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Toward a Theory of the North Atlantic Oscillation

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Pls. see these sites for further info.

http://www.atmos.ucla.edu/tcd/

http://www.environnement.ens.fr/

Motivation

- The ***North Atlantic Oscillation (NAO)*** is a leading mode of variability of the Northern Hemisphere and beyond.
- It affects ***the atmosphere and oceans*** on several time and space scales.
- Its ***predictive understanding*** could help interannual and decadal-scale climate prediction over and around the North Atlantic basin.
- The ***hierarchical modeling*** approach allows one to give proper weight to the understanding provided by the models *vs.* their realism, respectively.
- Back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.

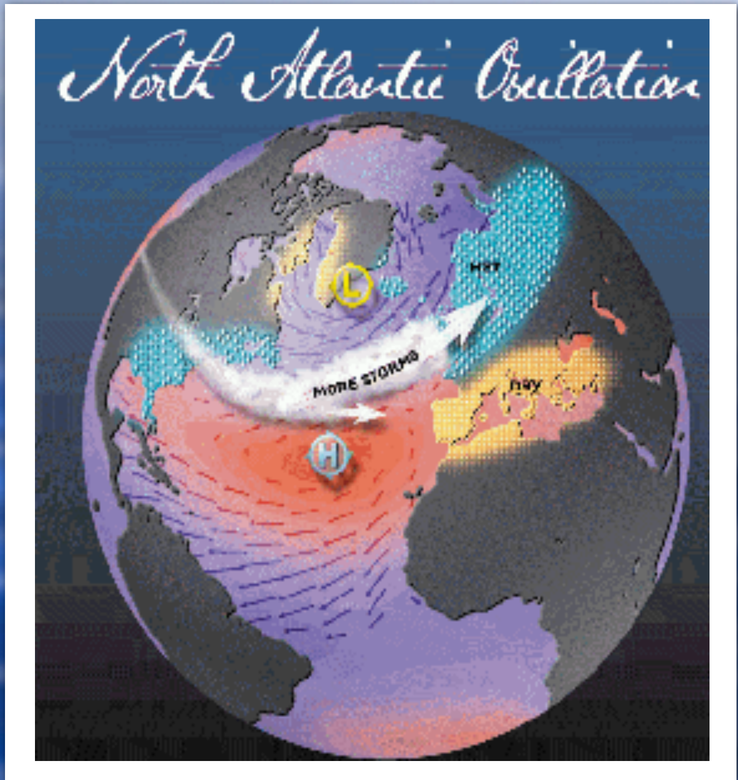
Joint work with S. Brachet, Y. Feliks and E. Simonnet

