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# Hierarchy of Models and Model Reduction in Climate Dynamics

Michael Ghil

Ecole Normale Supérieure, Paris, and
University of California, Los Angeles

*Joint work with*

Dmitri Kondrashov, UCLA;

Sergey Kravtsov, U. Wisconsin–Milwaukee;

Andrew Robertson, IRI, Columbia U.

http://www.atmos.ucla.edu/tcd/

# Global warming and
# its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …

*Source : IPCC* (2007),
*AR4, WGI, SPM*

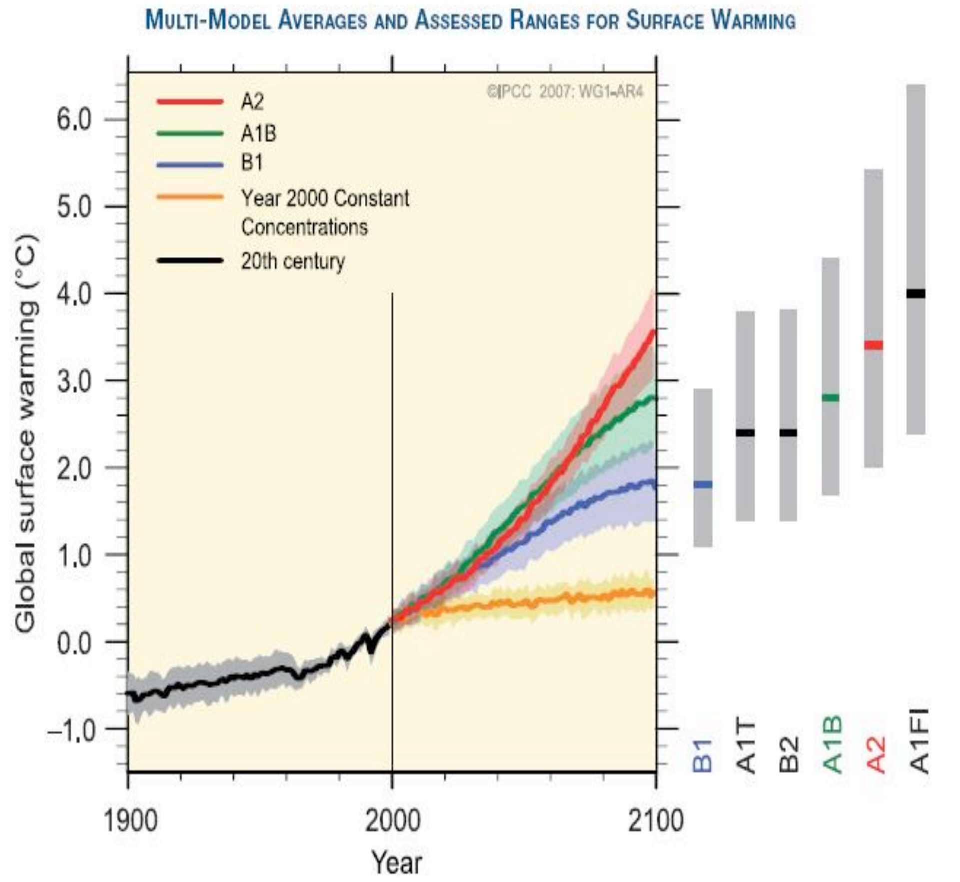
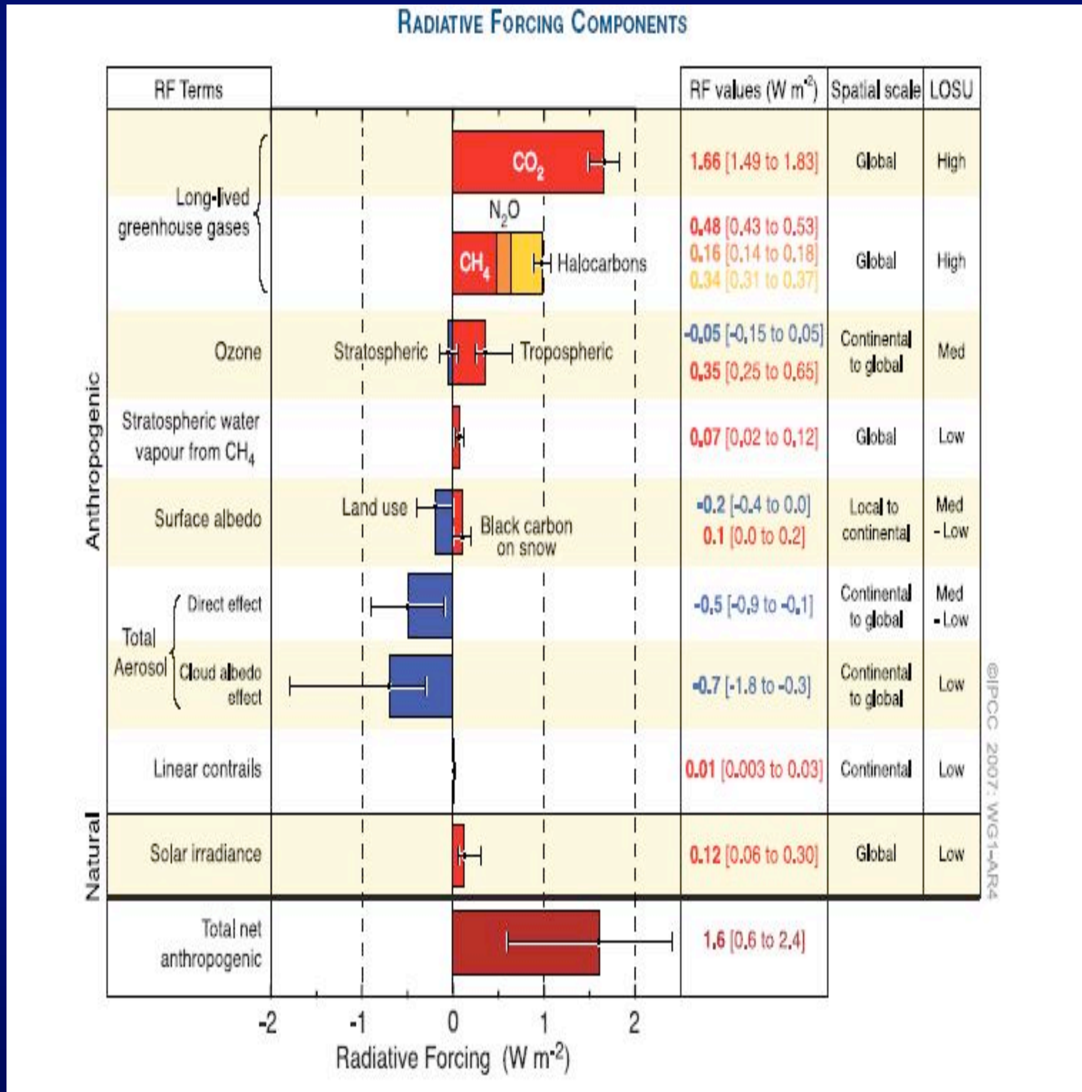



**Figure SPM.5.** *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* **likely** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* **likely** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

# GHGs rise

It's gotta do with us, at least a bit, ain't it?

But just how much?




*IPCC* (2007)

# Unfortunately, things aren't all that easy!

***What to do?***

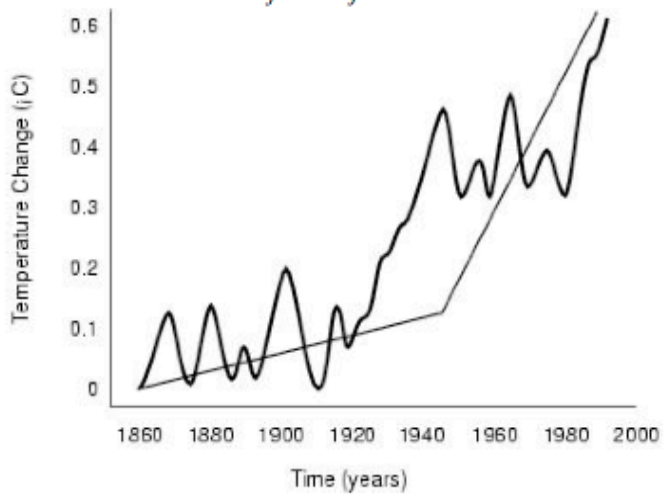
***Try to achieve better interpretation of, and agreement between, models …***

**Ghil, M.**, 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$c\frac{dT}{dt} = -kT + Q \qquad \begin{array}{l} k = \sum k_i - \text{feedbacks (+ve and -ve)} \\ Q = \sum Q_j - \text{sources \& sinks} \\ Q_j = Q_j(t) \end{array}$$




Linear response to $CO_2$ vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

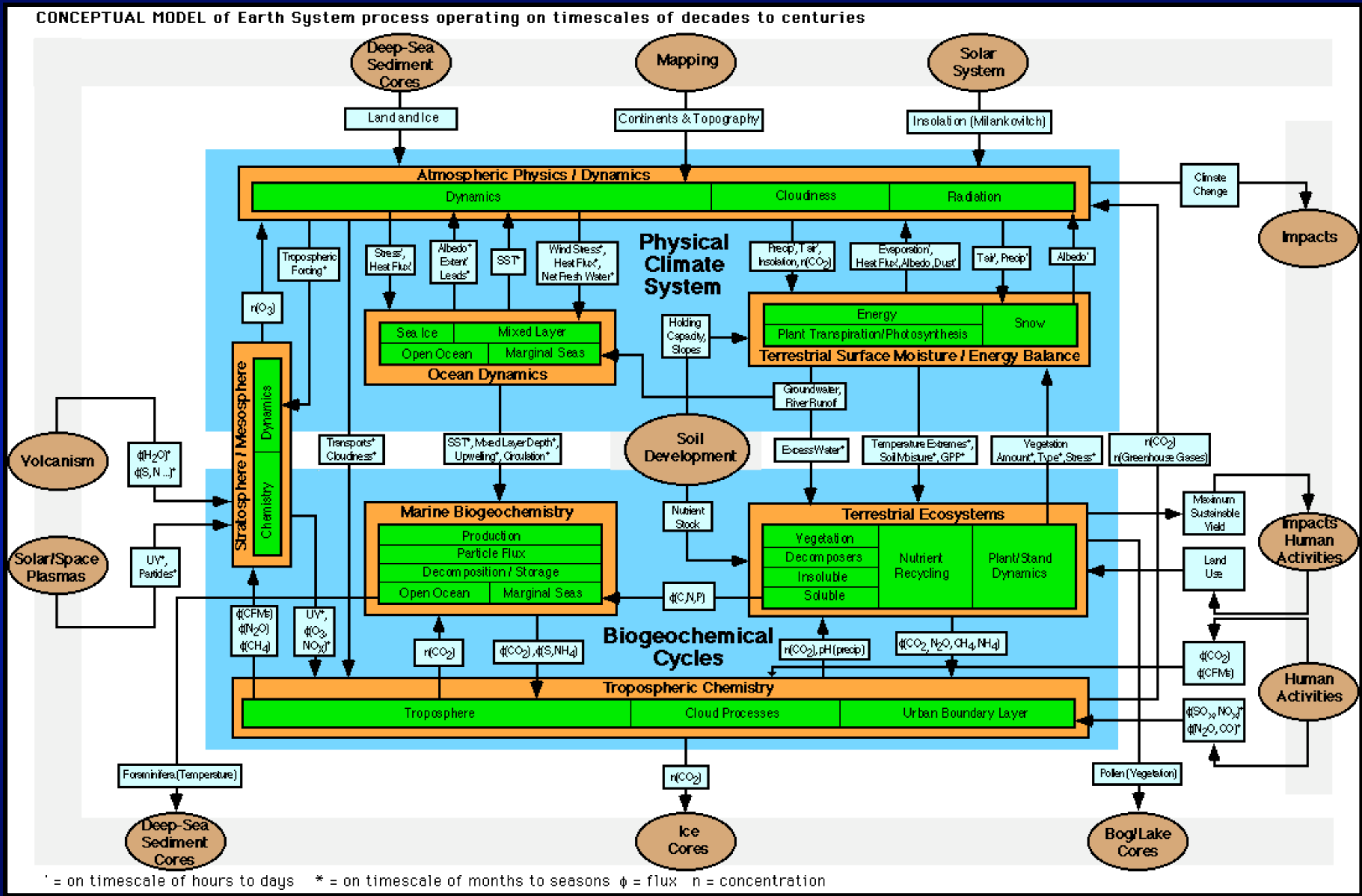
$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)$$

