State and Parameter Estimation for Coupled Ocean-Atmosphere Model

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Introduction

The El-Niño/Southern-Oscillation (ENSO) dominates interannual climate variability and plays a key role in seasonal-to-interannual prediction. Much is known by now about the main physical mechanisms that give rise to and modulate ENSO, but the values of several parameters that enter these mechanisms are unknown. Here we apply Extended Kalman Filtering (EKF) for both state and parameter estimation in an intermediate, nonlinear, coupled ocean--atmosphere model (ICM) of ENSO (Jin & Neelin, 1993).

EKF for parameter estimation

In the **"state augmentation"** method the parameters are treated as **additional state variables**. For simplicity, let us assume that there is only one unknown parameter μ in the discrete numerical model **F** for the state vector **x**. The underlying **"true"** natural system for **x** is then model **F** and in the simplest case persistence equation for μ, perturbed by noise **ε and ε^μ** with given covariances **Q=< ε(t) ε(t)^T >, Q_μ=< ε^μ(t) ε^μ(t)^T >**.

$$\bar{x}_k = \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} = \begin{pmatrix} F(x_{k-1}, \mu_{k-1}) \\ \mu_{k-1} \end{pmatrix} + \begin{pmatrix} \epsilon_k \\ \epsilon_k^\mu \end{pmatrix}$$

$$\bar{x}_k = \bar{M}_k \bar{x}_{k-1} + \bar{\epsilon}_k; \bar{M} = \frac{\partial \bar{F}}{\partial \bar{x}}$$

The **"augmented"** numerical model advances the forecast ("f") and propagates its error covariance matrix *P*:

$$\bar{x}_k^f = \bar{M}_k \bar{x}_{k-1}^f; \bar{P}_k^f = \bar{M}_k \bar{P}_{k-1}^f \bar{M}_k^T + \bar{Q}$$

The Kalman filter obtains analysis ("a") and reduces the error by assimilating state observations with specified error covariance **R**= **< ε^o(t) ε^o(t)^T >** :

$$y_k^o = (H\ 0)\begin{pmatrix} x_k \\ \mu_k \end{pmatrix} + \epsilon^0 = \bar{H}\bar{x}_k + \epsilon^0$$

$$\bar{x}_k^a = \bar{x}_k^f + \bar{K}(y_k^o - \bar{H}\bar{x}_k^f); \bar{P}_k^a = (I - \bar{K}\bar{H})\bar{P}_k^f$$

$$\bar{K} = \bar{P}^f \bar{H}^T (\bar{H}\bar{P}^f \bar{H}^T + R)^{-1}$$

- Parameters are non observable directly, BUT state innovations drive parameter changes via the **state-parameter cross-covariance**:

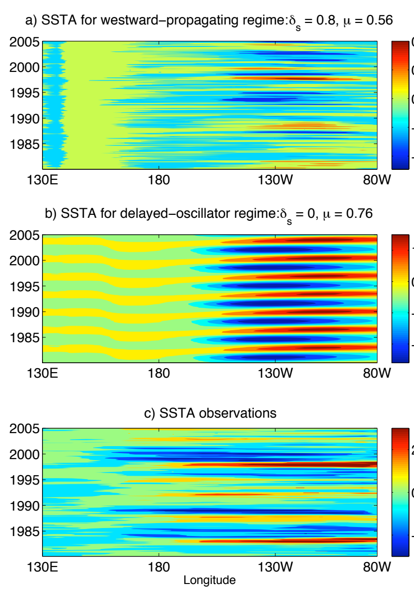
$$\bar{P}^f = \begin{pmatrix} P_{xx}^f & P_{x\mu}^f \\ P_{\mu x}^f & P_{\mu\mu}^f \end{pmatrix}; \bar{K} = \begin{pmatrix} P_{xx}^f H^T \\ P_{\mu x}^f H^T \end{pmatrix} (H P_{xx}^f H^T + R)^{-1}$$

- The **"augmented state"** approach can be easily generalized for various Kalman filter approximations: EnKF, reduced-state KF.