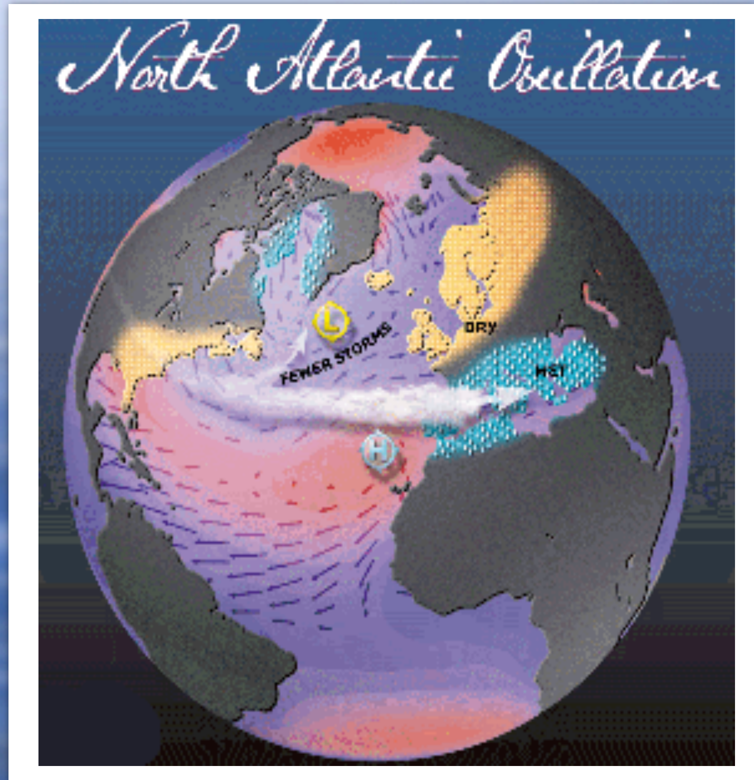
Outline

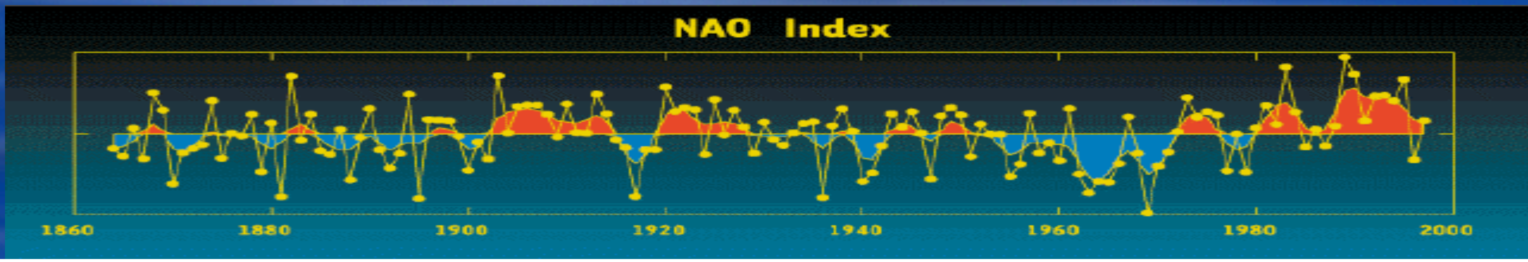
- Introduction: the NAO and the oceans' wind-driven circulation
- The low-frequency variability of the double-gyre circulation
  - – bifurcations in a toy model
    - => multiple equilibria, periodic and chaotic solutions
  - – some intermediate model results
- Atmospheric impacts
  - – simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
- A very promising coupled O–A model
- Conclusions
  - – The coupled climate system: is it the tail or the dog?
  - – Natural climate variability: a source of decadal predictability
- Some references

The North Atlantic Oscillation (NAO)

