

**Collaborative Research: Robust Climate Projections
*and (ending) Stochastic Stability of Dynamical Systems/
(LOI) Stochastic Models and Low-Frequency Modes***

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Outline

- The IPCC process: results and further questions.
- Natural climate variability as a source of uncertainties
 - sensitivity to initial data → error growth
 - sensitivity to model formulation → see below!
- Uncertainties and how to fix them
 - structural in/stability
 - statistical stability (current project results)
- Hierarchy of models
 - “toy” models – Lorenz model (Chekroun *et al.* poster)
 - an ENSO-DDE model (Zaliapin *et al.* poster)
 - the ICTP-AGCM (Bracco *et al.* poster)
- Applications to IPCC-type problems
 - climate sensitivity and climate response (Chekroun *et al.* poster, bis)
 - interdecadal predictions (Kondrashov *et al.* poster;
pls. see next talk, by A. W. Robertson *et al.*)

Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast ...

Source : IPCC (2007),
AR4, WGI, SPM

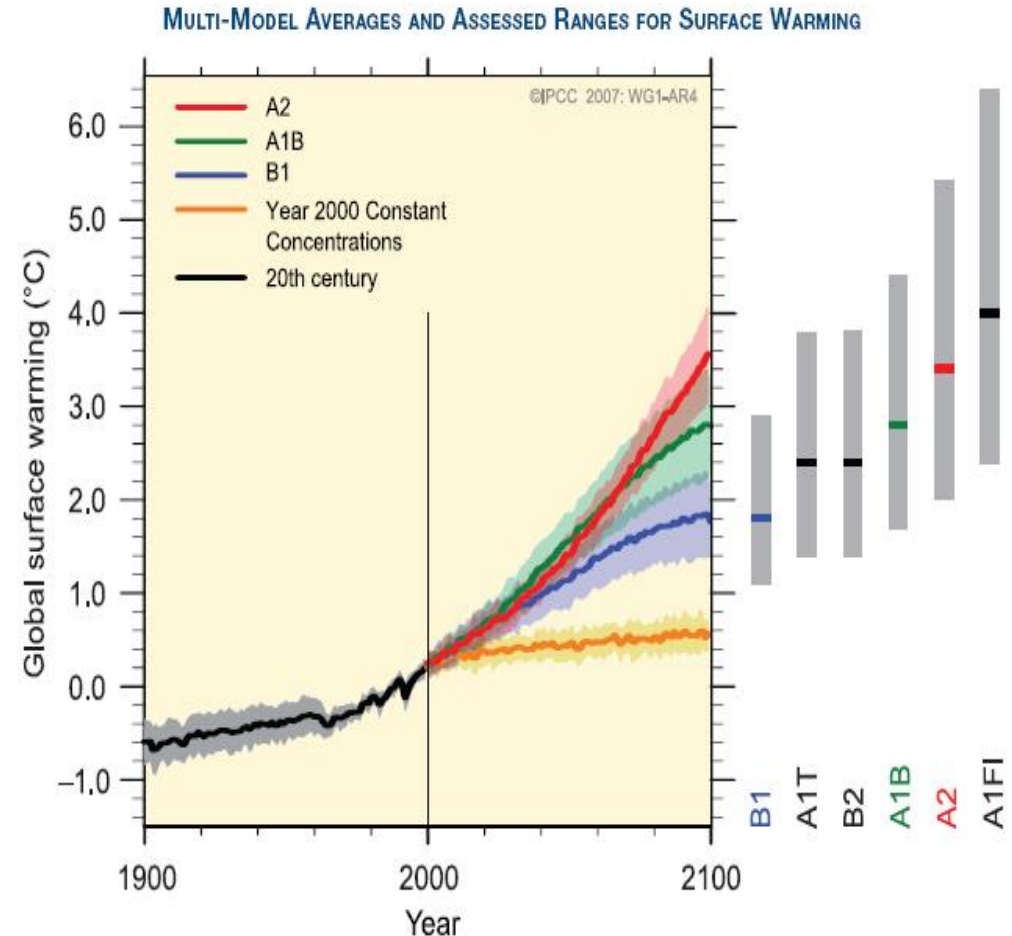


Figure SPM.5. Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ± 1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the likely range assessed for the six SRES marker scenarios. The assessment of the best estimate and likely ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. (Figures 10.4 and 10.29)