# So what's it gonna be like, by 2100?

**Table SPM.2.** *Recent trends, assessment of human influence on the trend and projections for extreme weather events for which there is an observed late-20th century trend. {Tables 3.7, 3.8, 9.4; Sections 3.8, 5.5, 9.7, 11.2–11.9}*

| Phenomenon[a] and direction of trend | Likelihood that trend occurred in late 20th century (typically post 1960) | Likelihood of a human contribution to observed trend[b] | Likelihood of future trends based on projections for 21st century using SRES scenarios |
|---|---|---|---|
| Warmer and fewer cold days and nights over most land areas | *Very likely*[c] | *Likely*[d] | *Virtually certain*[d] |
| Warmer and more frequent hot days and nights over most land areas | *Very likely*[e] | *Likely (nights)*[d] | *Virtually certain*[d] |
| Warm spells/heat waves. Frequency increases over most land areas | *Likely* | *More likely than not*[f] | *Very likely* |
| Heavy precipitation events. Frequency (or proportion of total rainfall from heavy falls) increases over most areas | *Likely* | *More likely than not*[f] | *Very likely* |
| Area affected by droughts increases | *Likely* in many regions since 1970s | *More likely than not* | *Likely* |
| Intense tropical cyclone activity increases | *Likely* in some regions since 1970 | *More likely than not*[f] | *Likely* |
| Increased incidence of extreme high sea level (excludes tsunamis)[g] | *Likely* | *More likely than not*[f,h] | *Likely*[i] |

# F. Bretherton's "horrendogram" of Earth System Science



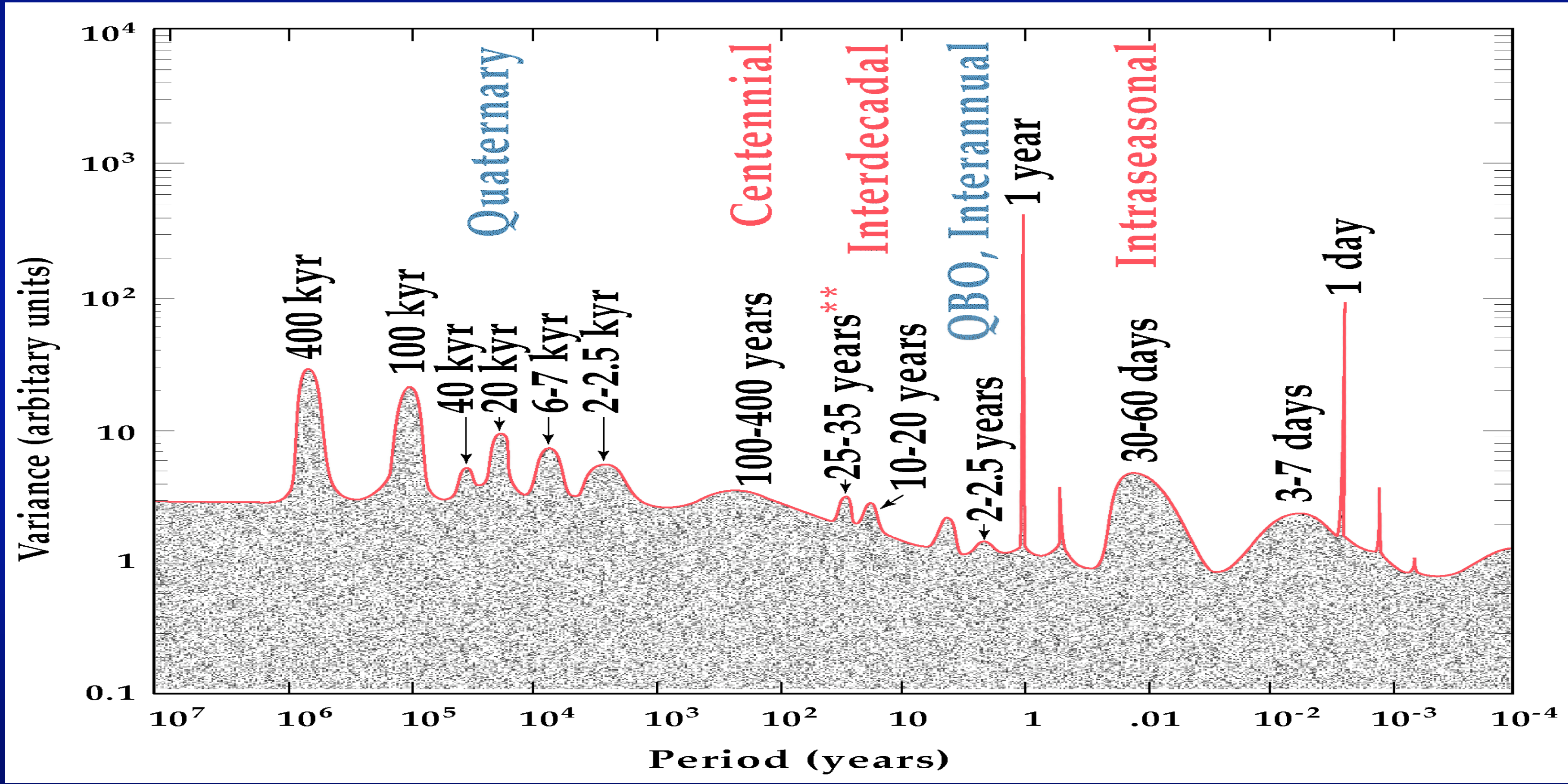

Earth System Science Overview, *NASA Advisory Council*, 1986

# Composite spectrum of climate variability

**Standard treatement of frequency bands:**

*1. High frequencies – white (or "colored") noise*

*2. Low frequencies – slow ("adiabatic") evolution of parameters*

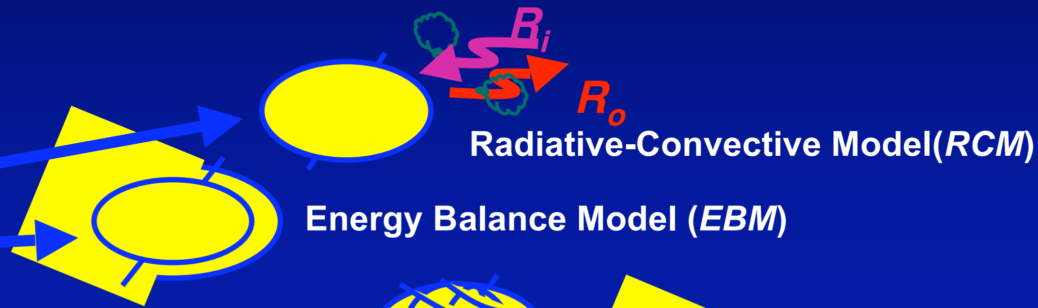



From **Ghil (2001, EGEC)**, after **Mitchell* (1976)**

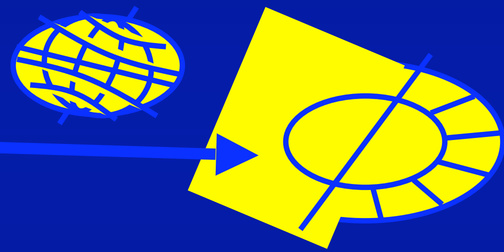
*** "No known source of deterministic internal variability"**

## Climate models (atmospheric & coupled) : A classification

- *Temporal*
  - stationary, (quasi-)equilibrium
  - transient, climate variability




- *Space*
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
    - horizontal
    - meridional plane
  - Simple and intermediate 2-D & 3-D models



- *Coupling*
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

**Hierarchy:** from the simplest to the most elaborate,
iterative comparison with the observational data

## Linear inverse model (LIM)

• We aim to use data in order to estimate the two matrices, **B** and **Q**, of the stochastic linear model:

$$d\mathbf{X} = B\mathbf{X} \cdot dt + d\xi(t), \tag{1}$$

where **B** is the (constant and stable) dynamics matrix, and **Q** is the lag-zero covariance of the vector white-noise process $d\xi(t)$.
• More precisely, the two matrices **B** and **Q** are related by a fluctuation-dissipation relation:

$$\mathbf{BC}(0) + \mathbf{C}(0)\mathbf{B}^t + \mathbf{Q} = 0, \tag{2}$$

where $\mathbf{C}(\tau) = \mathbb{E}\{\mathbf{X}(t+\tau)\mathbf{X}(t)\}$ is the lag-covariance matrix of the process $\mathbf{X}(t)$, and $(\cdot)^t$ indicates the transpose.
• One then proceeds to estimate the Green's function $\mathbf{G}(\tau) = \exp(\tau\mathbf{B})$ at a given lag $\tau_0$ from the sample $\mathbf{C}(\tau)$ by

$$\mathbf{G}(\tau_0) = \mathbf{C}(\tau_0)\mathbf{C}^{-1}(0).$$

## Nonlinear stochastic model (MTV)–I

- Let $\mathbf{z}$ be a **vector** decomposed into a **slow** ("climate") and a **fast** ("weather") vector of variables, $\mathbf{z} = (\mathbf{x}, \mathbf{y})$.

We model $\mathbf{x}$ deterministically and $\mathbf{y}$ stochastically, via the following **quadratic nonlinear dynamics**

$$\frac{d\mathbf{x}}{dt} = L_{11}\mathbf{x} + L_{12}\mathbf{y} + B^1_{11}(\mathbf{x},\mathbf{x}) + B^1_{12}(\mathbf{x},\mathbf{y}) + B^1_{22}(\mathbf{y},\mathbf{y}),$$
$$\frac{d\mathbf{y}}{dt} = L_{21}\mathbf{x} + L_{22}\mathbf{y} + B^2_{11}(\mathbf{x},\mathbf{x}) + B^2_{12}(\mathbf{x},\mathbf{y}) + B^2_{22}(\mathbf{y},\mathbf{y}).$$

- In stochastic modeling, the explicit nonlinear self-interaction for the variable $\mathbf{y}$, i.e. $B^2_{22}(\mathbf{y},\mathbf{y})$, is represented by a linear stochastic operator:

$$B^2_{22}(\mathbf{y},\mathbf{y}) \approx -\frac{\Gamma}{\varepsilon}\mathbf{y} + \frac{\sigma}{\sqrt{\varepsilon}}\dot{\mathbf{W}}(t),$$

where $\Gamma$ and $\sigma$ are matrices and $\dot{\mathbf{W}}(t)$ is a **vector-valued** white-noise.

## Nonlinear stochastic model (MTV)–II

• The parameter $\varepsilon$ measures the ratio of the correlation time of the weather and the climate variables, respectively,
and $\varepsilon \ll 1$ corresponds to this ratio being very small.
• Using the scaling $t \to \varepsilon t$, we derive the stochastic climate model:

$$\frac{d\mathbf{y}}{dt} = \frac{1}{\varepsilon}(L_{11}\mathbf{x} + L_{12}\mathbf{y} + B^1_{11}(\mathbf{x},\mathbf{x}) + B^1_{12}(\mathbf{x},\mathbf{y})),$$

$$\frac{d\mathbf{y}}{dt} = \frac{1}{\varepsilon}(L_{21}\mathbf{x} + L_{22}\mathbf{y} + B^2_{11}(\mathbf{x},\mathbf{x}) + B^2_{12}(\mathbf{x},\mathbf{y})) - \frac{\Gamma}{\varepsilon^2}\mathbf{y} + \frac{\sigma}{\varepsilon}\dot{\mathbf{W}}(t).$$

• In practice, the climate variables are determined by a variety of procedures, including leading-order empirical orthogonal functions (EOFs), zonal averaging in space, low-pass and high-pass time filtering, or a combination of these procedures.

# References for LIM & MTV

## Linear Inverse Models (LIM)

Penland, C., 1989: Random forcing and forecasting using principal oscillation pattern analysis. *Mon. Wea. Rev.*, **117**, 2165–2185.

Penland, C., 1996: A stochastic model of Indo-Pacific sea-surface temperature anomalies. *Physica D*, **98**, 534–558.

Penland, C., and M. Ghil, 1993: Forecasting Northern Hemisphere 700-mb geopotential height anomalies using empirical normal modes. *Mon. Wea. Rev.*, **121**, 2355–2372.

Penland, C., and L. Matrosova, 1998: Prediction of tropical Atlantic sea-surface temperatures using linear inverse modeling. *J. Climate*, **11**, 483–496.

## Nonlinear reduced models (MTV)

Majda, A. J., I. Timofeyev, and E. Vanden-Eijnden, 1999: Models for stochastic climate prediction. *Proc. Natl. Acad. Sci. USA*, **96**, 14687–14691.

Majda, A. J., I. Timofeyev, and E. Vanden-Eijnden, 2001: A mathematical framework for stochastic climate models. *Commun. Pure Appl. Math.*, **54**, 891–974.

Majda, A. J., I. Timofeyev, and E. Vanden-Eijnden, 2002: A priori test of a stochastic mode reduction strategy. *Physica D*, **170**, 206–252.

Majda, A. J., I. Timofeyev, and E. Vanden-Eijnden, 2003: Systematic strategies for stochastic mode reduction in climate. *J. Atmos. Sci.*, **60**, 1705–1722.