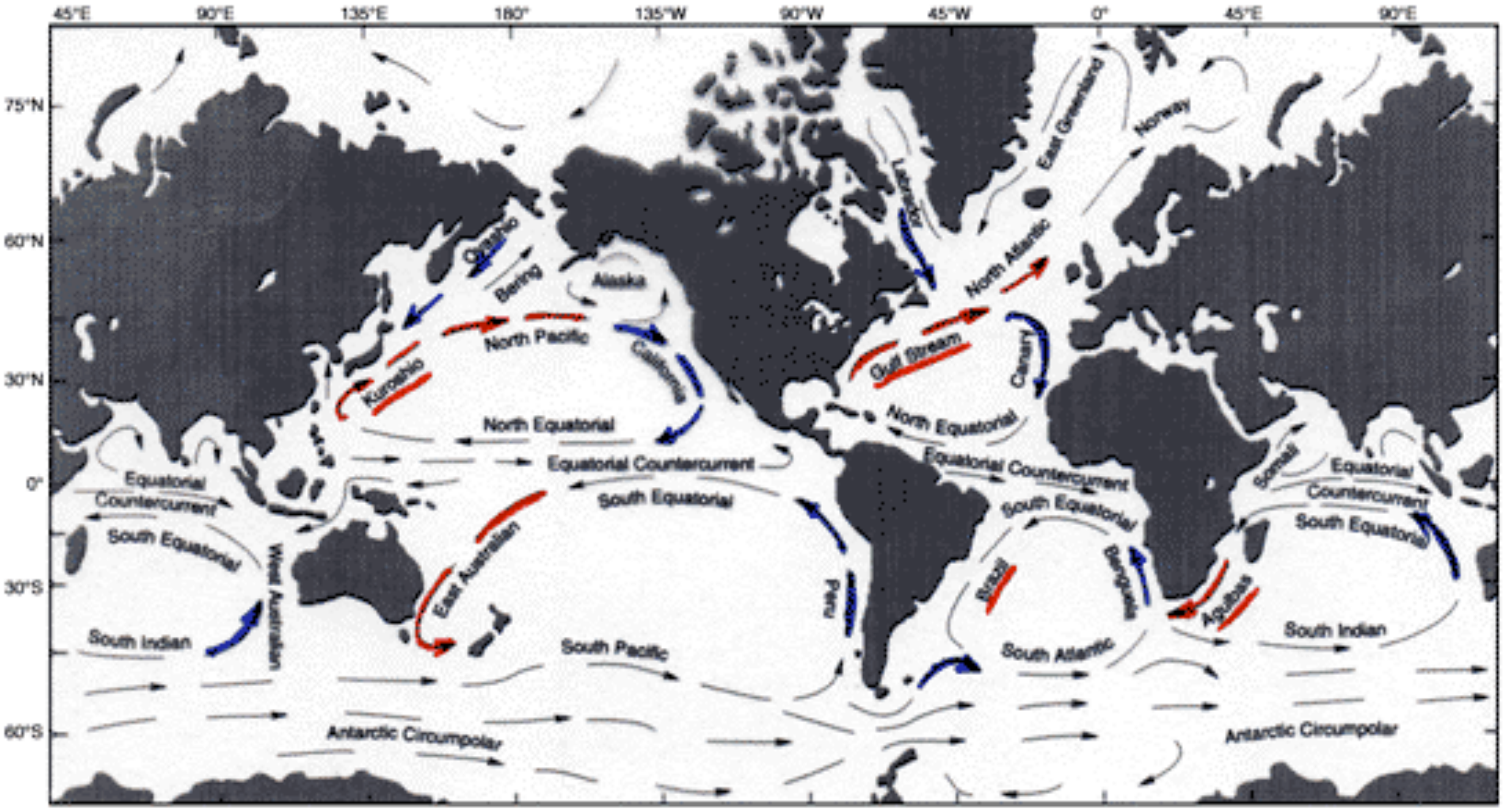
Positive phase *Negative phase*







An example of bifurcations and hierarchical modeling: The oceans' wind-driven circulation