Climate and Its Sensitivity

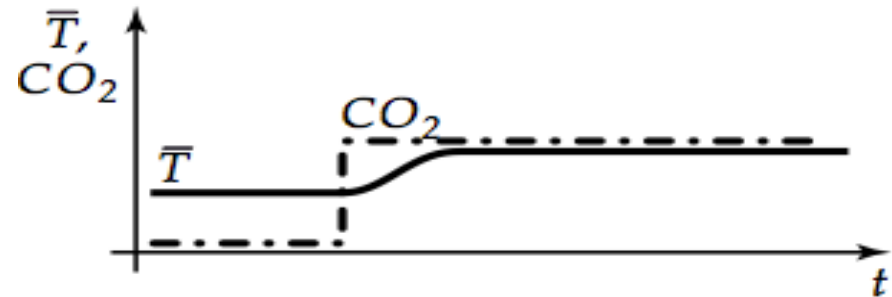
Let's say CO₂ doubles:

How will “climate” change?

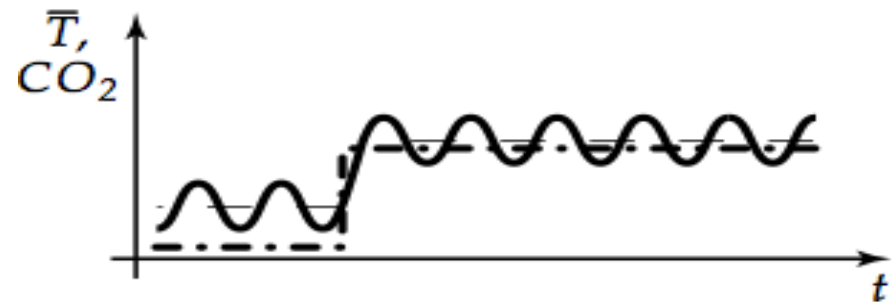
1. Climate is in **stable equilibrium** (fixed point); if so, **mean temperature** will just shift gradually to its new equilibrium value.
2. Climate is **purely periodic**; if so, **mean temperature** will (maybe) shift gradually to its new equilibrium value. But how will the **period, amplitude and phase** of the **limit cycle** change?
3. And how about some “real stuff” now: **chaotic + random**?

Ghil (Encycl. Global Environmental Change, 2002)

a) *Equilibrium sensitivity*



b) *Nonequilibrium sensitivity*



Can we, nonlinear dynamicists, help?

The uncertainties
might be *intrinsic*,
rather than mere
“tuning problems”

If so, maybe
*stochastic structural
stability* could help!

Might fit in nicely with
recent taste for
“stochastic
parameterizations”