Franzke, C., and Majda, A. J., 2006: Low-order stochastic mode reduction for a prototype atmospheric GCM. *J. Atmos. Sci.*, **63**, 457–479.

# Motivation

- Sometimes we have data but no models.
- Linear inverse models (LIM) are good least-square fits to data, but don’t capture all the processes of interest.
- Difficult to separate between the slow and fast dynamics (MTV).
- We want models that are as simple as possible, but not any simpler.

## Criteria for a good data-derived model

- Fit the data, as well or better than LIM.
- Capture interesting dynamics: regimes, nonlinear oscillations.
- Intermediate-order deterministic dynamics.
- Good noise estimates.

# Key ideas

- Nonlinear dynamics: $$\dot{\mathbf{x}} = \mathbf{L}\mathbf{x} + \mathbf{N}(\mathbf{x}).$$
- Discretized, quadratic:

$$dx_i = (\mathbf{x}^{\mathrm{T}}\mathbf{A}_i\mathbf{x} + \mathbf{b}_i^{(0)}\mathbf{x} + c_i^{(0)})\,dt + dr_i^{(0)}; \qquad 1 \le i \le I.$$

- Multi-level modeling of red noise:

$$\begin{aligned}
dx_i &= (\mathbf{x}^{\mathrm{T}}\mathbf{A}_i\mathbf{x} + \mathbf{b}_i^{(0)}\mathbf{x} + c_i^{(0)})\,dt + r_i^{(0)}\,dt,\\
dr_i^{(0)} &= \mathbf{b}_i^{(1)}[\mathbf{x}, \mathbf{r}^{(0)}]\,dt + r_i^{(1)}\,dt,\\
dr_i^{(1)} &= \mathbf{b}_i^{(2)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}]\,dt + r_i^{(2)}\,dt,\\
&\dots\\
dr_i^{(L)} &= \mathbf{b}_i^{(L)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}, \dots, \mathbf{r}^{(L)}]\,dt + dr_i^{(L+1)}; \qquad 1 \le i \le I.
\end{aligned}$$

# Nomenclature

*Response* variables:

$$\{y^{(n)}\}\ (1 \le n \le N) \equiv \{y^{(1)}, \ldots, y^{(N)}\}$$

*Predictor* variables:

$$\{x^{(n)}\}\ (1 \le n \le N) \equiv \{x^{(1)}, \ldots, x^{(N)}\}$$

- Each $y^{(n)}$ is *normally distributed* about $\widehat{y}^{(n)}$
- Each $x^{(n)}$ is *known exactly. Parameter set* $\{a_p\}$:

$$\widehat{y} = f(x;\, a_1, \ldots, a_P)$$ – known dependence of *f* on $\{x^{(n)}\}$ and $\{a_p\}$.

*REGRESSION*: Find $\{a_p\}\ (1 \le p \le P)$

# LIM extension #1

- Do a least-square fit to a *nonlinear function of the data*:

*J response variables*: $y_i^{(n)} \equiv (x_i^{(n+1)} - x_i^{(n)})/\Delta t$

*Predictor variables* (example – quadratic polynomial of *J* original predictors):

$$\widehat{y}_i = a_{0,\,i} + \sum_{j=1}^{J} a_{j,\,i} x_j + \sum_{j=1}^{J} \sum_{k \geq j} \tilde{a}_{jk,\,i} x_j x_k$$

*Note*: Need to find many more regression coefficients than for LIM; in the example above $P = J + J(J+1)/2 + 1 = O(J^2)$.

# Regularization

- *Caveat*: If the number *P* of regression parameters is comparable to (*i.e.*, it is not much smaller than) the number of data points, then the least-squares problem may become ill-posed and lead to unstable results (overfitting) ==> One needs to transform the predictor variables to *regularize* the regression procedure.

- Regularization involves *rotated predictor variables:* the orthogonal transformation looks for an “optimal” linear combination of variables.

- “Optimal” = (i) rotated predictors are nearly uncorrelated; and
  (ii) they are maximally correlated with the response.

- Canned packages available.

# LIM extension #2

- *Motivation*: Serial correlations in the residual.

Main level, $l = 0$: $$(x^{n+1} - x^n)/\Delta t = a_{x,0} x^n + r_0^n$$

Level $l = 1$: $$(r_0^{n+1} - r_0^n)/\Delta t = a_{x,1} x^n + a_{r_0,1} r_0^n + r_1^n$$

… and so on …

Level $L$: $$r_{L-1}^{n+1} - r_{L-1}^n = \Delta t [a_{x,L} x^n + \ldots] + \Delta r_L$$

- $\Delta r_L$ – Gaussian random deviate with appropriate variance
- If we suppress the dependence on $x$ in levels $l = 1, 2, \ldots L$, then the model above is formally identical to an ARMA model.

# Empirical Orthogonal Functions (EOFs)

- We want models that are as simple as possible, but not any simpler: use leading **empirical orthogonal functions** for data compression and capture as much as possible of the useful (predictable) variance.
- Decompose a spatio-temporal data set **D**($t$,$s$)($t$ = 1,…,$N$; $s$ = 1…,$M$) by using **principal components (PCs**) – $\mathbf{x}_i(t)$ and **empirical orthogonal functions (EOFs)** – $\mathbf{e}_i(s)$: diagonalize the $M$ x $M$ spatial covariance matrix **C** of the field of interest.

$$\mathbf{C} = \frac{1}{N}(\mathbf{D} - <\mathbf{D}>)^{\mathrm{t}}(\mathbf{D} - <\mathbf{D}>)$$
$$\mathbf{C}\lambda_i = \lambda_i e_i,\ x_i = (\mathbf{D} - <\mathbf{D}>)e_i$$

- EOFs are optimal patterns to capture most of the variance.
- Assumption of robust EOFs.
- EOFs are statistical features, but may describe some dynamical (physical) mode(s).

# Empirical mode reduction (EMR)–I

- *Multiple predictors*: Construct the reduced model using $J$ leading PCs of the field(s) of interest.

- *Response variables*: one-step time differences of predictors; step = sampling interval = $\Delta t$.

- Each response variable is fitted by an *independent* multi-level model:
  The *main level* $l = 0$ is *polynomial* in the predictors;
  all the other levels are linear.

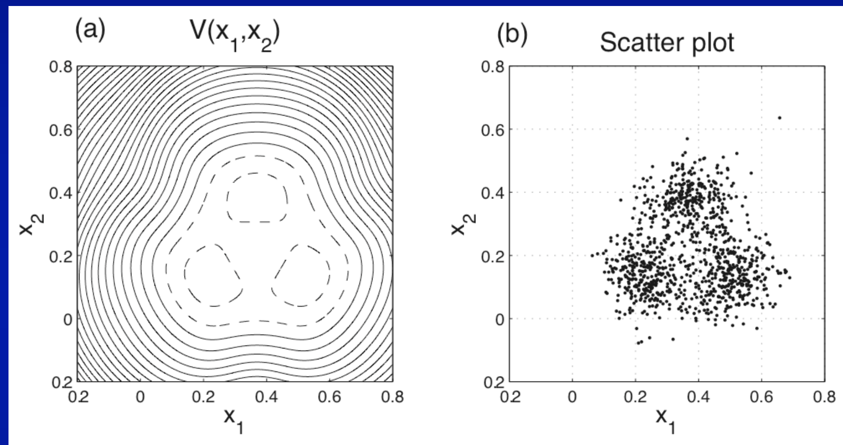
# Empirical mode reduct'n (EMR) – II

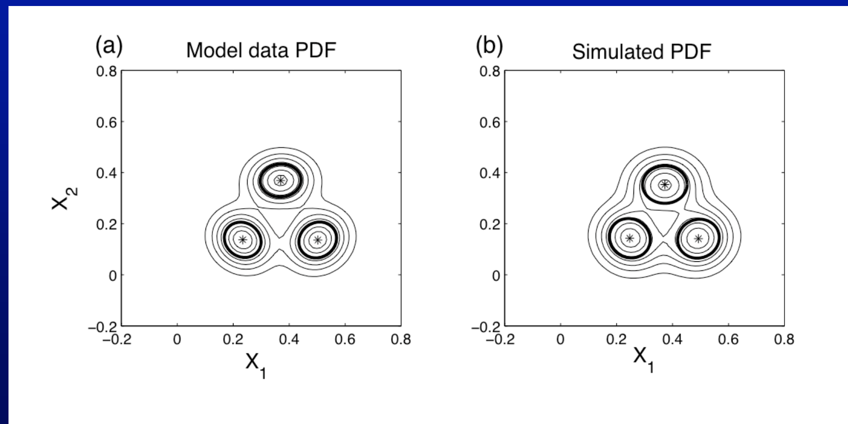
- The number *L* of levels is such that each of the last-level residuals (for each channel corresponding to a given response variable) is "white" in time.
- Spatial (cross-channel) correlations of the last-level residuals are retained in subsequent regression-model simulations.
- The number *J* of PCs is chosen so as to optimize the model's performance.
- Regularization is used at the main (nonlinear) level of each channel.

# Illustrative example: Triple well

$$d\mathbf{x}(t) = -\nabla V(\mathbf{x})dt + \sigma\, d\mathbf{b}$$

- $V(x_1,x_2)$ is *not polynomial*!
- Our *polynomial* regression model produces a time series whose statistics are nearly identical to those of the full model!!
- Optimal order is $m = 3$; regularization required for polynomial models of order $m \geq 5$.



# NH LFV in QG3 Model – I

*The QG3 model* (Marshall and Molteni, *JAS*, 1993):

- Global QG, T21, 3 levels, with topography; perpetual-winter forcing; ~1500 degrees of freedom.

- Reasonably realistic NH climate and LFV:
  (i) multiple planetary-flow regimes; and
  (ii) low-frequency oscillations
  (submonthly-to-intraseasonal).

- Extensively studied: A popular "numerical-laboratory" tool to test various ideas and techniques for NH LFV.

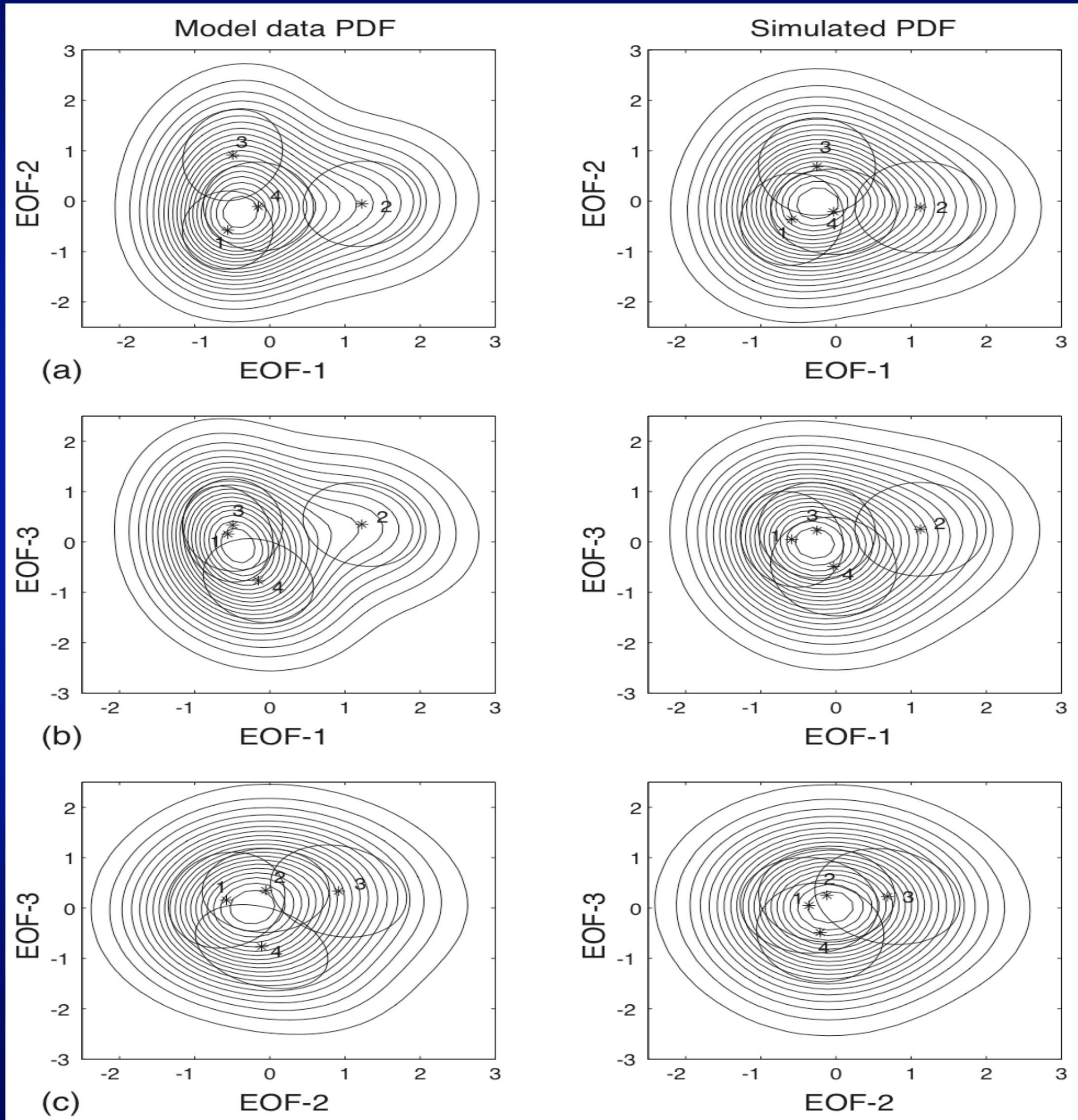
# NH LFV in QG3 Model – II

*Output*: daily streamfunction ($\Psi$) fields (≈ $10^5$ days)