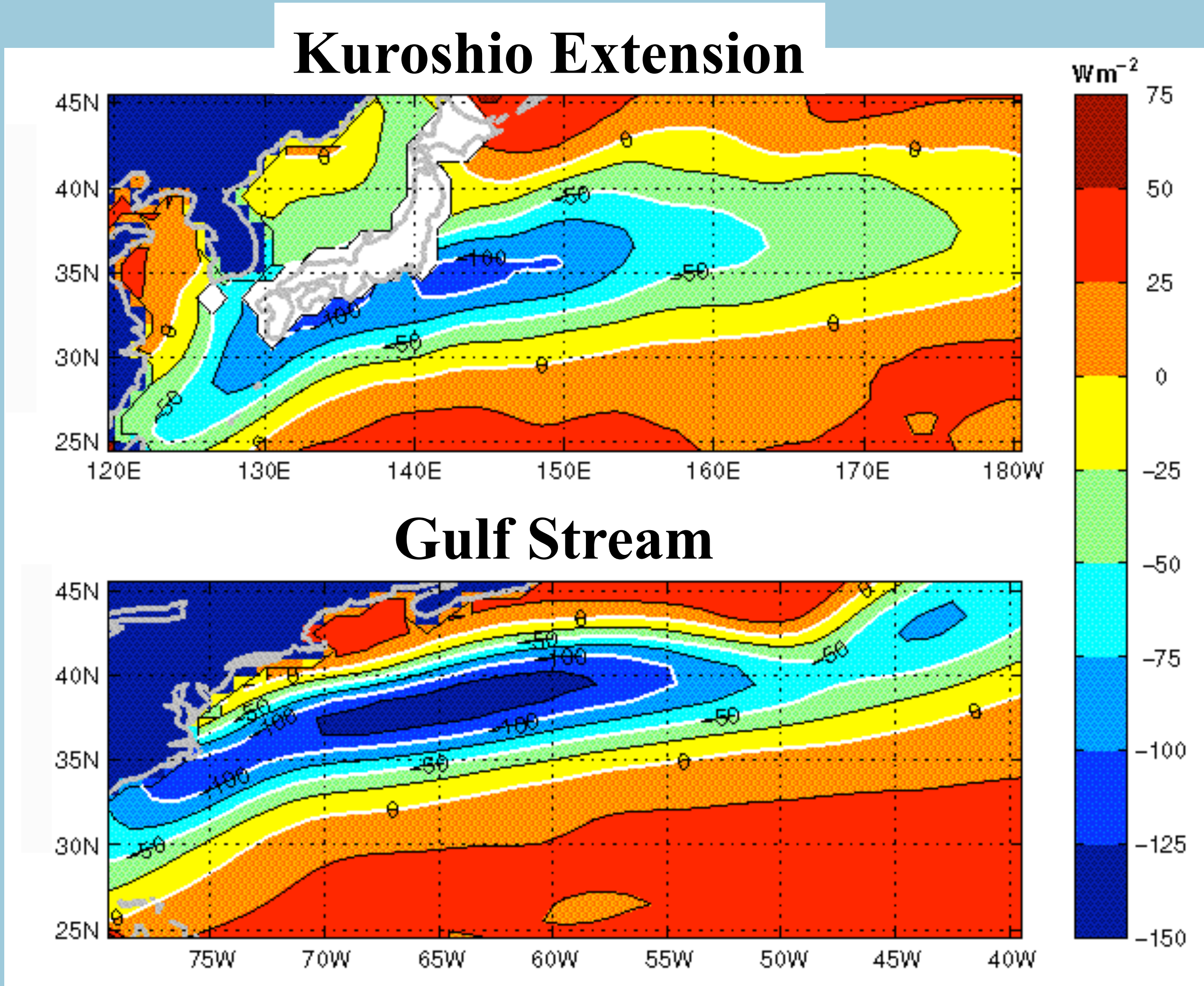


J. Apel (1987), Principles of Ocean Physics

The mean surface currents are (largely) wind-driven

Annual Mean Net Surface Heat Flux

Large heat loss balanced by poleward heat transport (advection)
Latent heat flux is large relative to sensible.



