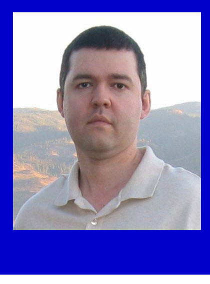
A coupled oscillator concept for modeling El-Niño/Southern Oscillation (ENSO) variability:
What has been learned and what comes next

DOE Integrated Climate Change Modeling Science Team Meeting
March 29 – April 2, 2010



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1. INTRODUCTION

The physical growth mechanism of ***El-Niño/Southern-Oscillation (ENSO)*** is quite well understood: is due to the positive atmospheric feedbacks on equatorial SST anomalies via the surface wind stress, cf. Bjerknes [1969]. Still, ENSO's unstable quasi-periodic behavior prevents its robust predictions, even at subannual lead times. Conceptual numerical modeling plays a prominent role in understanding ENSO variability and developing forecasts. Despite the existence and importance of comprehensive numerical models, ***much of our theoretical understanding of ENSO comes from relatively simple models***. Initiated in the 1980s, the study of such conceptual models has significantly contributed to shedding new light on many aspects of ENSO, including its quasi-periodic behavior, onset of instabilities, phase locking, power spectrum, and interdecadal variability.

This project focuses on theoretical and numerical exploration of a ***conceptual modeling approach*** that deals with a simplified picture of ENSO dynamics yet allows one to achieve a rather ***comprehensive understanding of ENSO's underlying mechanisms*** and their interplay. The projects explores a *deterministically chaotic, nonlinear* paradigm to explain the complexities of ENSO dynamics by the nonlinear interplay of its principal internal mechanisms.

2. SUMMARY OF RESULTS

We studied a forced delay differential equation (DDE) for ENSO variability. The model combines two key mechanisms that participate in ENSO dynamics: (i) ***delayed negative feedback*** caused by oceanic waves, and (ii) ***seasonal forcing***. The main results are summarized by Ghil *et al.* [2008b] and Zaliapin and Ghil [2010]. We have developed an appropriate software and described the model behavior in the three-dimensional (3-D) space of its physically relevant parameters – oceanic wave delay τ, strength κ of ocean-atmosphere coupling, and amplitude b of seasonal forcing – and established two regimes of variability, ***stable and unstable***, separated by a sharp neutral curve in parameter space. A detailed numerical exploration of model solutions has found (i) Numerous scenarios relevant to the ENSO physics, including quasi-periodic ***El Niño/La Niña events***, ***interdecadal variability,*** and patterns reminiscent of ***Madden-Julian oscillations*** or ***westerly wind bursts***; (ii) The ***phase locking*** of solutions to the seasonal cycle: local temperature maxima and minima tend to occur at the same position within this cycle, which is a characteristic feature of the observed El Niño events; (iii) Parametric ***instabilities*** in the location of extrema; (iv) Co-existence of ***multiple solutions*** for the same parameter values in certain parameter ranges; (v) Scenario by which the model goes from simple (period-1) to more complicated (period-*k*) solutions. Furthermore, we have applied to our model the concept of ***pullback attractor*** (PBA, [Ghil *et al.*, 2008a]) and demonstrated that its dynamics – whether periodic or quasi-periodic – occurs on a two-dimensional torus. This behavior reflects the ***competition between two oscillatory mechanisms***: an external one due to the seasonal forcing and an internal one due to the delayed feedbacks. Such an interpretation is much harder to obtain from the complex, parameter-sensitive dynamics of the model using more traditional, theoretical and numerical, approaches. We expect to see similar behavior in much more detailed and realistic models, where it is harder to describe its causes as completely.

3. WHAT'S NEXT

The results and collaborations established within this project have prepared us to address the following problems: (1) Explore the quasi-biennial and quasi-quadrennial modes of variability associated with ENSO; and more generally, study the modes of low-frequency ENSO variability in a full hierarchy of models; (2) Further develop the pullback attractor (PBA) approach to ENSO modeling. This includes a study of PBA geometry and its dependence on the model parameters as well as a study of the system's physical measure on the PBA; (3) Expand the present study to the analysis of ENSO variability under global climate change (e.g., global warming); and (4) Explore more realistic conceptual models of ENSO that will include positive delayed feedbacks on the thermocline depth (temperature) and oceanic wave.