Model and Observations

An upper-ocean, reduced-gravity model of the Tropical Pacific and a steady-state atmospheric response to the sea surface temperature (SST). Model behavior is very sensitive to two key parameters: (i) the ocean-atmosphere coupling coefficient between SST and wind stress anomalies μ, which measures the degree of nonlinearity and (ii) the surface-layer coefficient δ_s, which determines the period of the model's self-sustained oscillation in the absence of seasonal forcing. Depending on the values of these parameters, the spatio-temporal pattern of model solutions is either that of a **delayed oscillator** or of **a westward propagating mode**.

What are the "optimal" μ, δ_s?



To successfully apply observational SSTs from the IRI/LDEO Climate Data Library (1975-2005) in our parameter estimation scheme, we had to mach the climatology of the model and observations.

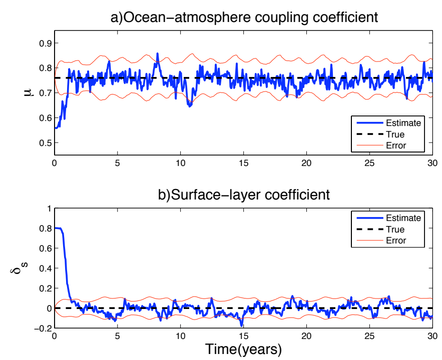
Identical twins (synthetic) data

-obtain ``observations" (i.e., synthetic data) from model run with **"truth"** parameter values **μ = 0.8, δ_s =0 (delayed oscillator)**

- start with **"wrong"** model parameters: **μ = 0.56, δ_s =0.8 (westward propagating mode)** and assimilate **"truth"** observations.

- goal: recover model parameters of **"truth"** by assimilating **SST "TAO data"** from 15 equatorial Pacific locations that resembles the **Tropical Atmosphere Ocean (TAO)** project array.

- the model errors are assumed to be mainly in the atmospheric wind stress.



- Since a smooth estimation of the parameter is often required, a small value of its model error tends to be a good choice.
- convergence depends on the initial uncertainty of the parameters; parameter estimates do not depend significantly on model error of the state and/or observational errors;
- once converged, parameters fluctuate about their **"true"** values and are bound within the error of the EKF's estimated variance.
- parameter estimation can reduce error in non-observed variables (here ocean currents).

Acknowledgments

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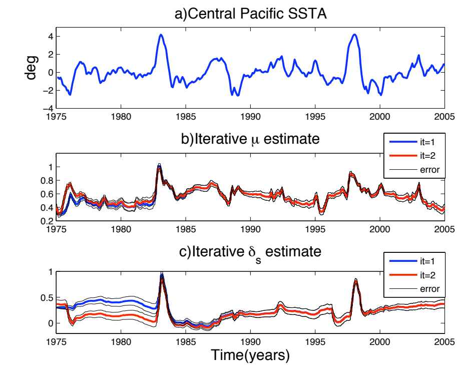
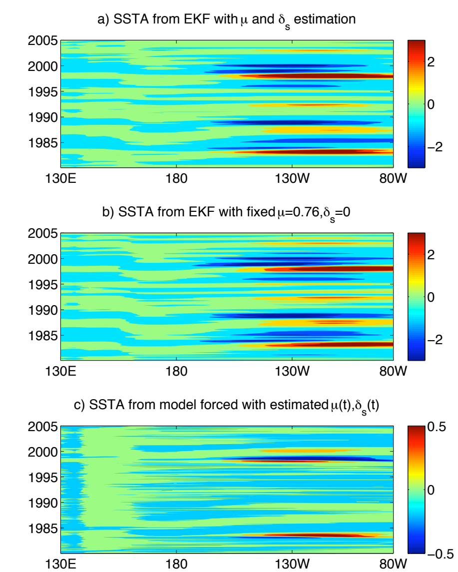
Observational data



-Iterative estimation in a loop ensures optimality in the earlier part of the record and independence from initial parameter guesses

-Parameters switch very fast between the two distinct modes of ENSO. Rapid adjustments occur for strong ENSO events.



- Results suggest that our ICM is too idealized to represent the complex evolution patterns of the observed SST, but it is skillful when the ENSO signals are strong.

http://www.atmos.ucla.edu/tcd/