Southhampton Oceanography Centre

Kelly, Jan 2009

Applied Physics Laboratory
University of Washington

Western North Atlantic Circulation

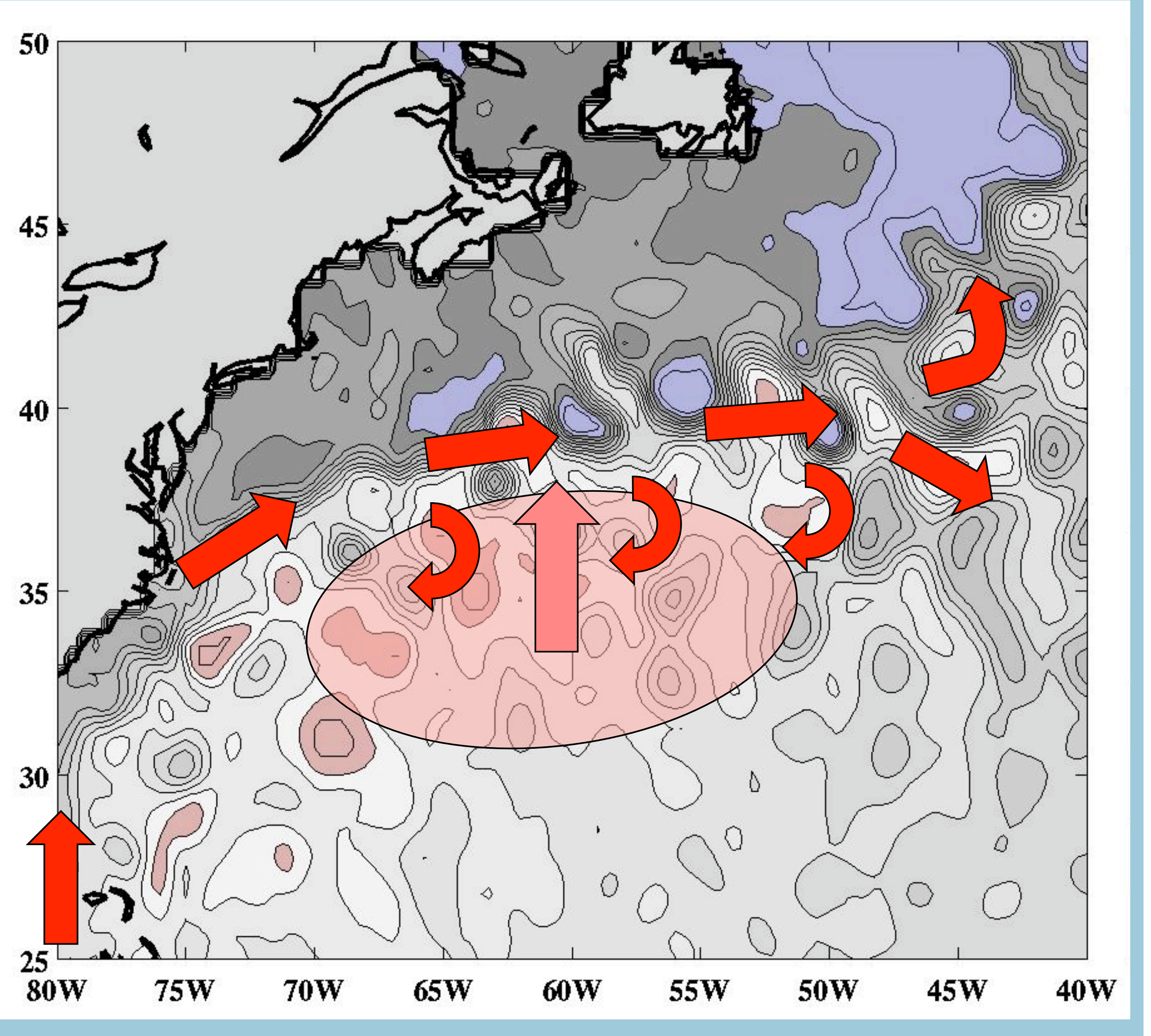
Florida Current brings warm water north

Gulf Stream separates & recirculates

Recirculation creates heat reservoir

Heat fluxed to atmosphere

Kelly, Jan 2009

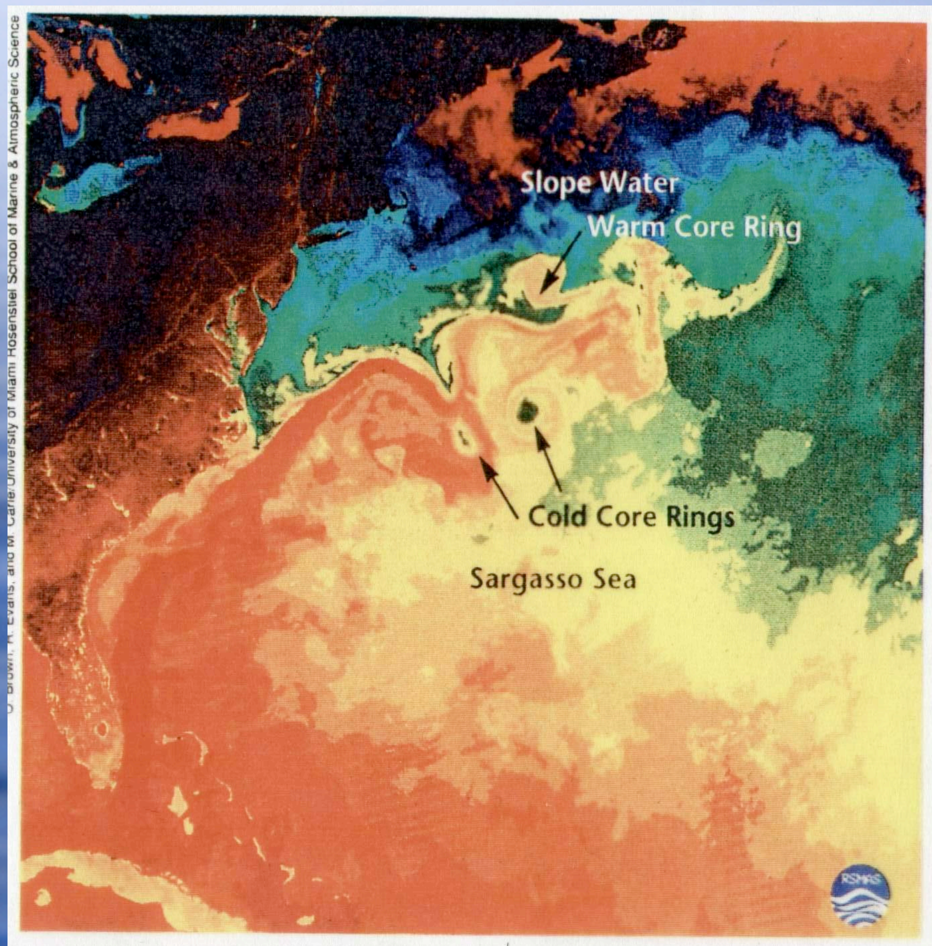


Applied Physics Laboratory
University of Washington

The gyres and the eddies

Many scales of motion, dominated in the mid-latitudes by
(i) ***the double-gyre circulation***;
and (ii) ***the rings and eddies***.

Much of the focus of physical oceanography over the ‘70s to ‘90s has been with the “***meso-scale***”: the meanders, rings & eddies, and the associated two-dimensional and quasi-geostrophic ***turbulence***.