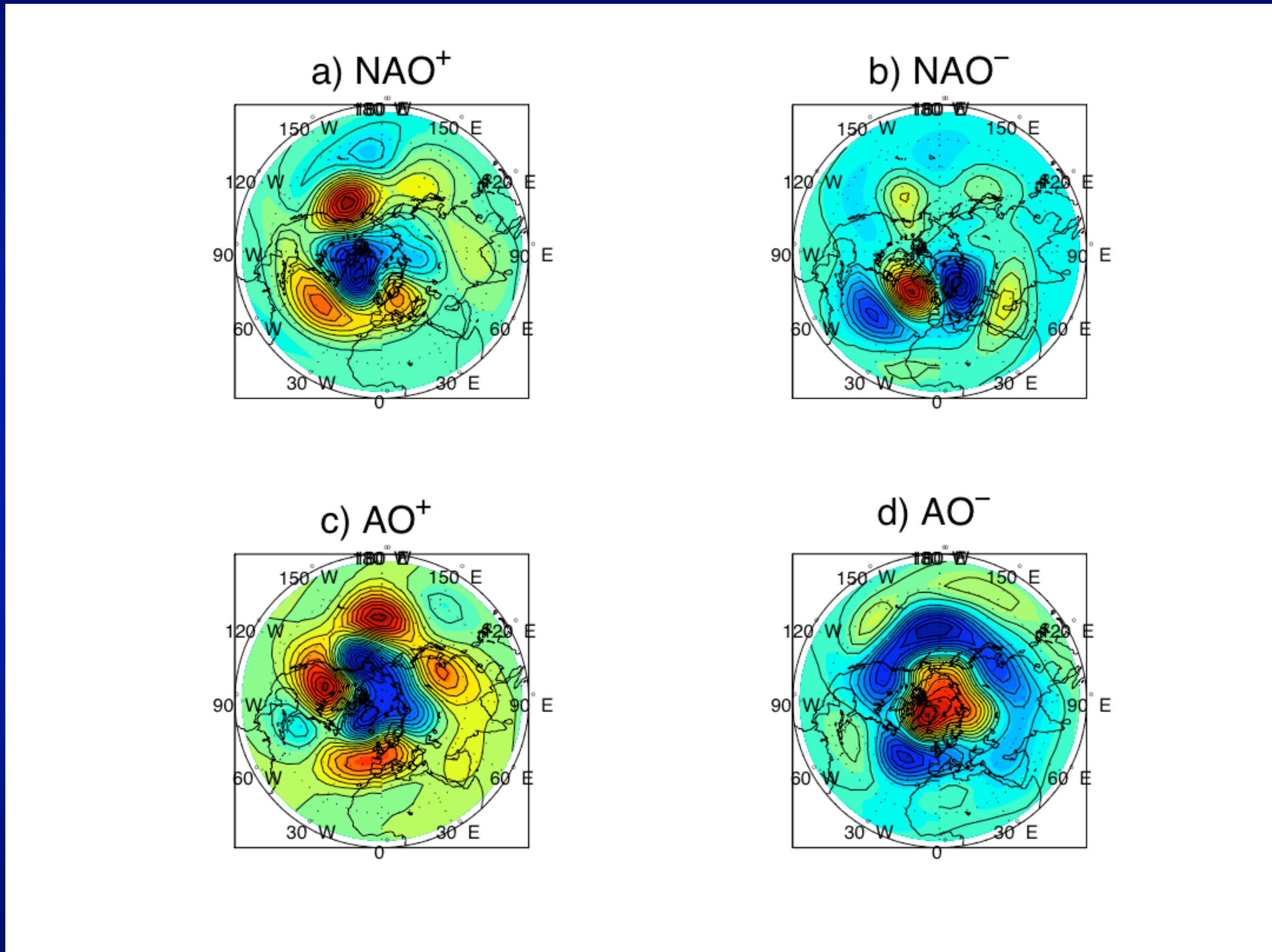
*Regression model*:

- 15 variables, 3 levels ($L$ = 3), quadratic at the main level
- Variables: Leading PCs of the middle-level $\Psi$
- No. of degrees of freedom = 45 (a factor of 40 less than in the QG3 model)
- Number of regression coefficients $P$ = (15+1+15•16/2+30+45)•15 = 3165 (<< $10^5$)
- Regularization via PLS applied at the main level.

# NH LFV in QG3 Model – III



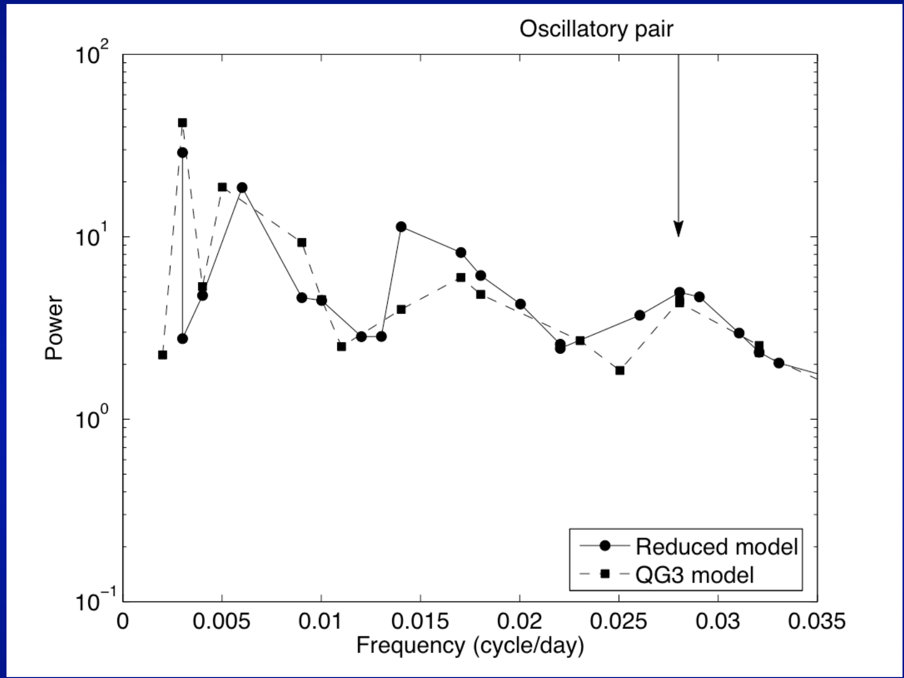
# NH LFV in QG3 Model – IV




The correlation between the QG3 map and the EMR model's map exceeds 0.9 for each cluster centroid.

# NH LFV in QG3 Model – V

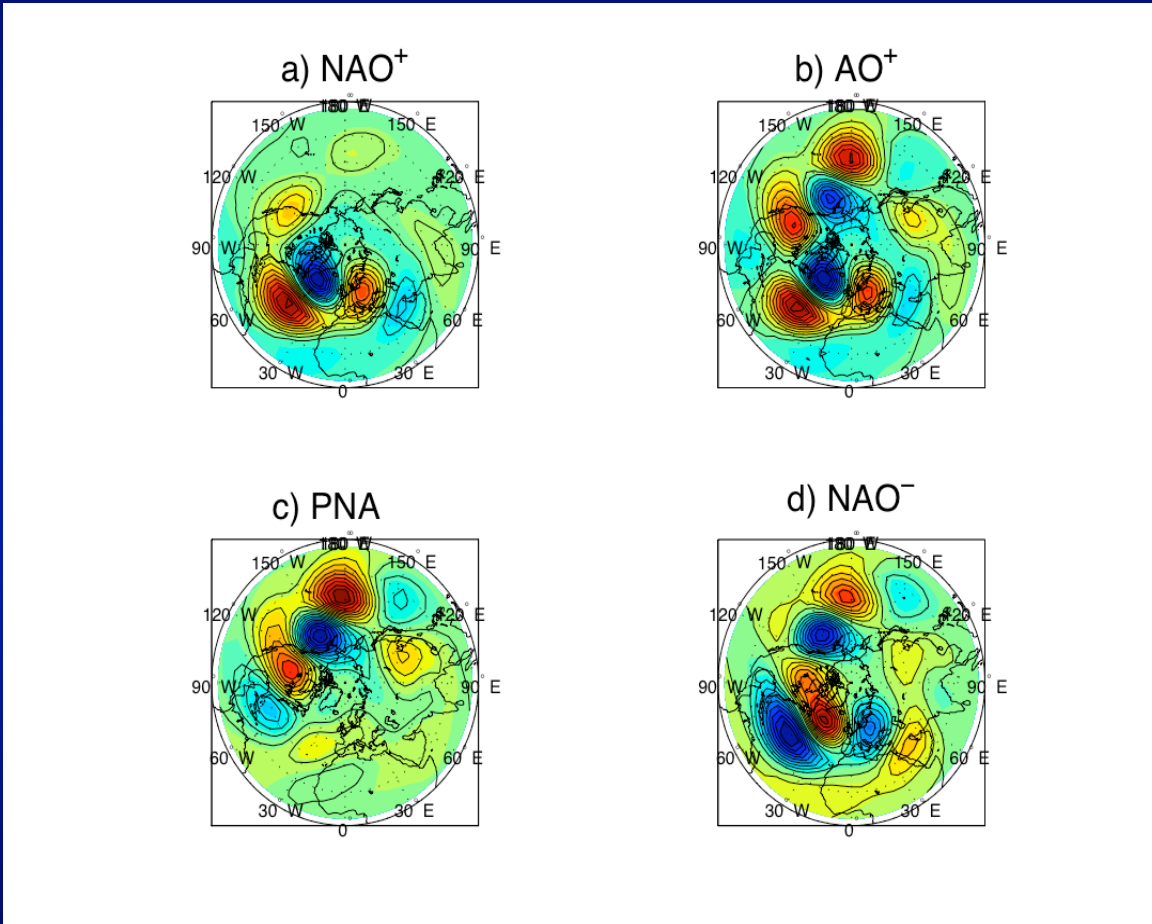
- Multi-channel SSA (M-SSA) identifies 2 oscillatory signals, with periods of 37 and 20 days.




- Composite maps of these oscillations are computed by identifying 8 phase categories, according to M-SSA reconstruction.

# NH LFV in QG3 Model – VI

Composite 37-day cycle:




QG3 and EMR results are virtually identical.

# NH LFV in QG3 Model – VII

*Regimes vs. Oscillations:*

- Fraction of regime days as a function of oscillation phase.
- Phase speed in the (RC vs. ΔRC) plane – both RC and ΔRC are normalized so that a linear, sinusoidal oscillation would have a constant phase speed.