4. MODEL: FORMULATION AND SOLUTION PROPERTIES

We consider the Ghil *et al.* [2008b] model, summarized here as follows:

$$\frac{d}{dt}h(t) = -\tanh\left[\kappa h(t-\tau)\right] + b\cos(2\pi t)$$

- $h(t)$: Thermocline depth deviations from the annual mean in the eastern Pacific
- $\tanh[\ldots]$: Wind-forced ocean waves (Eastward Kelvin, Westward Rossby)
- κ: Strength of the atmosphere-ocean coupling
- τ: Delay due to finite wave velocity
- $b\cos(2\pi t)$: Seasonal cycle forcing

Theorem 1 (Existence, uniqueness, continuous dependence); see Ghil et al. [2008b]

$$\frac{dh(t)}{dt} = -\tanh\left[\kappa h(t-\tau)\right] + b\cos(2\pi t),\ t \ge 0 \qquad (1)$$

$$h(t) = \varphi(t),\ t \in [-\tau, 0) \qquad (2)$$

The IVP (1-2) has a unique solution on $[0,\infty)$ for any set $(\kappa, b, \tau, \varphi)$.
This solution depends continuously on initial data $\varphi(t)$, delay τ, and the rhs of (1) (in an appropriate norm).

Corollary 1 (Detection of unstable solutions)

A **discontinuity** in a solution profile (see **Fig. 7** below) indicates existence of an **unstable solution** that separates the attractor basins of two **stable solutions**.

5. INSTABILITIES

Ghil *et al.* [2008b] have discovered two domains in the model's parameter space: stable and unstable. The **stable domain** is characterized by unique period-1 solutions: no multiple solutions exist in this domain. The **unstable domain** exhibits physically plausible behavior with quasi-periodic solutions; in this domain multiple solutions obtain almost everywhere. This panel illustrates **instabilities in the trajectory maximum**; similar results are obtained for other trajectory statistics: minimum, mean, variance, *etc.*