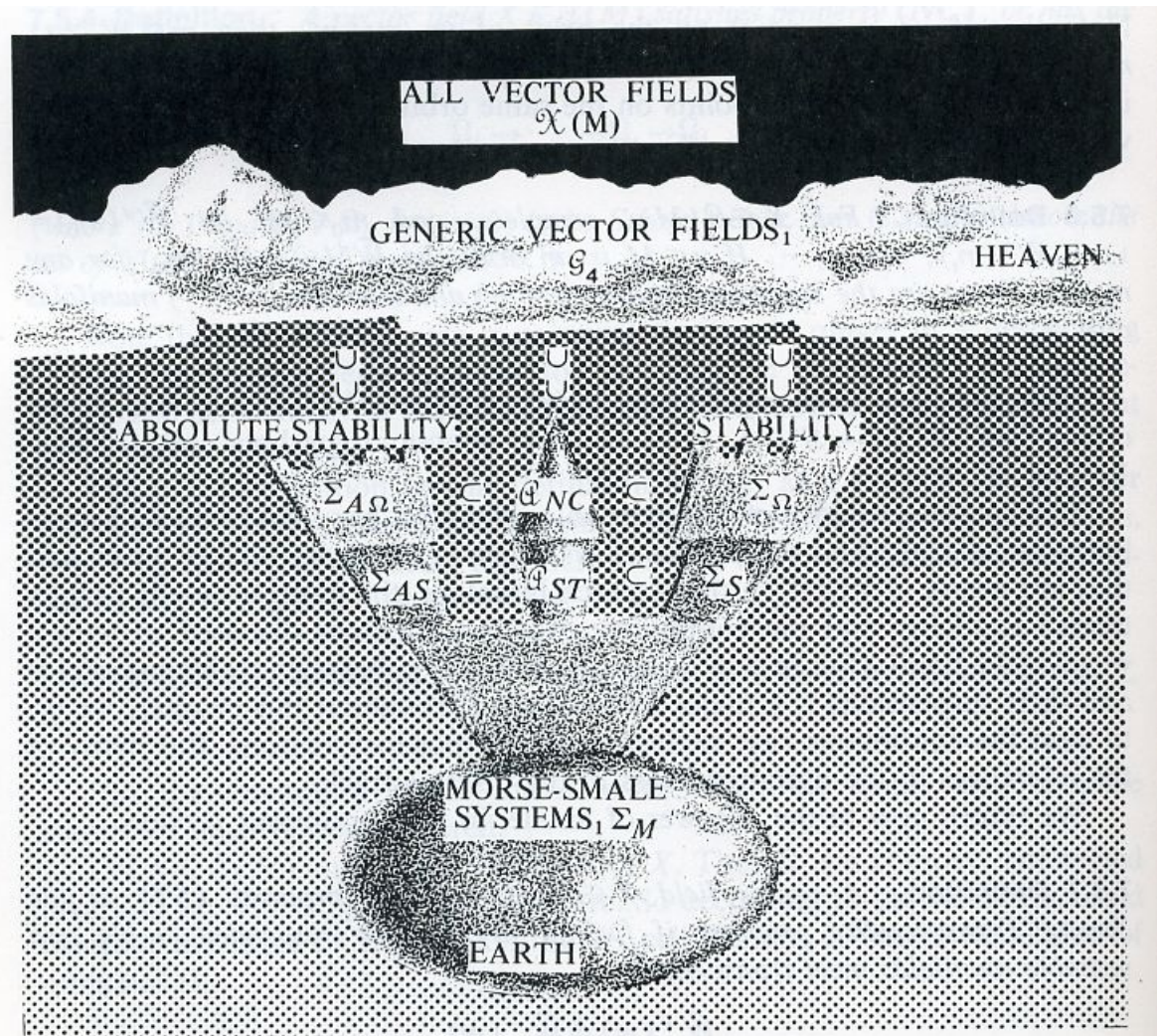


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from Abraham & Marsden, 1978)

The Ruelle response formula

- From a mathematical point of view, climate sensitivity can be analyzed in terms of **sensitivity of SRB measures**.
- The **thermodynamic formalism à la Ruelle, in the RDS context**, helps to understand the response of **systems out-of-equilibrium**, to changes in the parameterizations (Kifer, Liu, Gundlach,...).
- **The Ruelle response formula**: Given an SRB measure μ of an autonomous chaotic system $\dot{x} = F(x)$, an observable $\Phi : X \rightarrow \mathbb{R}$, and a smooth **time-dependent perturbation** G_t , then the time-dependent variations $\delta_t \mu$, of μ is given by:

$$\delta_t \langle \mu, \Phi \rangle = \int_{-\infty}^t d\tau \int \mu(dx) G_\tau(x) \cdot \nabla_x (\Phi \circ \varphi_{t-\tau}(x)),$$

where φ_t is the flow of the unperturbed system $\dot{x} = F(x)$ and $\langle \mu, \Phi \rangle := \int \Phi(x) d\mu(x)$.

- **This formula permits to compute the response of the system without ensemble of long-run simulations!**

The susceptibility function

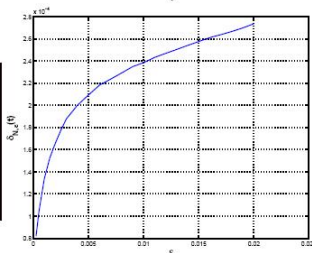
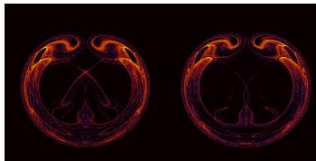
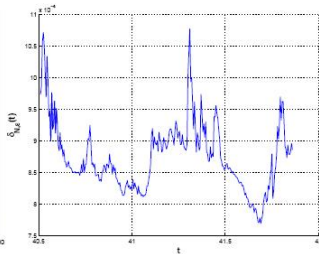
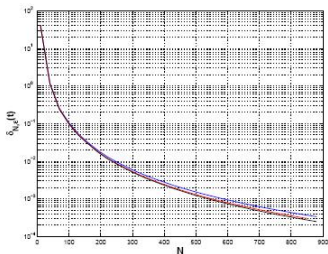
- In the case of perturbation of the form $G_t(x) = \phi(t)G(x)$, the Ruelle response formula can be written:

$$\delta_t \langle \mu, \Phi \rangle = \int dt' \kappa(t - t') \phi(t'),$$

where κ is called the **response function**. The **Fourier transform** $\hat{\kappa}$ of the response function is called the **susceptibility function**.