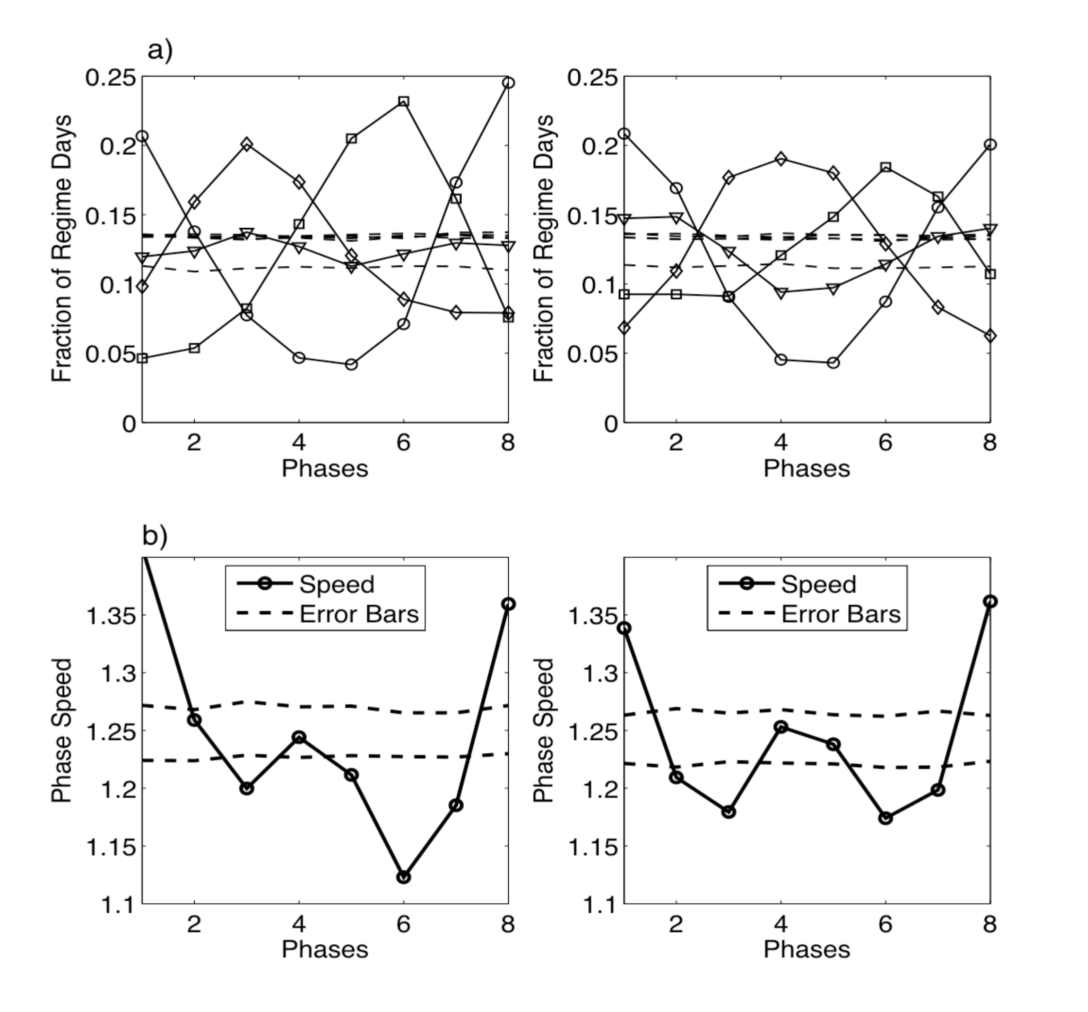
# NH LFV in QG3 Model – VIII

*Regimes vs. Oscillations:*

- Fraction of regime days:

  NAO⁻ (squares),
  NAO⁺ (circles),
  AO⁺ (diamonds);
  AO⁻ (triangles).

- Phase speed



# NH LFV in QG3 Model – IX

*Regimes vs. Oscillations*:

- Regimes $AO^+$, $NAO^-$ and $NAO^+$ are associated with anomalous slow-down of the 37-day oscillation's trajectory $\Rightarrow$ nonlinear mechanism.

- $AO^-$ is a stand-alone regime, not associated with the 37- or 20-day oscillations.

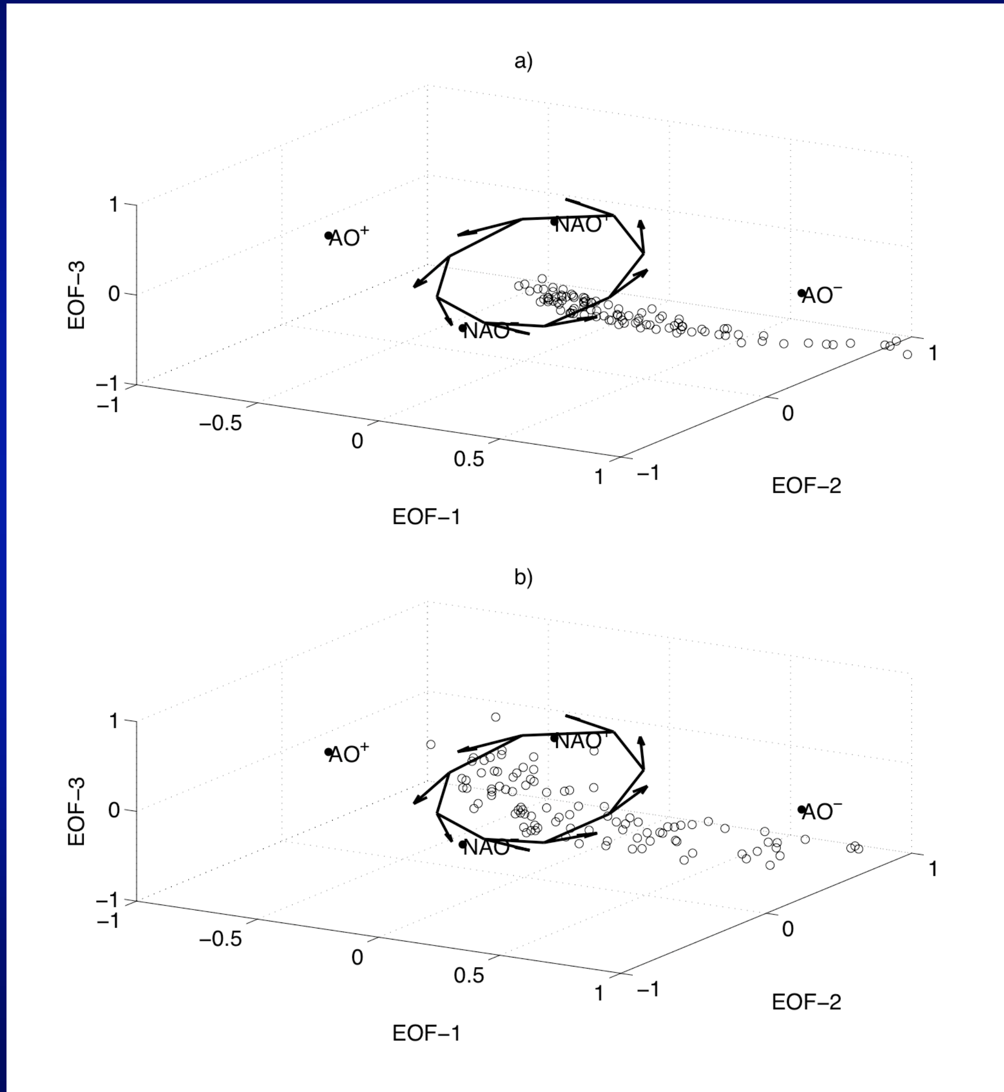
# NH LFV in QG3 Model – X

*Quasi-stationary states* of the EMR model's deterministic component.

Tendency threshold

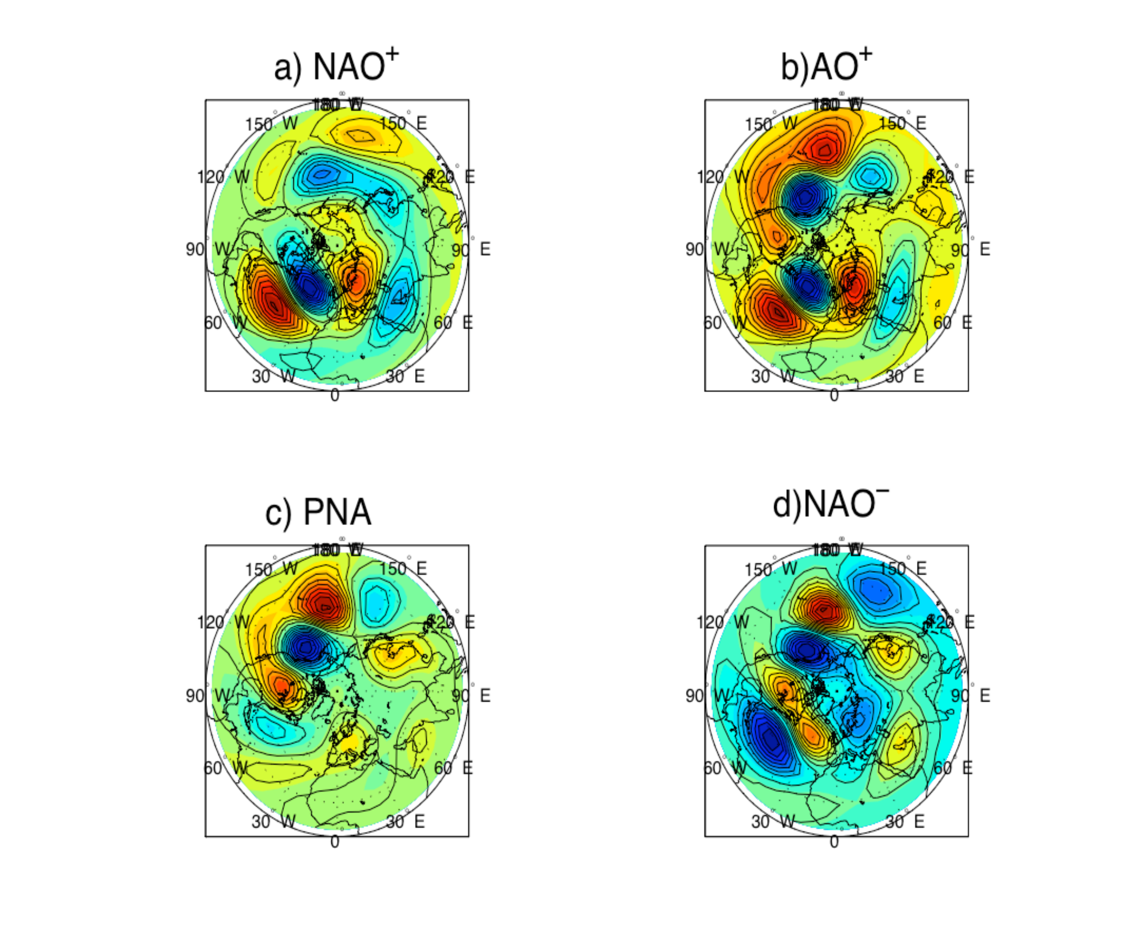
a) $\beta = 10^{-6}$; and

b) $\beta = 10^{-5}$.



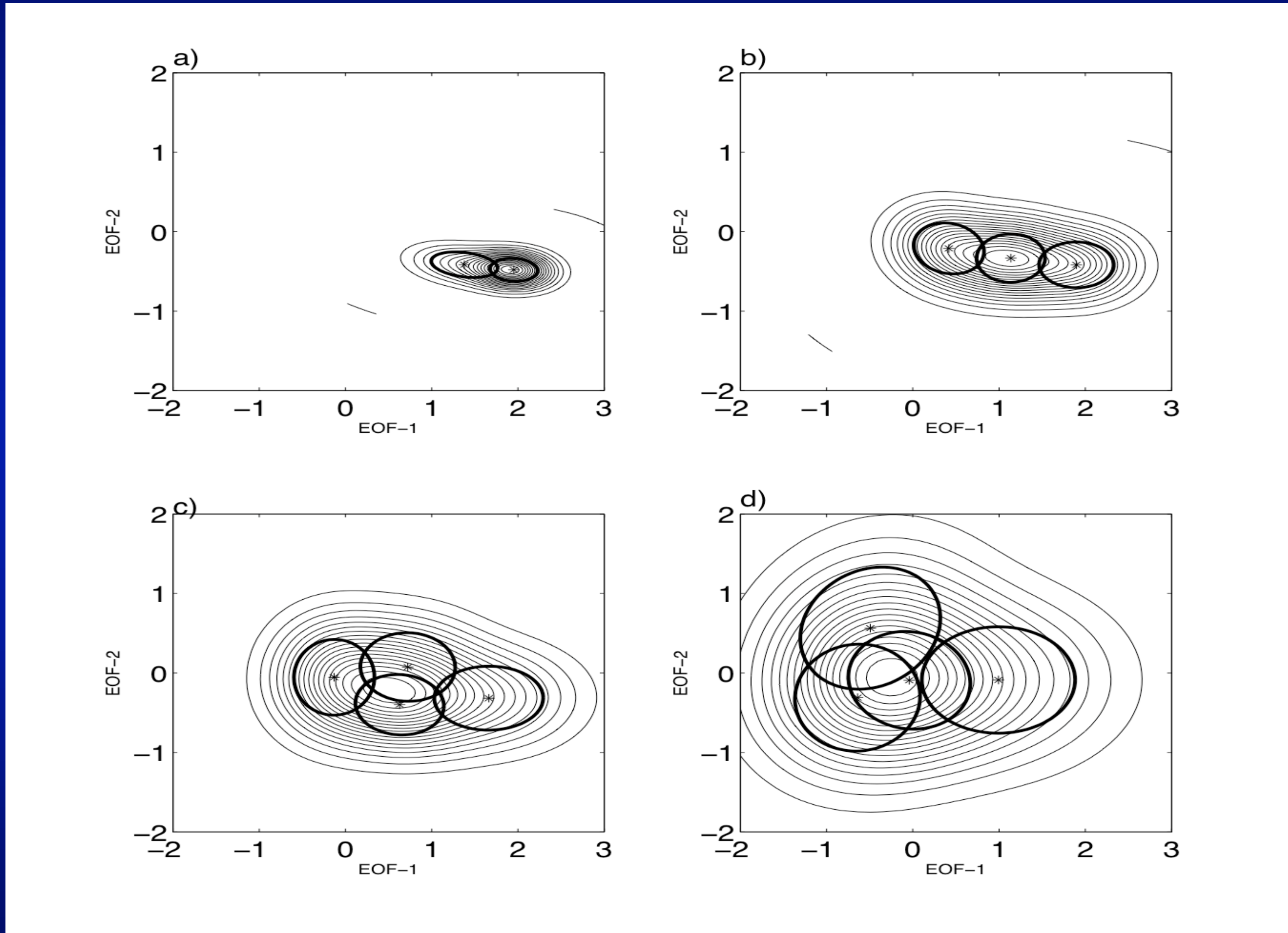
# NH LFV in QG3 Model – XI

*37-day eigenmode of the regression model linearized about climatology**




* Very similar to the composite 37-day oscillation.