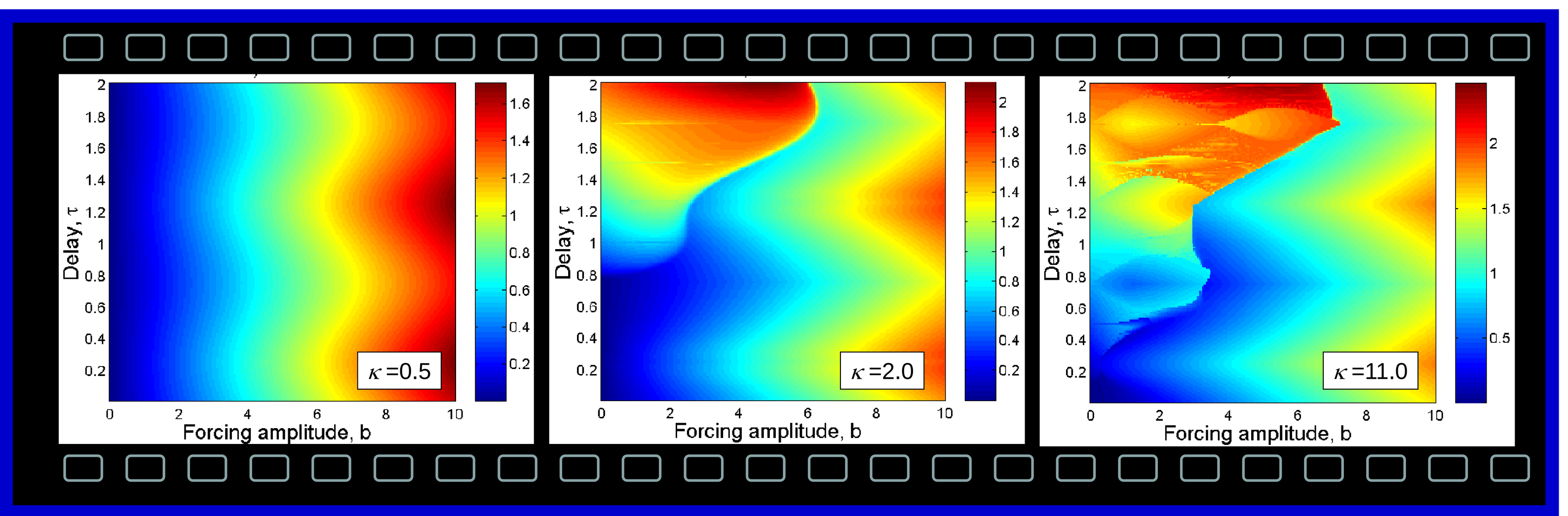
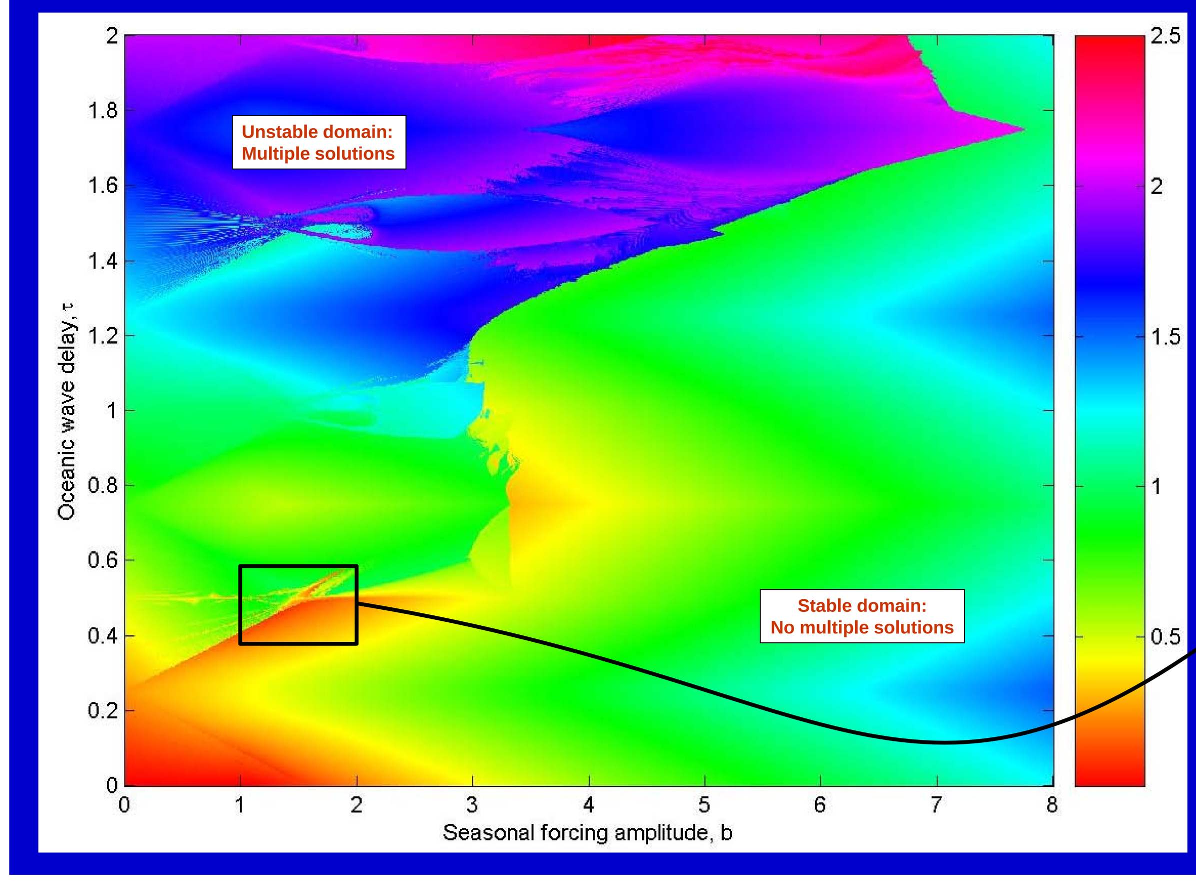


Figure 1: Instabilities in the trajectory maximum onset as the ocean-atmosphere coupling κ increases