- In this case $\delta_t \langle \hat{\mu}, \Phi \rangle(\xi) = \hat{\kappa}(\xi) \hat{\phi}(\xi)$ and since the r.h.s. is a product, there are no frequencies in the linear response that are not present in the signal.
- In general, the situation can be more complicated and the theory gives the following criteria of high-sensitivity:
 - Ⓞ: **Poles of the susceptibility function $\hat{\kappa}(\xi)$ in the upper-half plane**
 \Rightarrow **High sensitivity of the systems response function $\kappa(t)$.**
- RDS theory offers a path for extending this criteria when **random perturbations** are considered.

Pathwise linear response: Numerical results

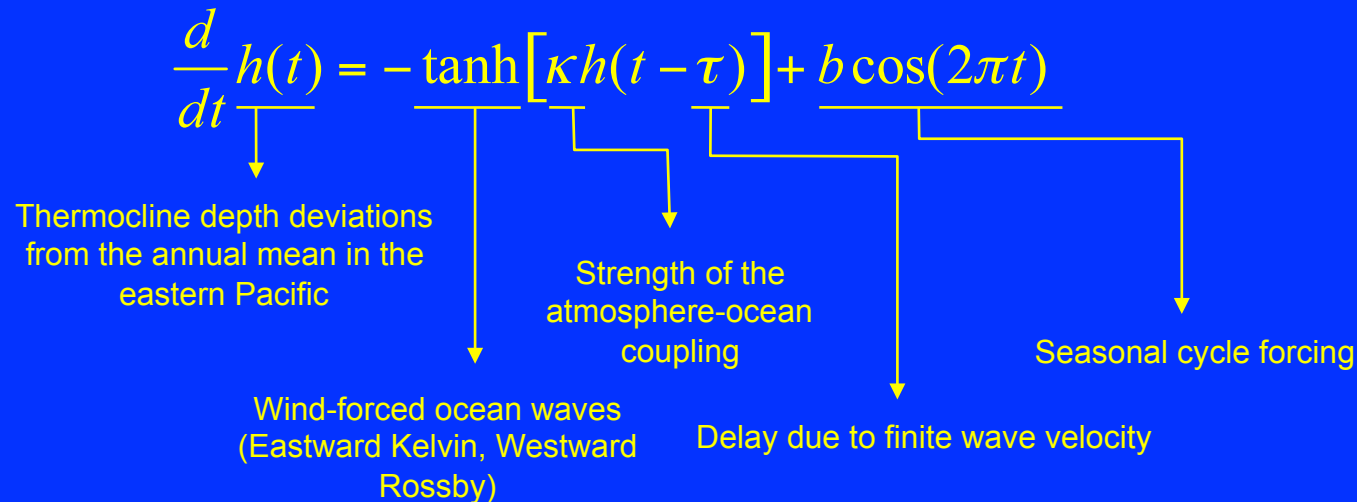


- Lower-right panel shows **pathwise linear response** in a stochastic Lorenz model (cf. poster Chekroun et al.).

A conceptual delay differential model for ENSO variability:

What has been learned and what comes next

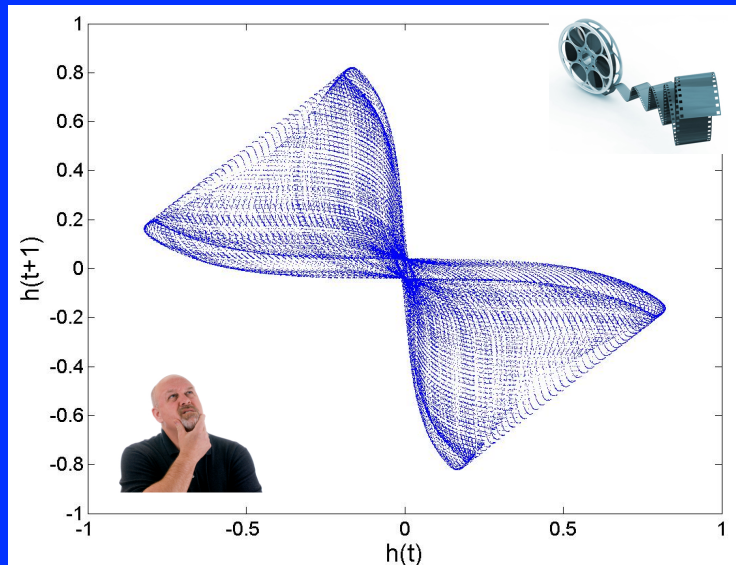
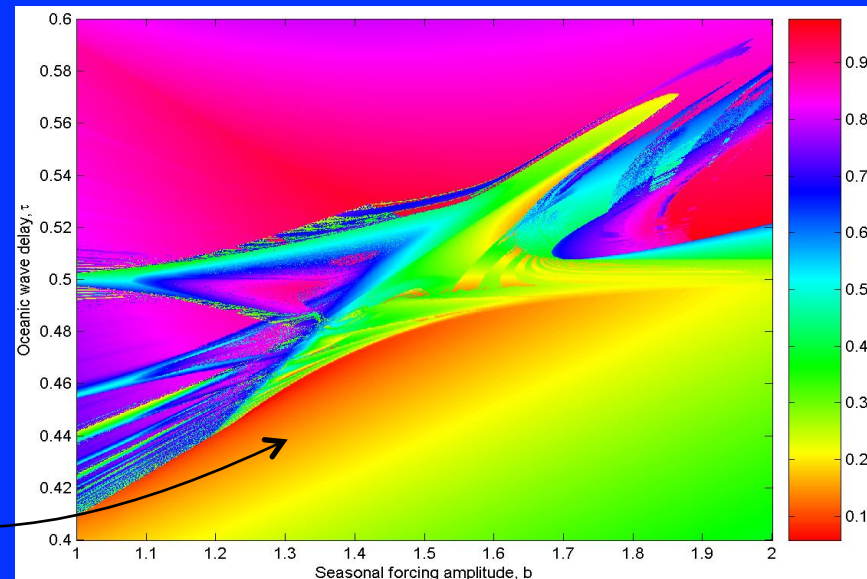
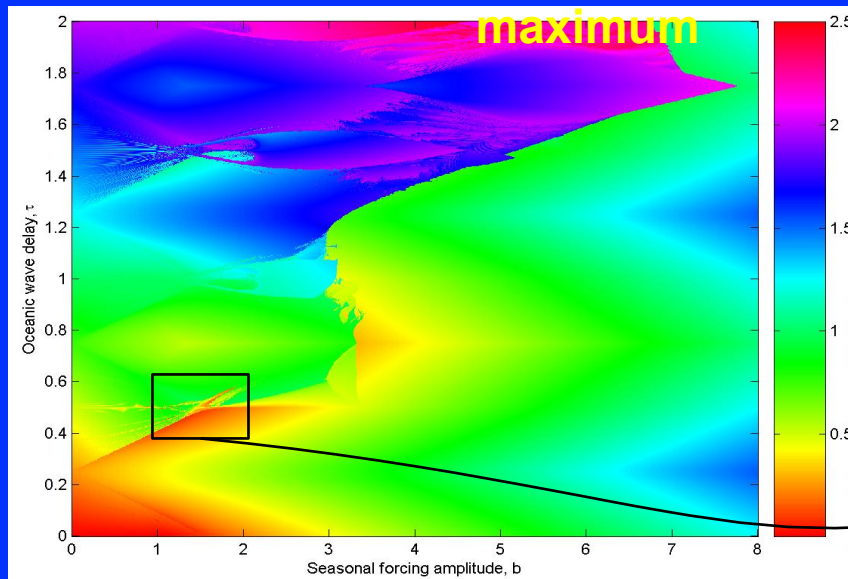
Poster 22, Wednesday, March 31, 11:35-1:30PM



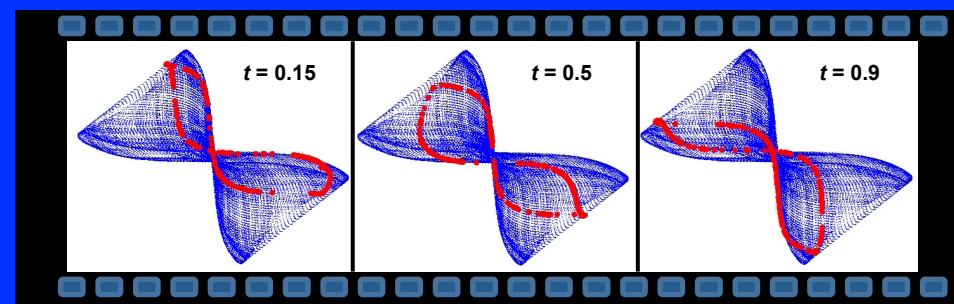
Main results:

- (1) Theory of this non-autonomous DDE (existence, uniqueness, continuous dependence, existence of pullback attractor);
- (2) Software (FORTRAN-90) for DDE numerical exploration (standard software cannot handle full parameter space exploration);
- (3) Numerical analysis of the model in its full 3-D space of physically relevant parameters;
- (4) Model explains: quasi-periodic ENSO behavior; intermittency of El Niño/La Niña events; phase locking (warm events around Christmas); interdecadal variability.