# NH LFV in QG3 Model – XII




Panels (a)–(d): noise amplitude ε = 0.2, 0.4, 0.6, 1.0.

# Conclusions on QG3 Model

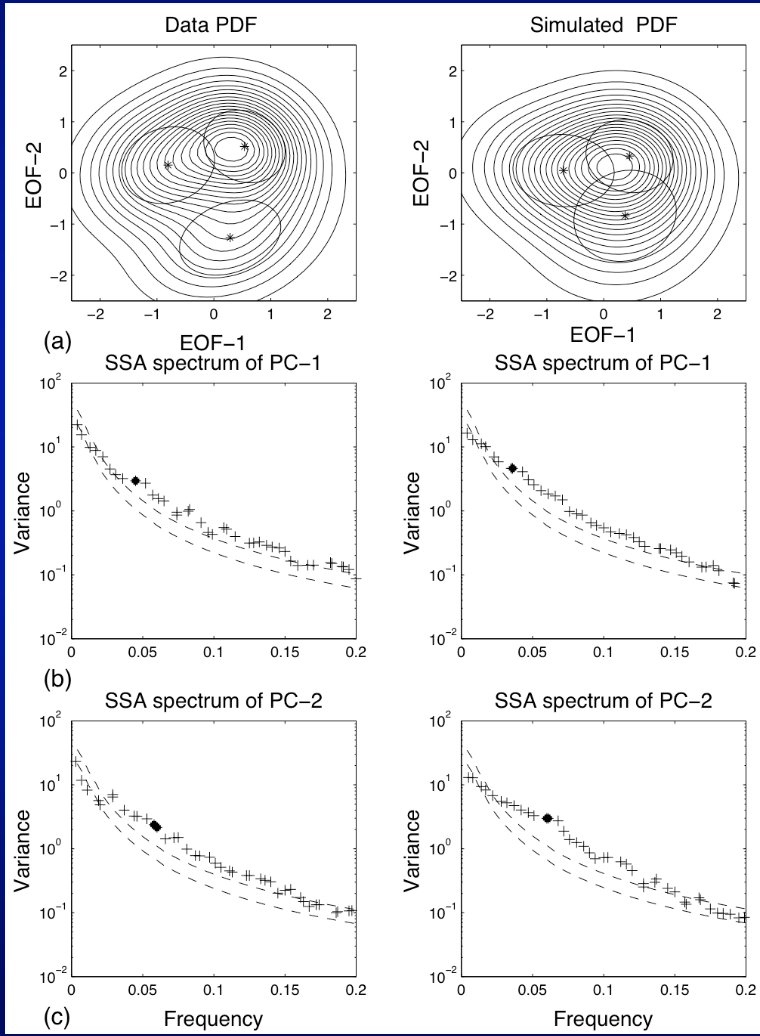
- Our ERM is based on 15 EOFs of the QG3 model and has *L* = 3 regression levels, *i.e.,* a total of 45 predictors (*).
- The ERM approximates the QG3 model's major statistical features (PDFs, spectra, regimes, transition matrices, etc.) strikingly well.
- The dynamical analysis of the reduced model identifies $AO^{-}$ as the model's unique steady state.
- The 37-day mode is associated, in the reduced model, with the least-damped linear eigenmode.
- The additive noise interacts with the nonlinear dynamics to yield the full ERM's (and QG3's) phase-space PDF.

(*) An ERM model with 4*3 = 12 variables only does not work!

# NH LFV – Observed Heights

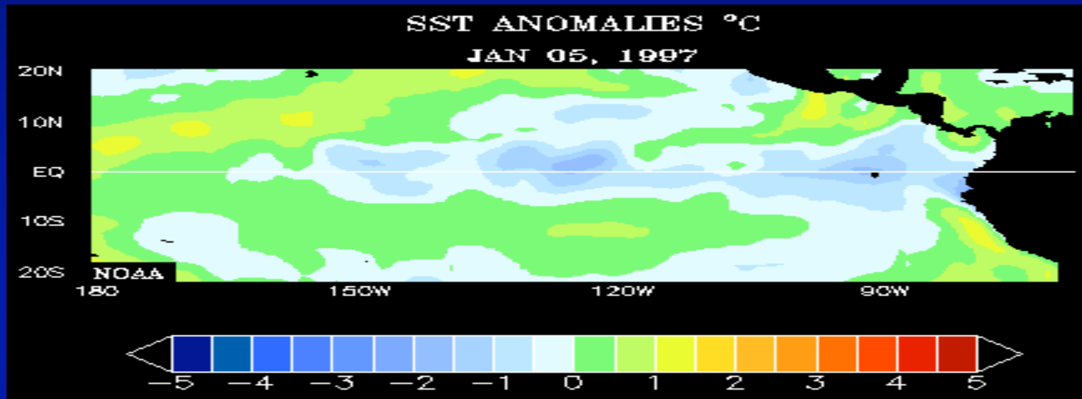
- 44 years of daily 700-mb-height winter data

- 12-variable, 2-level model works OK, but dynamical operator has unstable directions: “sanity checks” required.



# Spatio-temporal evolution of ENSO episode

## 1997-98 El Niño Animation




Anomaly = (Current observation – Corresponding climatological value)
Base period for the climatology is 1950–1979

NOAA-CIRES Climate Diagnostics Center

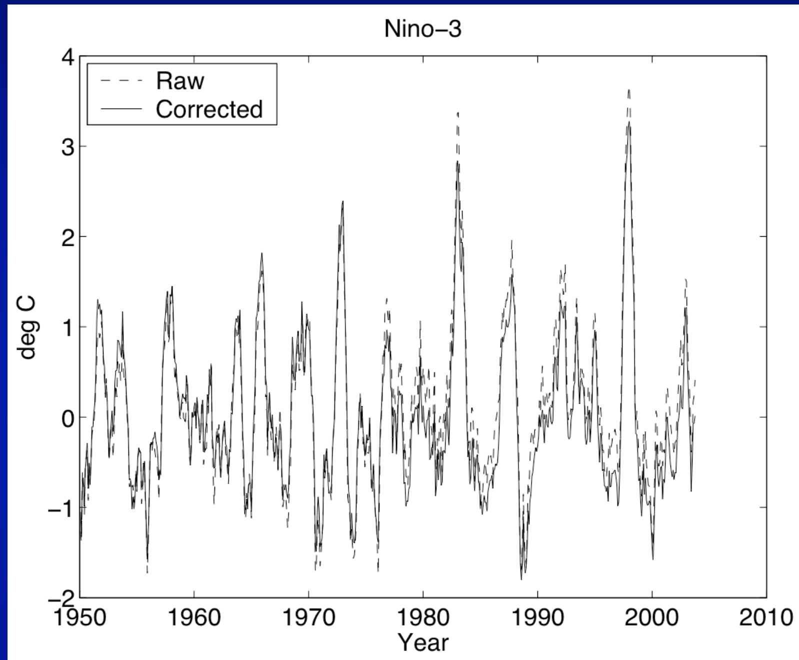
http://www.cdc.noaa.gov/map/clim/sst_olr/old_sst/sst_9798_anim.shtml

Courtesy of NOAA-CIRES Climate Diagnostics Center

# ENSO – I

*Data*:

- Monthly SSTs: 1950–2004, 30 S–60 N, 5x5 grid (Kaplan *et al*., 1998)




- 1976–1977 shift removed

- Histogram of SST data is skewed (warm events are larger, while cold events are more frequent): Nonlinearity important?

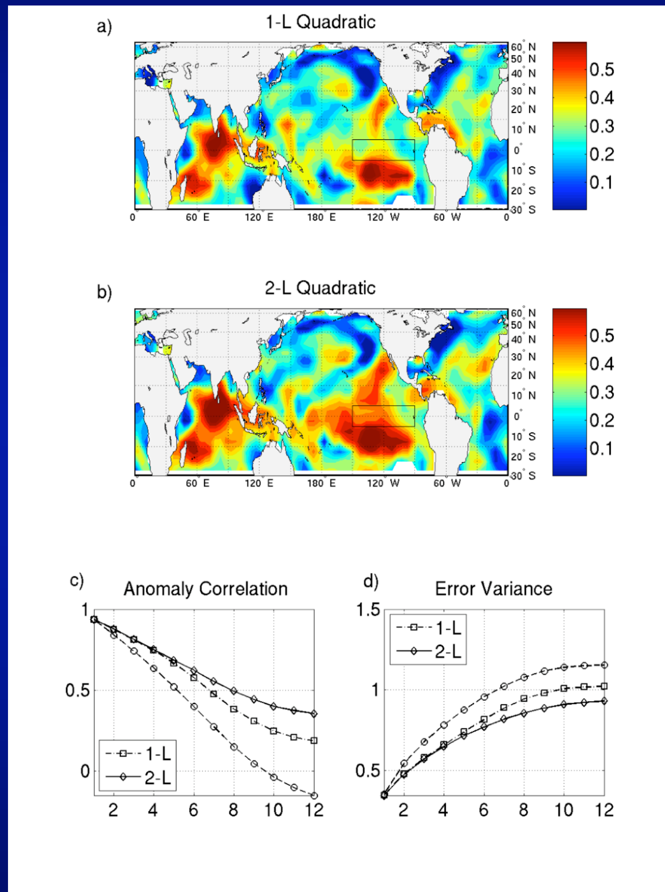
# ENSO – II

*Regression model:*

- $J$ = 20 variables (EOFs of SST)
- $L$ = 2 levels

- Seasonal variations included in the linear part of the main (quadratic) level.

- Competitive skill: Currently a member of a multi-model prediction scheme of the IRI, see: http://iri.columbia.edu/climate/ENSO/currentinfo/SST_table.html.



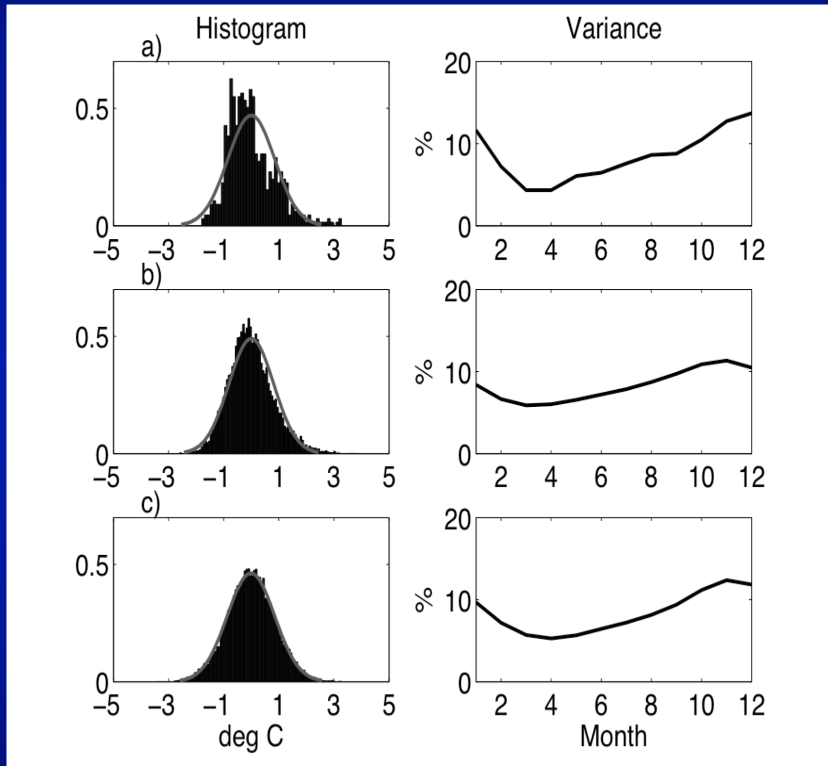
# ENSO – III

## PDF – skewed vs. Gaussian

- Observed

- Quadratic model (100-member ensemble)