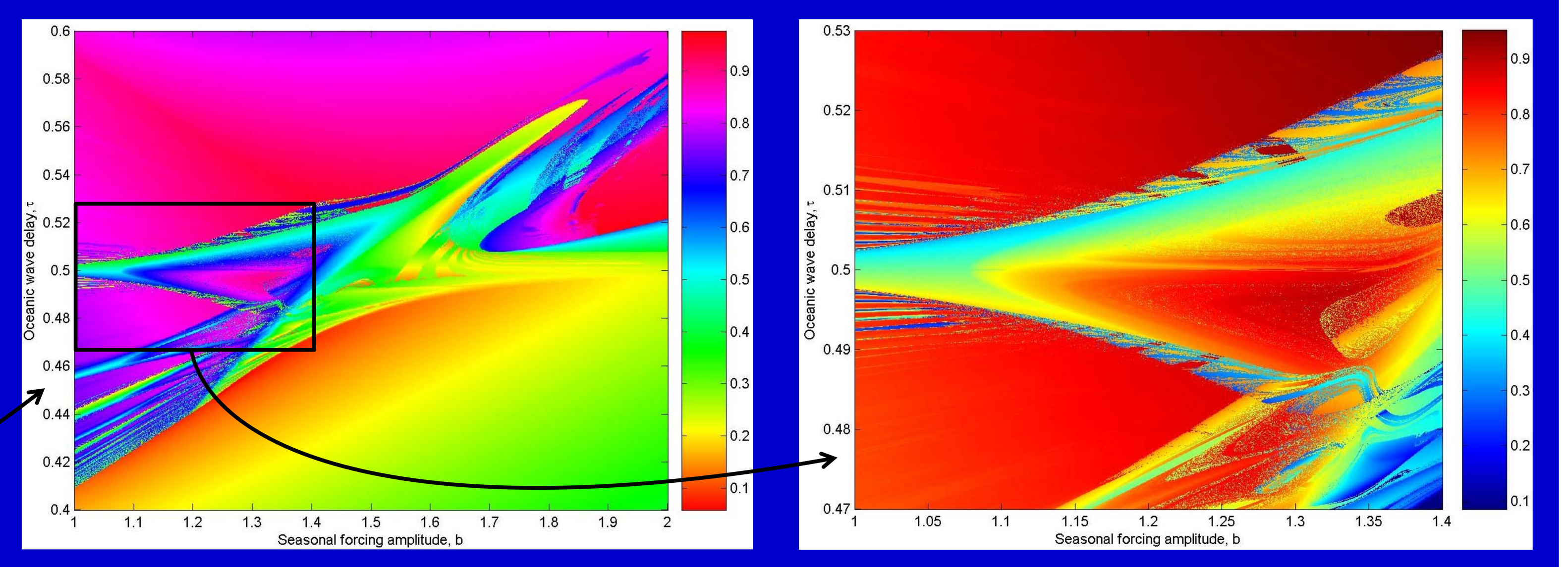


Figure 2: Maximal solution value as a function of (b,τ), for fixed κ = 100. These maps clearly illustrate the stable and unstable domains in the model's parameter space (cf. Ghil et al. [2008]).

6. BEHAVIOR OF EXTREMA (INTERMITTENCY)

Figure 3 shows the values of local maxima (red) and minima (blue) as a function of the oceanic wave delay τ. Ghil et al. [2008] have found that the solution period generally increases with τ. Such a period increase is associated with an increase of the number of distinct local extrema, each of which is observed once a year. This increase follows a pattern, shown in **Fig. 5**, that is typical of chaos in discrete-time dynamical systems (maps) and suggests that the DDE model's continuous-time solutions also evolve to larger and larger periods through a sequence of increasingly complex chaotic regimes.