1. Instabilities in the trajectory



2. Pullback attractor: A way to explore asymptotic behavior of driven systems



A systematic study of parameter dependence in the ICTP AGCM

Annalisa Bracco, GATech, J. David Neelin, UCLA, Hao Luo, GATech,
Jim McWilliams, UCLA, Joyce Meyerson, UCLA, Michael Ghil, UCLA

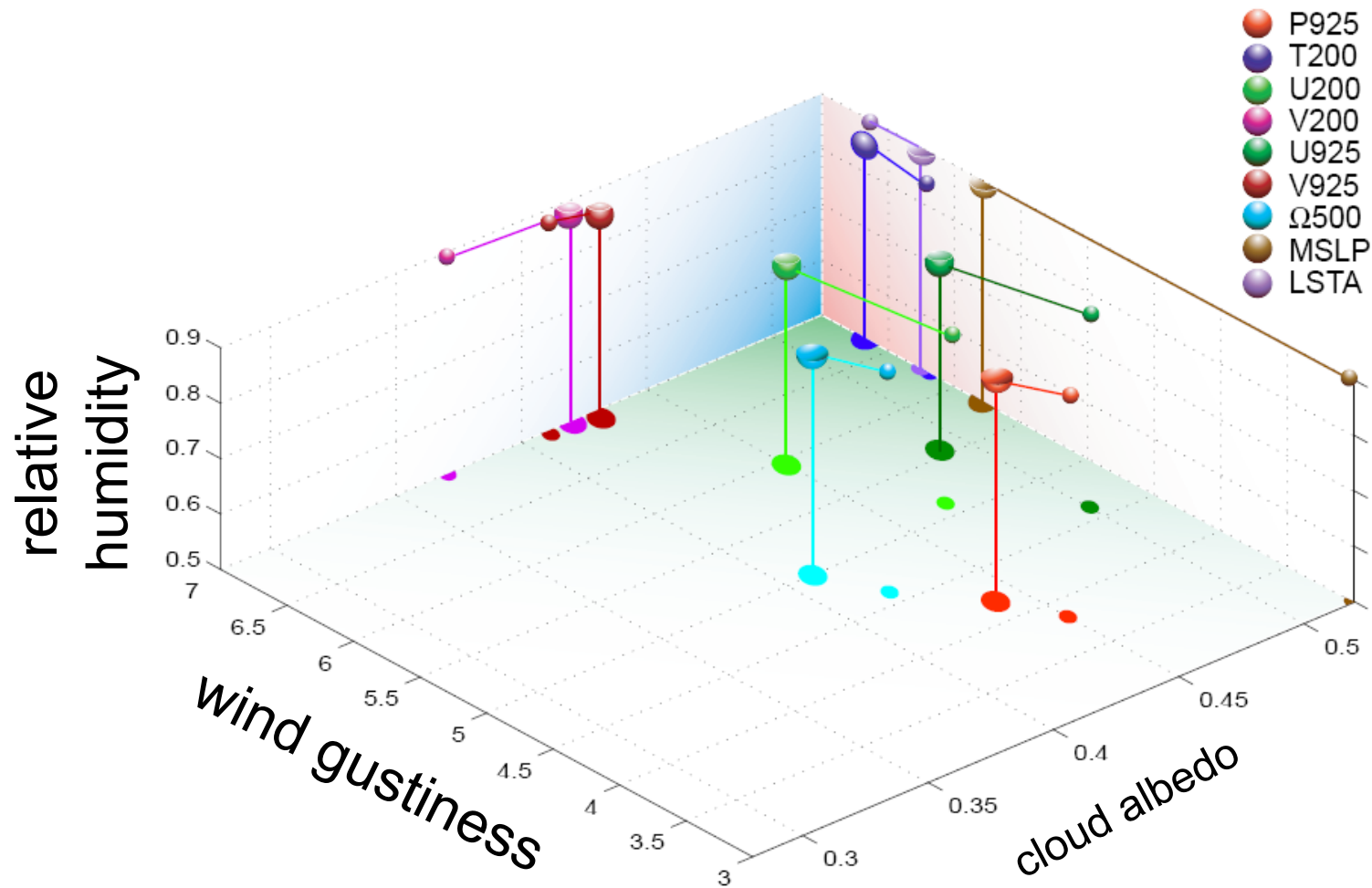
Goal:

To examine optimization strategies for a complex climate model, including the behavior of error measures used as contributions to a cost function

- Model and set-up: The ICTP AGCM
- Methods: Low order polynomial fits to the climate model outputs in the parameter space; compare linear and non-linear contributions.
- Evaluate multi-objective approach as opposed a cost function that uses pre-determined, user-supplied, weights to optimize different climate variables

Achievements:

For large scale measures (such as RMS of climate variables) we find that low order fitting procedures are quite successful. This leads to a constrained optimization problem simple enough to solve with standard algorithms. We then optimize for multiple objective functions associated with different climate variables---the location of the optima in parameter space quantifies the 'contradiction' between objectives



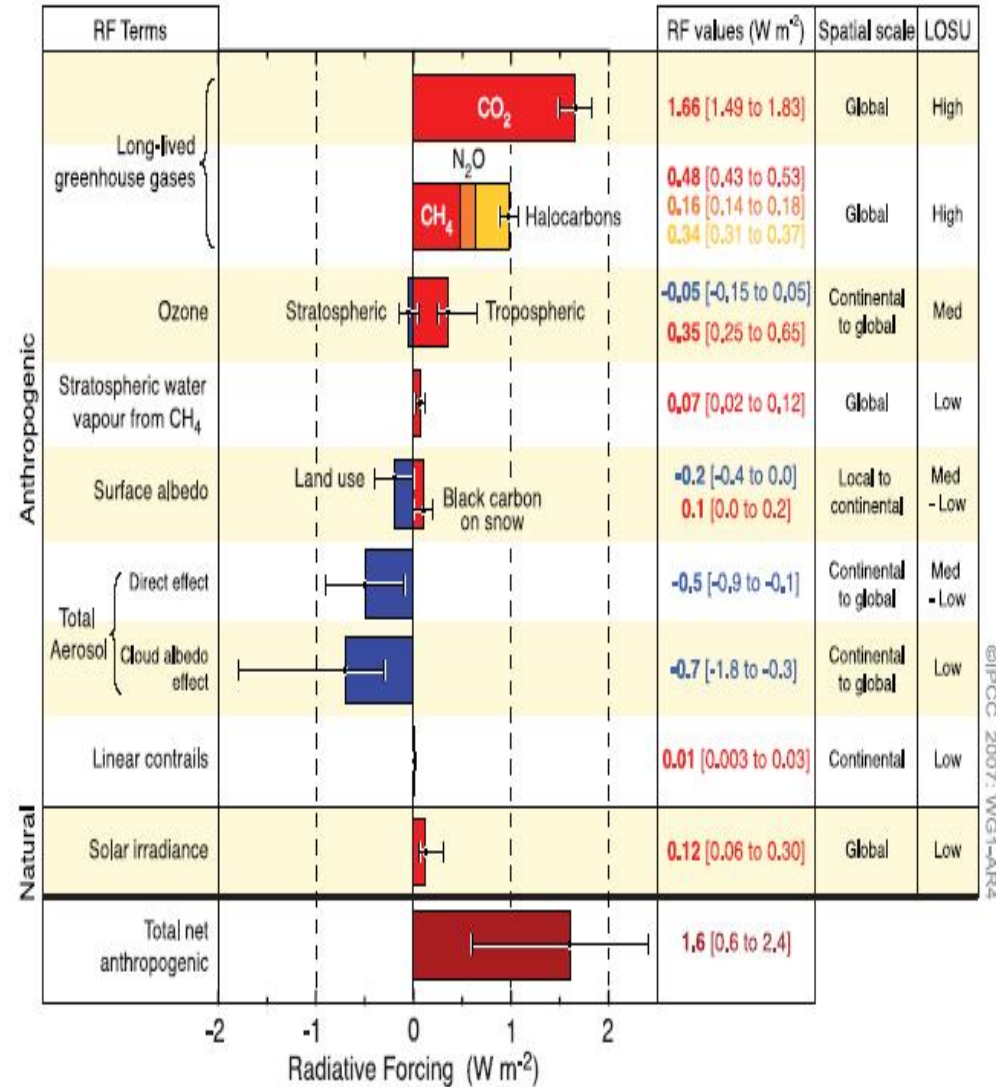
Global optima for the ICTP AGCM calculated for different objective functions optimized separately for various climate variables. It shows the contradiction between different objective functions (experienced by modelers as ‘one thing got better but the others got worse’). Optima can then be given a strict partial order to inform user decisions. Small spheres are obtained with a fitting procedure of order N (in the # of parameters); in most cases they give a reasonable approximation to an order N^2 procedure (larger spheres).

Reserve Slides

GHGs rise!