- Linear model (100-member ensemble)



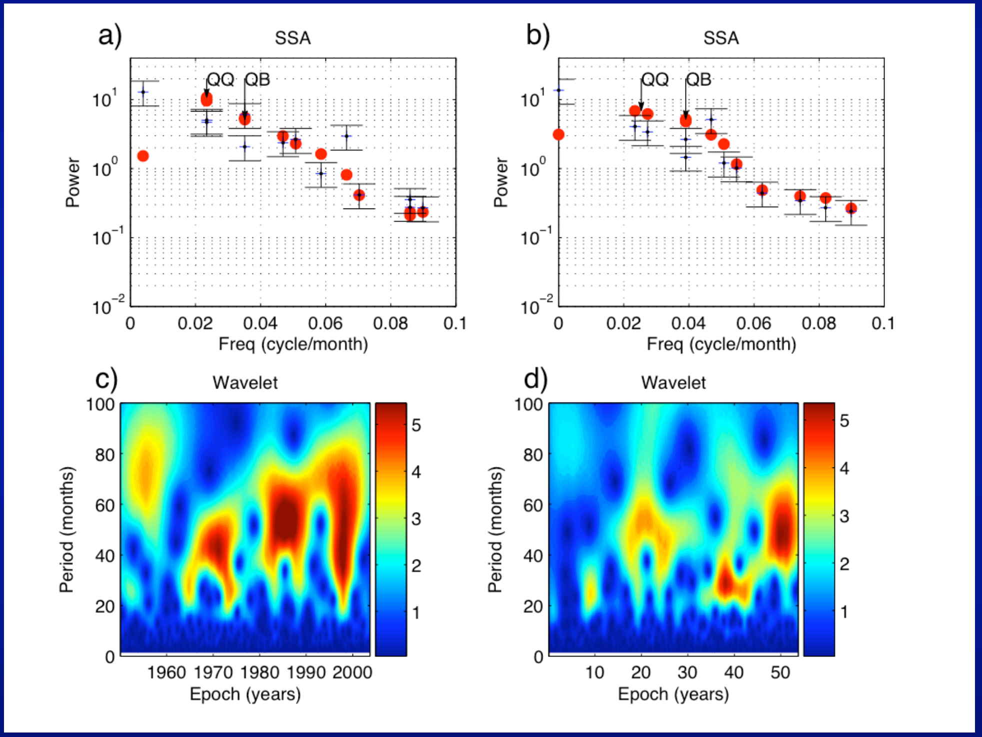

*The quadratic model has a slightly smaller RMS error in its extreme-event forecasts (not shown)*

# ENSO – IV

*Spectra:* Data Model

- SSA

- Wavelet

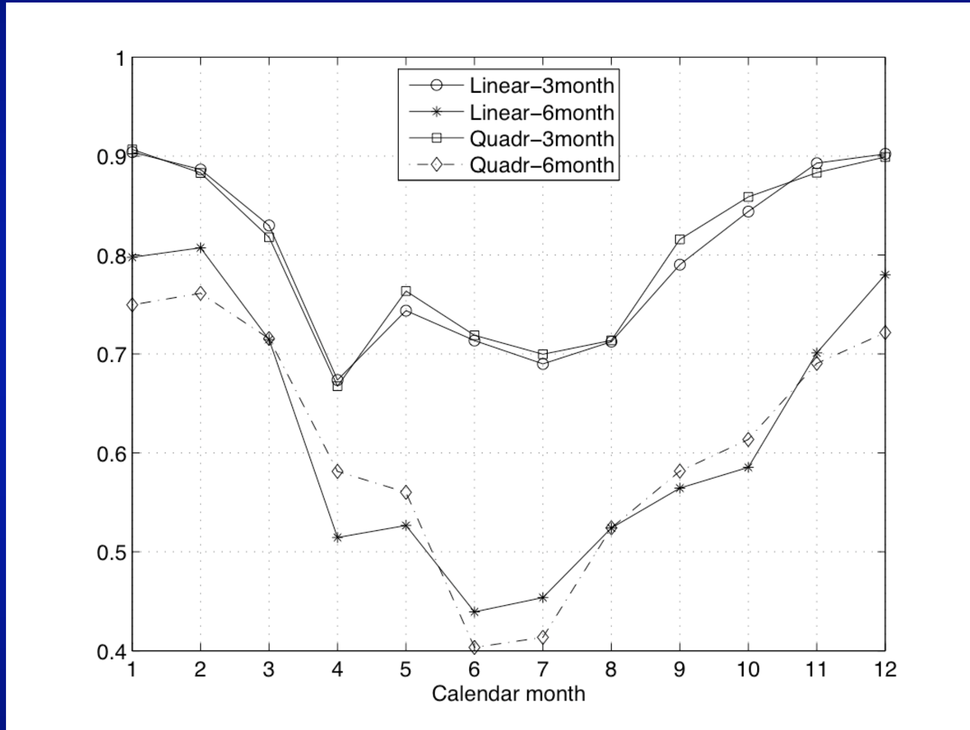



*ENSO's leading oscillatory modes, QQ and QB, are reproduced by the model, thus leading to a skillful forecast.*

# ENSO – V

*"Spring barrier":*

Hindcast skill vs. target month

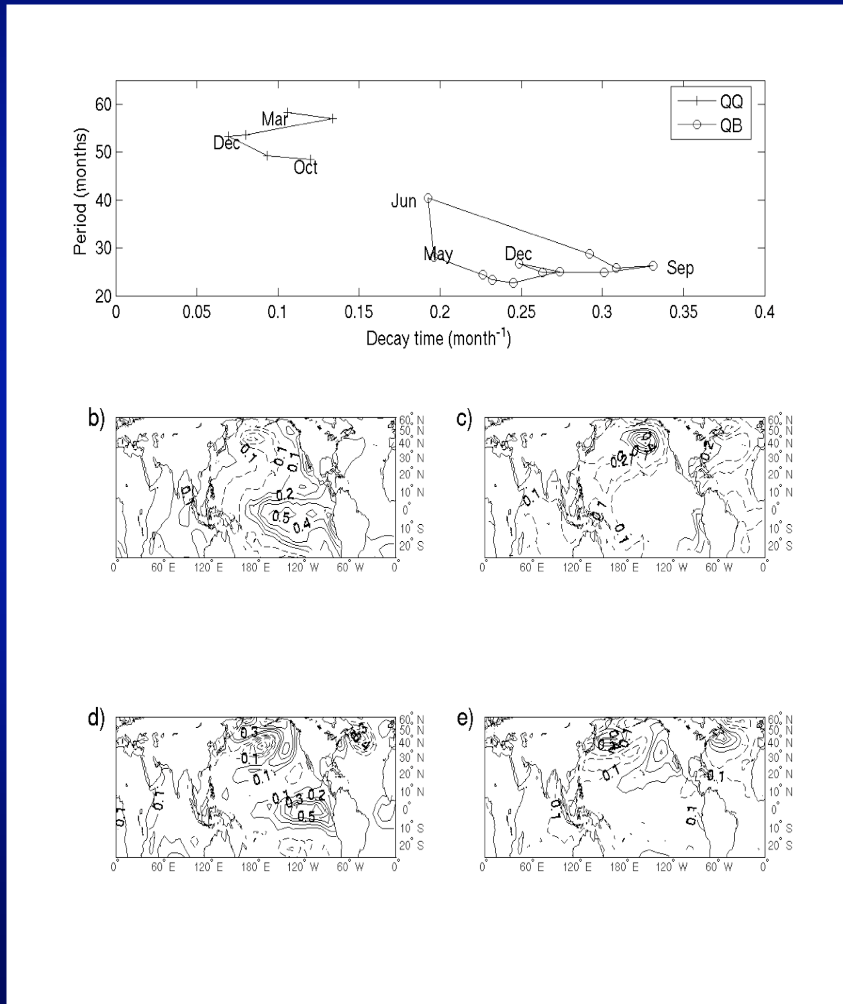



- SSTs for June are more difficult to predict.
- A feature of virtually all ENSO forecast schemes.
- SST anomalies are weaker in late winter through summer (why?), and signal-to-noise ratio is low.

# ENSO – VI

- Stability analysis, month-by-month, of the linearized regression model identifies weakly damped QQ mode (with a period of 48–60 mo), as well as strongly damped QB mode.

- QQ mode is least damped in December, while it is not identifiable at all in summer!



# ENSO – VII

*Floquet analysis*

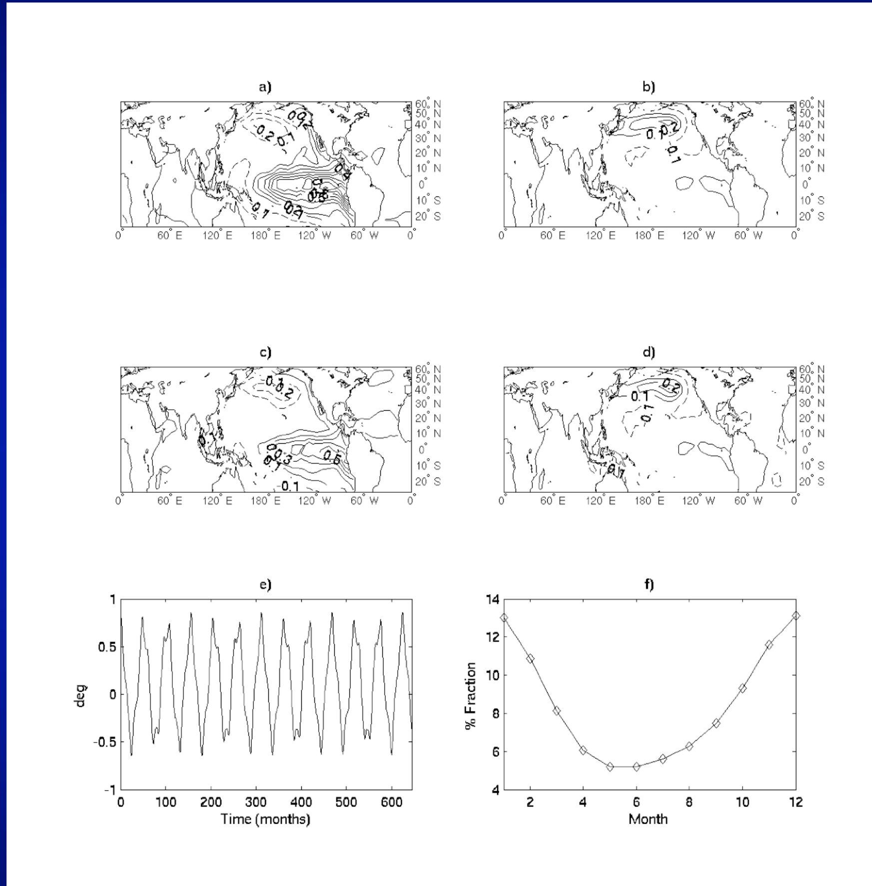
for seasonal cycle ($T$ = 12 mo):

$$\dot{\mathbf{x}} = \mathbf{L}(t)\mathbf{x}$$

$$\dot{\mathbf{\Phi}} = \mathbf{L}(t)\mathbf{\Phi}, \qquad \mathbf{\Phi}(0) = \mathbf{I}$$

$$\mathbf{M} \equiv \mathbf{\Phi}(T)$$

Floquet modes are related to the eigenvectors of the monodromy matrix ***M.***




QQ mode:

period = 52 months, damping: 11 months.