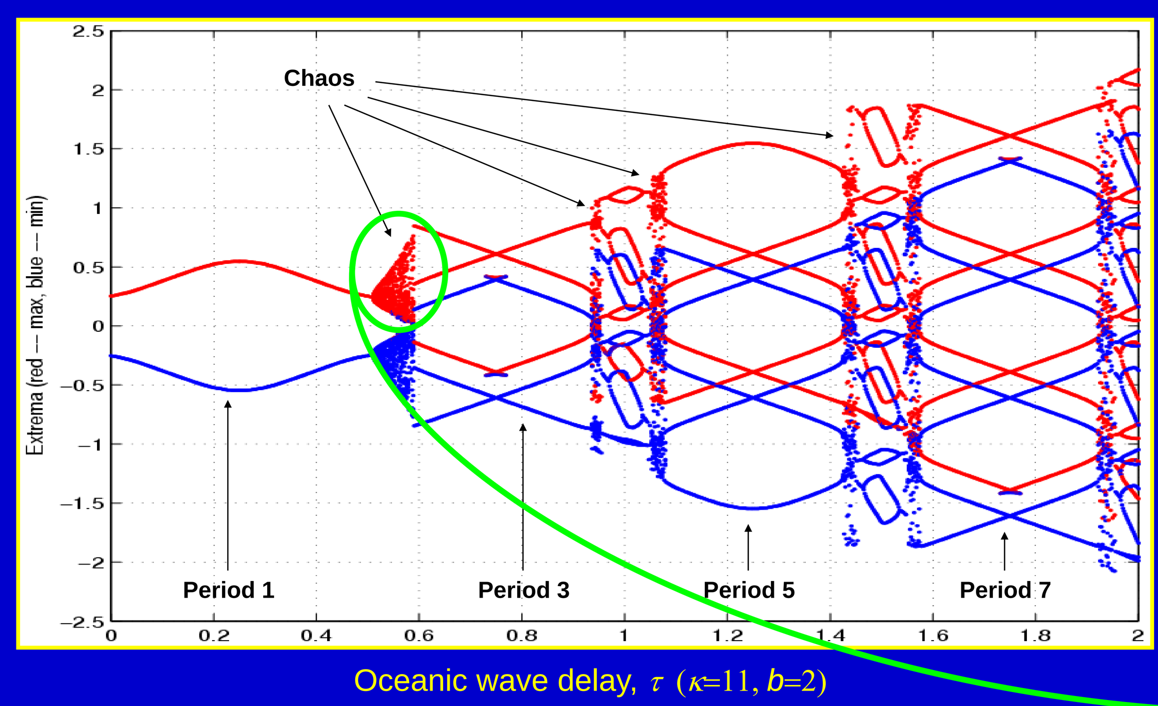


Figure 3: Values of local maxima (red) and minima (blue) as a function of oceanic wave delay. Periodic and chaotic behavior alternate intermittently.

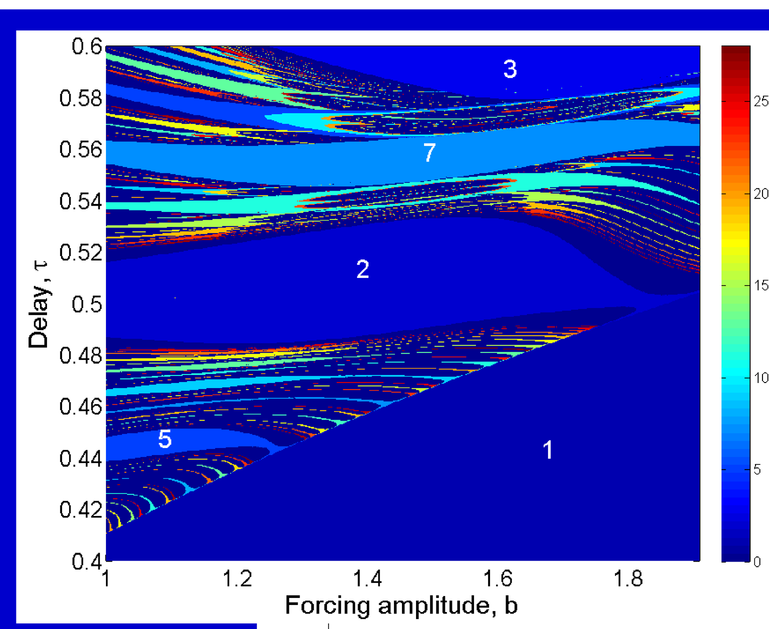


Figure 4: Period map, κ = 11

Numbers show the period within the largest constant-period tongues.