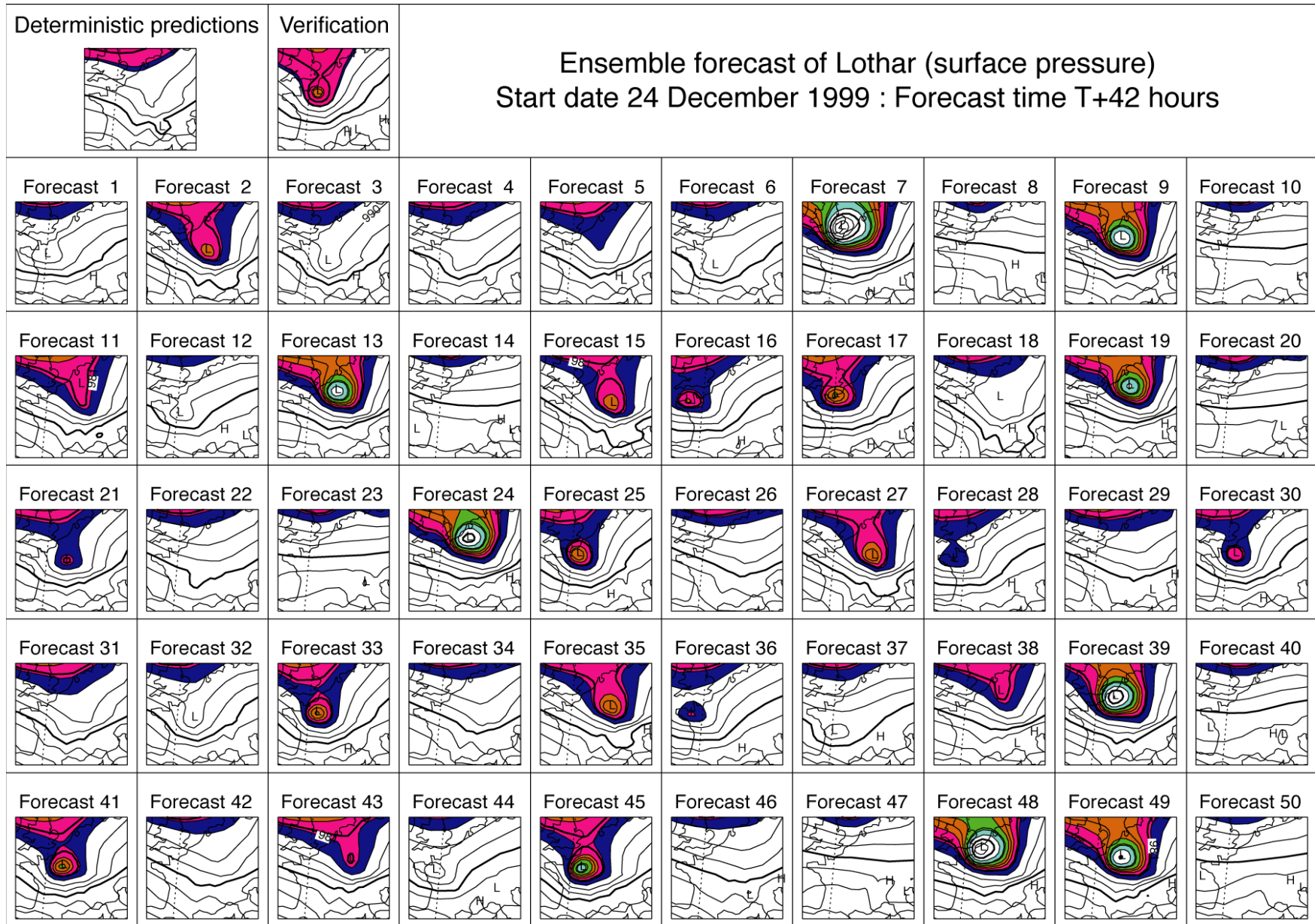
It's gotta do with us, at least a bit, ain't it?
But just how much?

RADIATIVE FORCING COMPONENTS



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IPCC (2007)



Courtesy Tim Palmer, 2009

Concluding remarks, I – RDS and RAs

Summary

- A change of paradigm for open, non-autonomous systems
- Random attractors are (i) spectacular, (ii) useful, and (iii) just starting to be explored for climate applications.

Work in progress

- Study the effect of specific **stochastic parametrizations** on model robustness.
- Applications to **intermediate models and GCMs**.
- Implications for **climate sensitivity**.
- Implications for **predictability?**

Concluding remarks, II – General

What do we know?

- It's getting warmer.
- We do contribute to it.
- So we should act as best we know and can!

What do we know less well?

- By how much?
 - Is it getting warmer ...
 - Do we contribute to it ...
- How does the climate system (atmosphere, ocean, ice, etc.) really work?
- How does natural variability interact with anthropogenic forcing?

What to do?

- Better understand the system and its forcings.
- Explore the models', and the system's, robustness and sensitivity
 - stochastic structural and statistical stability!
 - linear response = response function + susceptibility function!!

Some general references

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