# ENSO – VIII

*ENSO development*
*and non-normal growth of*
*small perturbations*

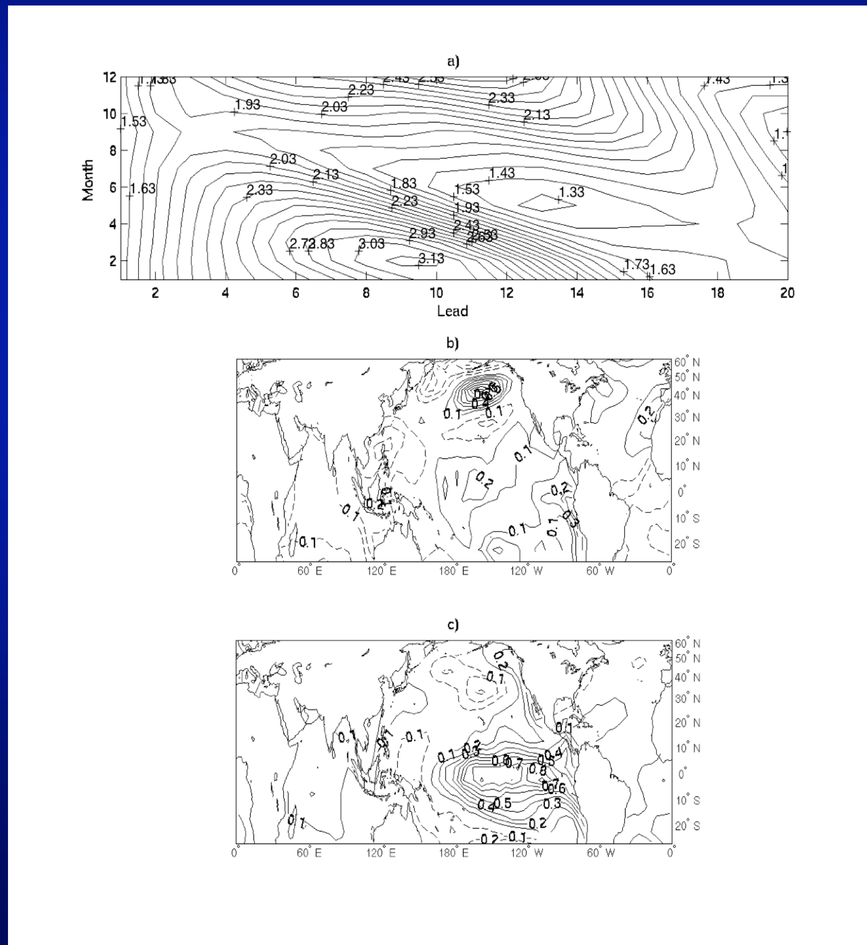
(Penland & Sardeshmukh, 1995;
Thompson & Battisti, 2000)

$$\mathbf{\Phi}(\tau) = \mathbf{U}\cdot\mathbf{S}\cdot\mathbf{V}^{\top}$$

*V* – optimal initial vectors
*U* – final pattern at lead $\tau$

- *Maximum growth*:

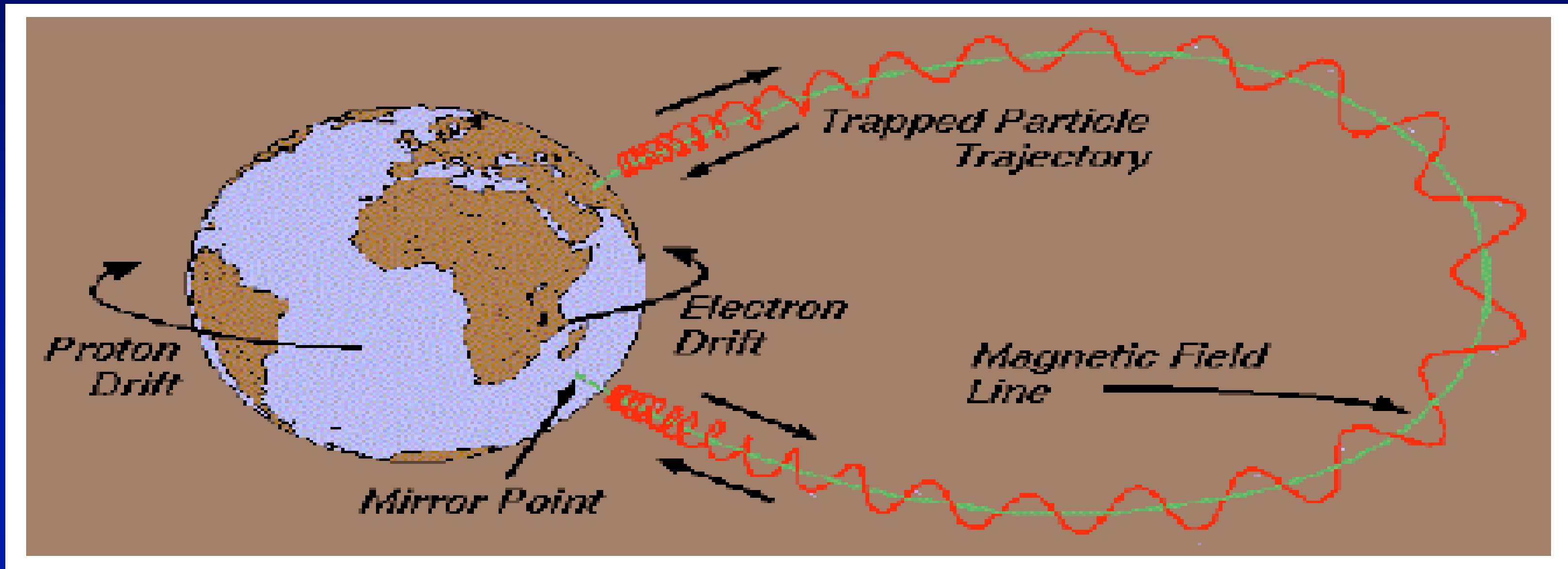
(b) start in Feb., (c) $\tau$= 10 months

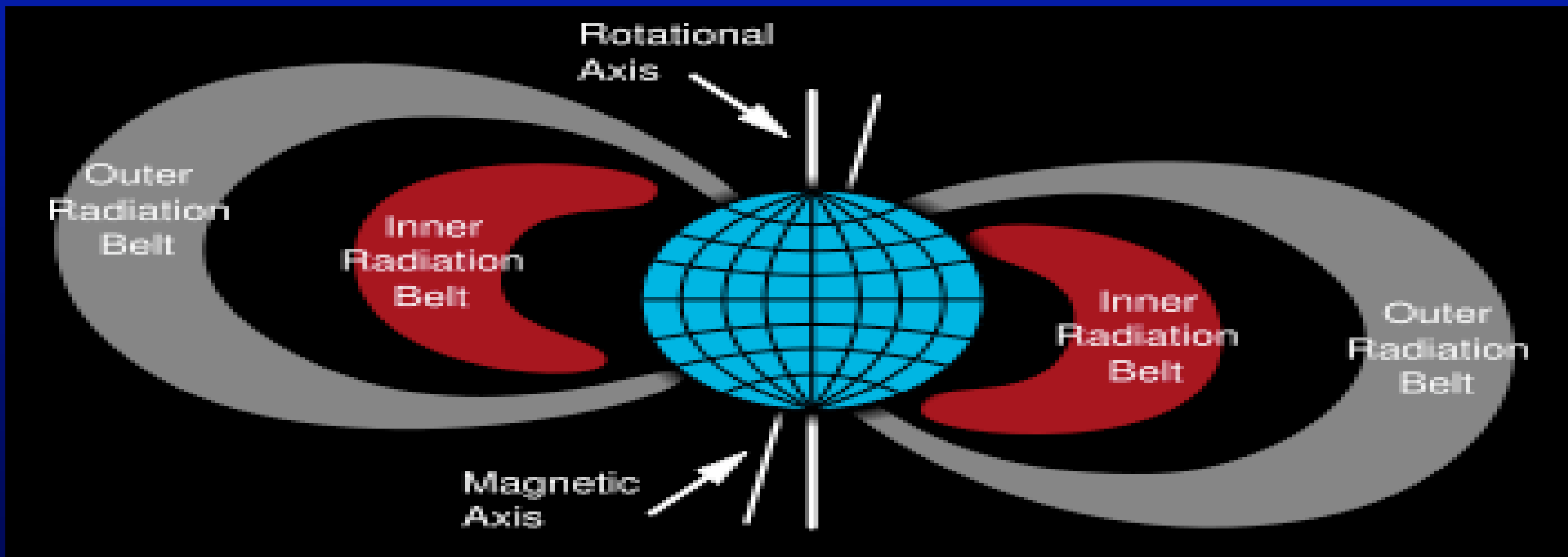


# Conclusions on ENSO model

- The quadratic, 2-level EMR model has competitive forecast skill.
- Two levels really matter in modeling “noise.”
- EMR model captures well the “linear,” as well as the “nonlinear” phenomenology of ENSO.
- Observed statistical features can be related to the EMR model’s dynamical operator.
- SST-only model: other variables? (A. Clarke)

# Van Allen Radiation Belts



# EMR for Radiation Belts – I

Radial diffusion code (Y. Shprits) – estimating phase space density $f$ and electron lifetime $\tau_L$:

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left[ D_{LL} L^{-2} \frac{\partial f}{\partial L} \right] - \frac{f}{\tau_{effective}}$$

$$D_{LL}(L, Kp(t)) = 10^{.506 Kp - 9.325} L^{10} \quad Kp = 1 \text{ to } 6$$

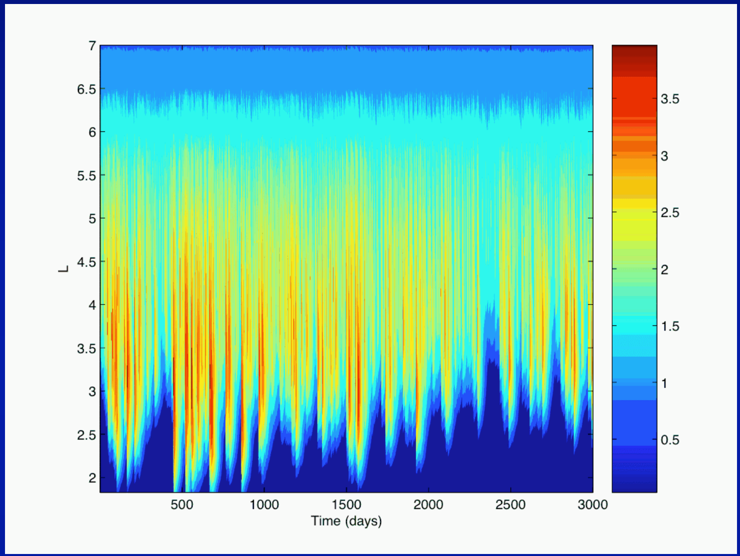
$$J = 8222.6 \cdot \exp(-7.068E), \quad L = 7$$

Different lifetime parameterizations for plasmasphere – out/in: $\tau_{Lo} = \zeta / K_p(t)$; $\tau_{Li}$=const.

- Test EMR on the model dataset for which we know the origin (“truth”) and learn something before applying it to real data.
- Obtain long time integration of the PDE model forced by historic **Kp** data to obtain data set for analysis.
- Calculate **PCs of log(fluxes)** and fit EMR.
- Obtain simulated data from the integration of reduced model and compare with the original dataset.

# EMR for Radiation Belts – II

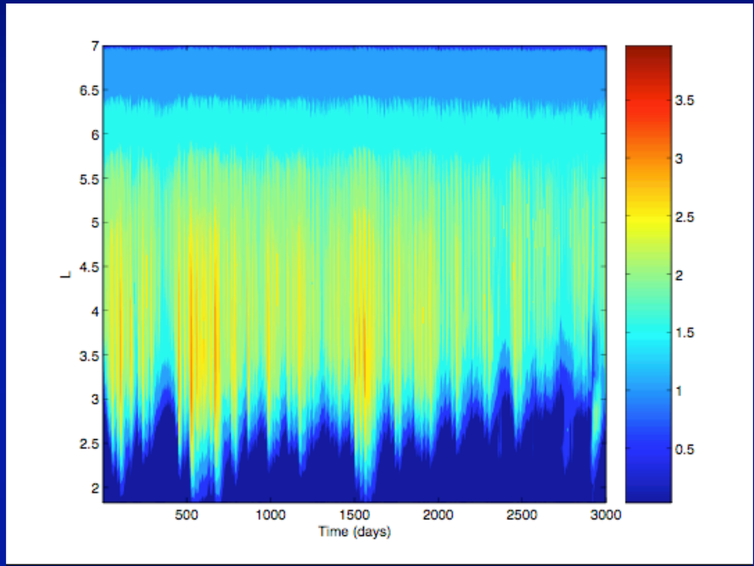
*Data*:




24000x26 dataset (3-hr resolution)

- Six leading PCs (account for 90% of the variance) ~ ENSO
- **Best** EMR model **is linear** with **3 levels**
- 6 spatial degrees of freedom (instead of 26).

*Model*:




- Random realization from **continuous** integration of EMR model forced by Kp.
- EMR model is **constant** in time
- stochastic component,
- deterministic part of EMR model has unstable eigenmodes.

# Concluding Remarks – I

- The generalized least-squares approach is well suited to derive nonlinear, reduced models (EMR models) of geophysical data sets; regularization techniques such as PCR and PLS are important ingredients to make it work.
- The multi-level structure is convenient to implement and provides a framework for dynamical interpretation in terms of the “eddy–mean flow” feedback (not shown).
- Easy add-ons, such as seasonal cycle (for ENSO, etc.).
- The dynamic analysis of EMR models provides conceptual insight into the mechanisms of the observed statistics.

# Concluding Remarks – II

*Possible pitfalls:*

- The EMR models are maps: need to have an idea about (time & space) scales in the system and sample accordingly.
- Our EMRs are parametric: functional form is pre-specified, but it can be optimized within a given class of models.
- Choice of predictors is subjective, to some extent, but their number can be optimized.
- Quadratic invariants are not preserved (or guaranteed) – spurious nonlinear instabilities may arise.

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