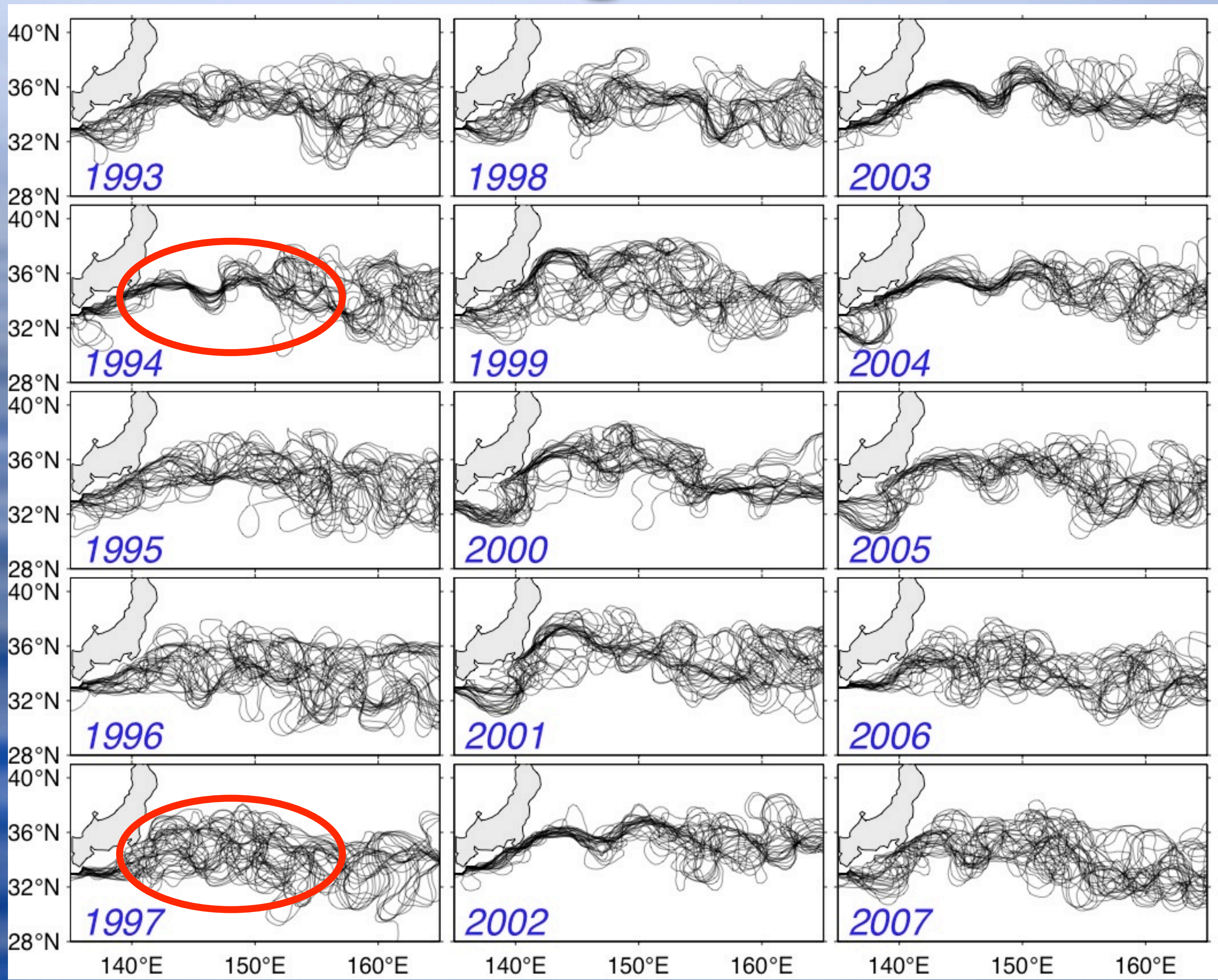


Based on SSTs, from satellite IR data

Kuroshio Extension (KE) Path Changes

Monthly paths from altimeter:

Stable vs. unstable periods



Qiu & Chen (*Deep-Sea Res.*, 2009)

