithms
- efficient parallelization, ….

How much DA *vs*. forecast?

- Design integrated observing–forecast–assimilation systems!

Observing system design

- **Need no more (independent) observations than *d-o-f* to be tracked:**
  - "features" (Ide & Ghil, 1997a, b, *DAO*);
  - instabilities (Todling & Ghil, 1994 + Ghil & Todling, 1996, *MWR*);
  - trade-off between mass & velocity field (Jiang & Ghil, *JPO*, 1993).
- The cost of **advanced** DA is **much** less than that of instruments & platforms:
  - at best use DA **instead** of instruments & platforms.
  - at worst use DA to determine which instruments & platforms (**advanced** OSSE)
- **Use any** observations, if forward modeling is possible (observing operator H)
  - satellite images, 4-D observations;
  - pattern recognition in observations and in phase-space statistics.

The DA Maturity Index of a Field

- **Pre-DA:** few data, poor models
  - The theoretician: Science is truth, don't bother me with the facts!
  - The observer/experimentalist: Don't ruin my beautiful data with your lousy model!!
- **Early DA:**
  - Better data, so-so models.
  - Stick it (the obs'ns) in – direct insertion, nudging.
- **Advanced DA:**
  - Plenty of data, fine models.
  - EKF, 4-D Var (2^{nd} duality).
- **Post-industrial DA:**

  (Satellite) images → (weather) forecasts, climate "movies" …

Conclusion

- No observing system without data assimilation and no assimilation without dynamics[a]

- Quote of the day: "You cannot step into the same river[b] twice[c]"
  (Heracleitus, *Trans. Basil. Phil. Soc. Miletus*, *cca.* 500 B.C.)

[a]of state and errors
[b]Meandros
[c] "You cannot do so even once" (subsequent development of "flux" theory by Plato, *cca.* 400 B.C.)
Τα παντα ρεει = Everything flows

General references

Bengtsson, L., M. Ghil and E. Källén (Eds.), 1981. *Dynamic Meteorology: Data Assimilation Methods*, Springer-Verlag, 330 pp.

Daley, R., 1991. *Atmospheric Data Analysis*. Cambridge Univ. Press, Cambridge, U.K., 460 pp.

Ghil, M., and P. Malanotte-Rizzoli, 1991. Data assimilation in meteorology and oceanography. *Adv. Geophys.*, **33**, 141–266.

Bennett, A. F., 1992. *Inverse Methods in Physical Oceanography*. Cambridge Univ. Press, 346 pp.

Malanotte-Rizzoli, P. (Ed.), 1996. *Modern Approaches to Data Assimilation in Ocean Modeling*. Elsevier, Amsterdam, 455 pp.

Wunsch, C., 1996. *The Ocean Circulation Inverse Problem*. Cambridge Univ. Press, 442 pp.

Ghil, M., K. Ide, A. F. Bennett, P. Courtier, M. Kimoto, and N. Sato (Eds.), 1997. *Data Assimilation in Meteorology and Oceanography: Theory and Practice*, Meteorological Society of Japan and Universal Academy Press, Tokyo, 496 pp.

"Miss Peterson, may I go home? I can't assimilate any more data today."

J.B. Handelsman (5/31/1969)