Notice a dominant period-2 tongue, related to the quasi-biennial ENSO oscillation.

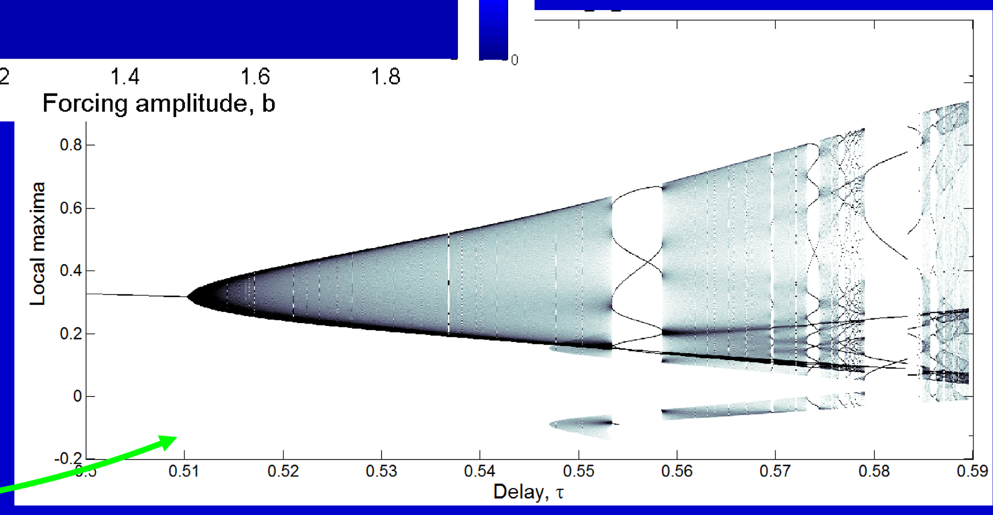


Figure 5: Distribution of the values of local maxima within a chaotic regime that separates period-1 and period-3 trajectories

7. TIME-DEPENDENT ATTRACTOR: PULLBACK APPROACH

The concept of ***attractor*** plays an important role in studying autonomous dynamical systems. Pullback Attractor (PBA) is a generalization of the concepts of ***attractor*** and ***strange attractor*** to non-autonomous (driven) systems; see Ghil *et al.* [2008a] and references therein. A PBA formalism has been applied to studying the DDE ENSO model, as well as a broader class of models.

Theorem 2 (Existence of PBA in a DDE system)

The class of DDE considered

Let $\omega \in C_b(\mathbb{R},\mathbb{R}^n)$ an almost periodic function. Let Ω be the closure of $\{\omega(t+\cdot); t \in \mathbb{R}\}$ for the topology of uniform convergence,

$$\dot{x} = f(x(t)) + J(x(t-\tau)) + \omega(t),\ \omega \in P \qquad (2)$$

supplemented by

$$x(s) = \psi(s),\ -\tau \le s \le 0$$

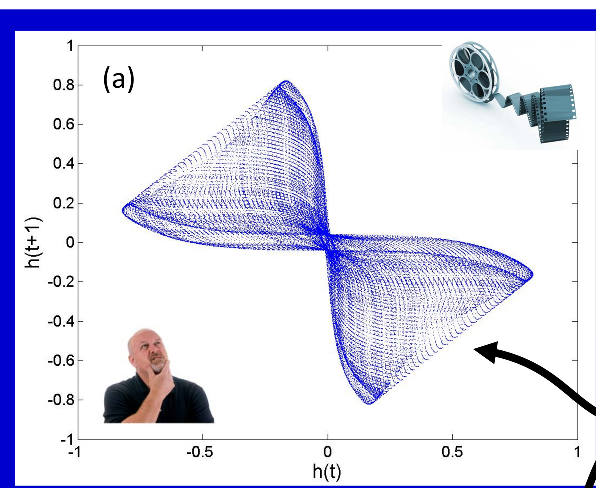
with $\tau > 0$ and $\psi \in C^0(\mathbb{R},\mathbb{R}^n)$.