"Limited-contour" analysis for atmospheric low-frequency variability

10-day sequences of
subtropical jet paths:
blocked vs. zonal
flow regimes

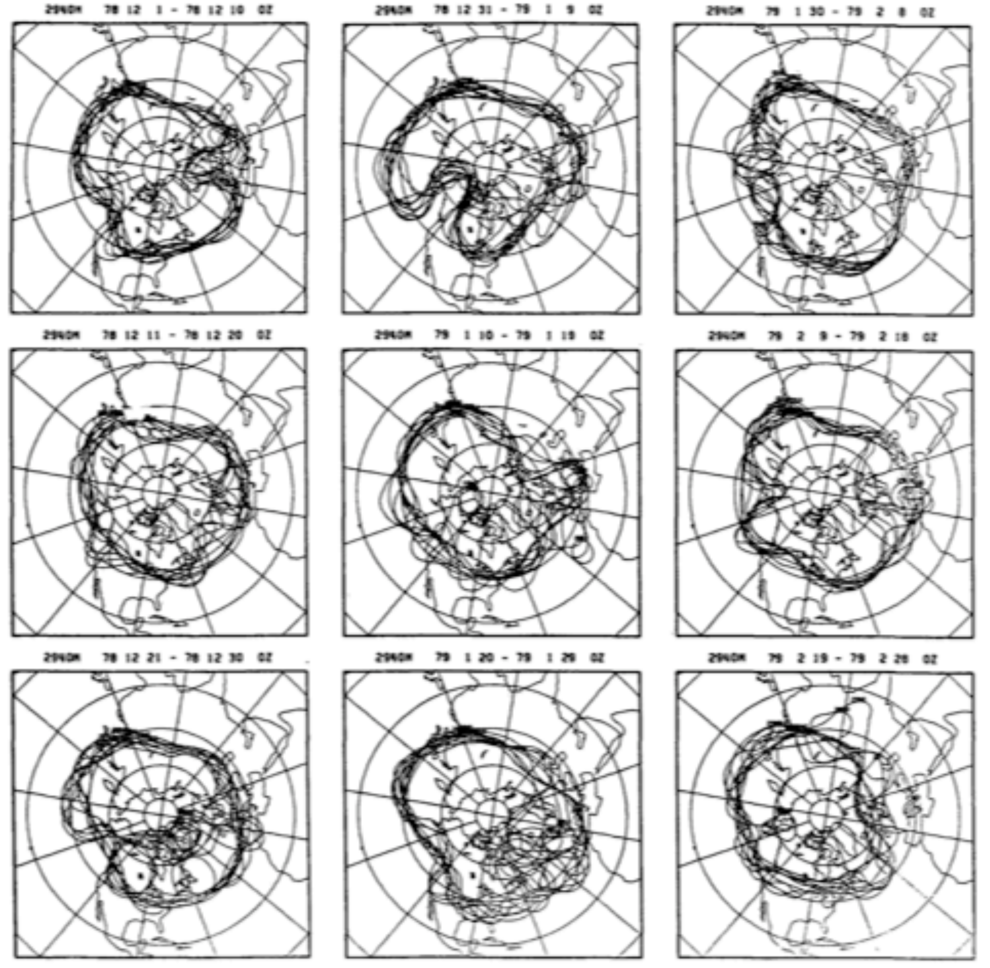
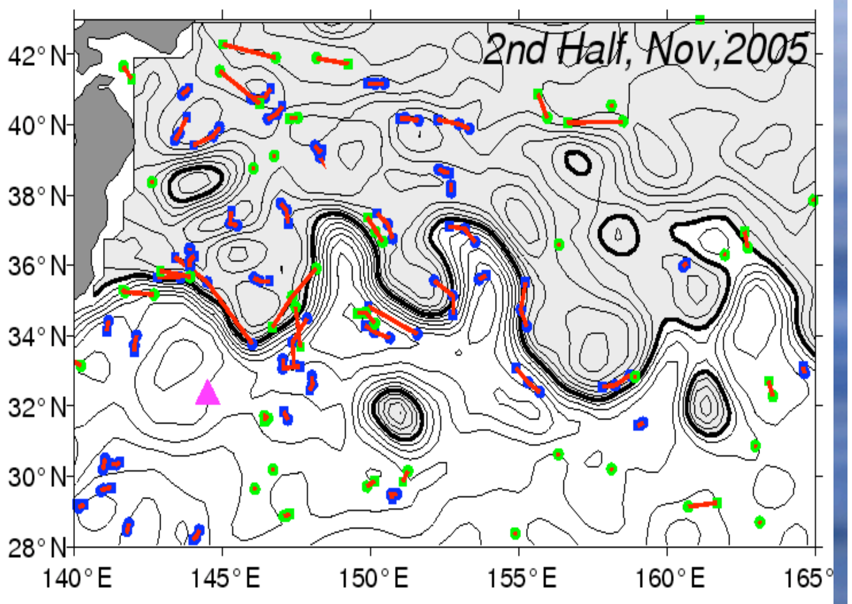