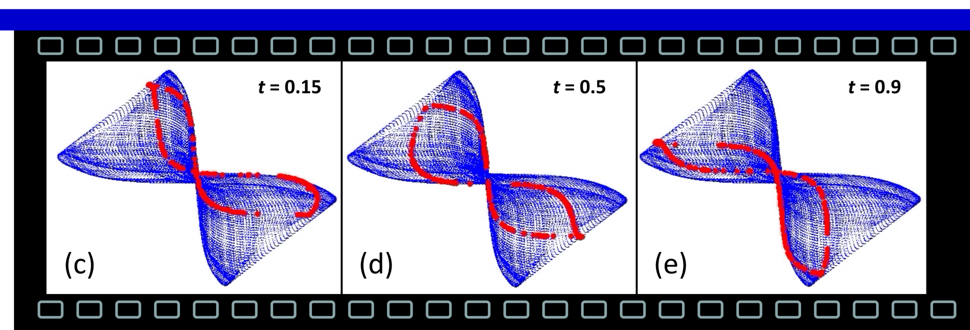
Existence of PBA

Theorem. Assume that there exist $\alpha > 0$, and $\beta \ge 0$, s.t.

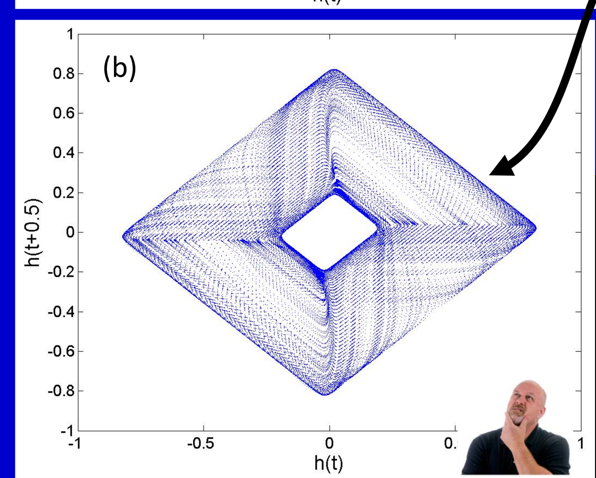
$$\langle f(x), x\rangle \le -\alpha\|x\|^2 + \beta,\ \forall\, x \in \mathbb{R}^n,$$

and that $J : \mathbb{R}^n \to \mathbb{R}^n$ is continuous and bounded *i.e.*
$\exists\, M > 0$, s.t. $\|J(\xi)\| \le M$ for all $\xi \in \mathbb{R}^n$.
Then there exists a global pullback attractor of (2).





- Numerical system's forward attractor may suggest that the dynamics is quasi-periodic or weakly chaotic.
  - Numerical system's PBA, shown in red over the static attractor in the "movie frames", clearly shows that the dynamics --- periodic or quasi-periodic --- occurs on a 2-D torus within the three-dimensional space generated by the "product" of the 2-D manifolds.



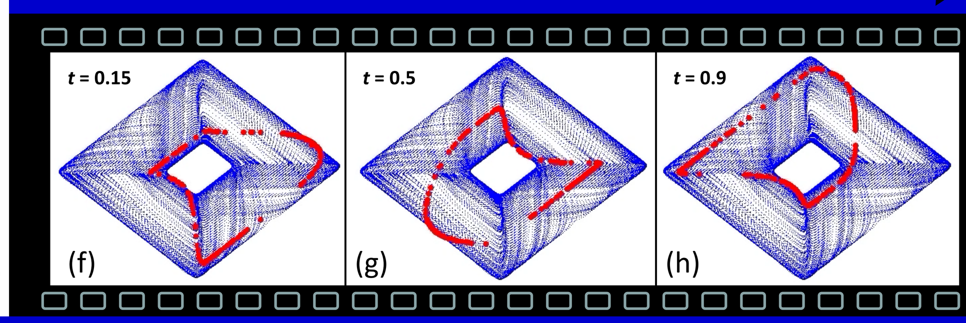


Figure 6: Forward attractor (a,b) and pullback attractor (c-h) for the DDE ENSO model.

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ACKNOWLEDGMENT

This work is part of a DOE-CCPP collaborative research project on "Robust Climate Projections and Stochastic Stability of Dynamical Systems," supported by DOE Grants DE-FG02-07ER64439 and DE-FG02-07ER64440.