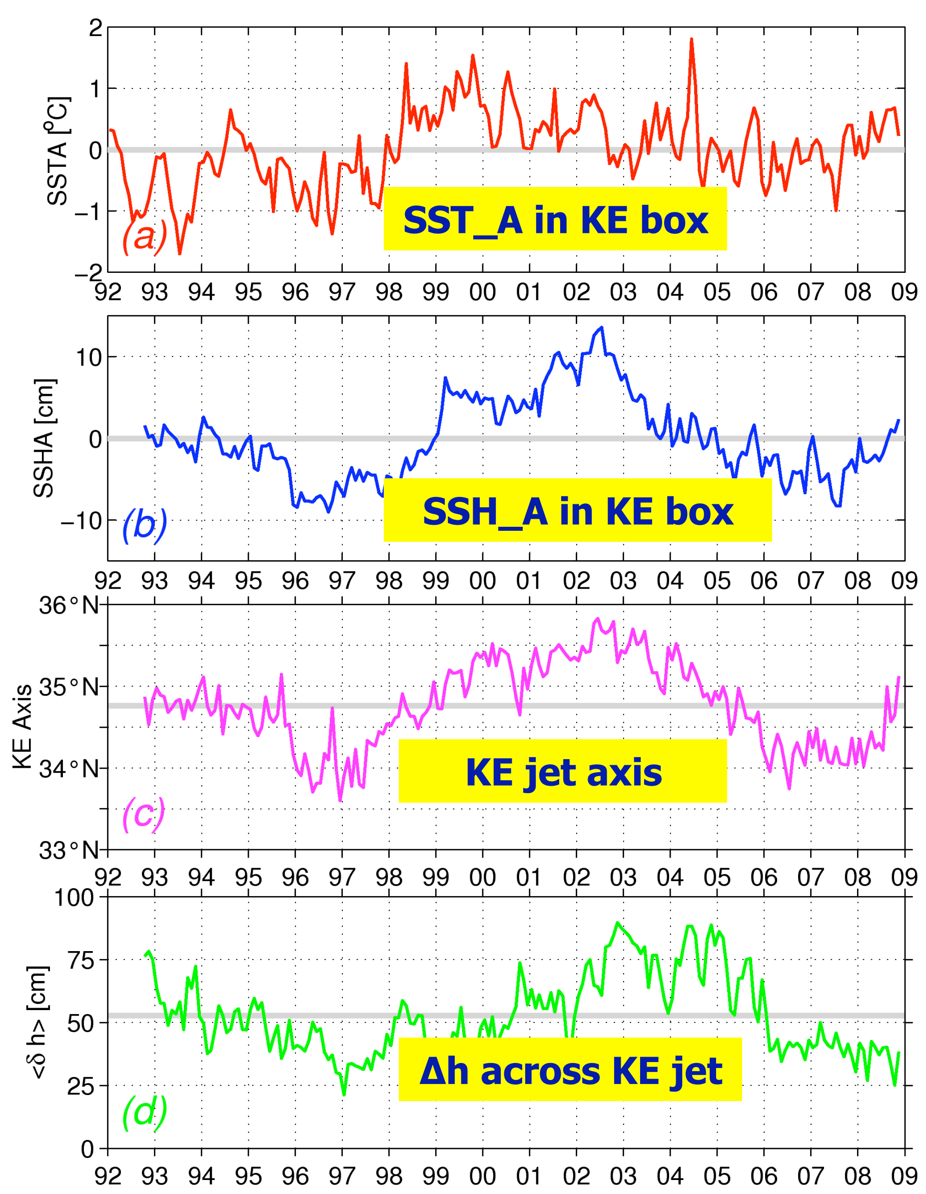
FIG. 1. Limited contour analysis of Northern Hemisphere (NH) flows. Daily contours of a prescribed height (2940 m in this case—roughly corresponding to the jet axis) are superimposed for successive 10-day intervals during NH winter 1978/79. Persistence is illustrated by some of the panels (see text for details).

Kimoto & Ghil, *JAS*, 1993a

Kuroshio Extension (KE) box

SST anomalies are largely caused by strength changes of the KE jet



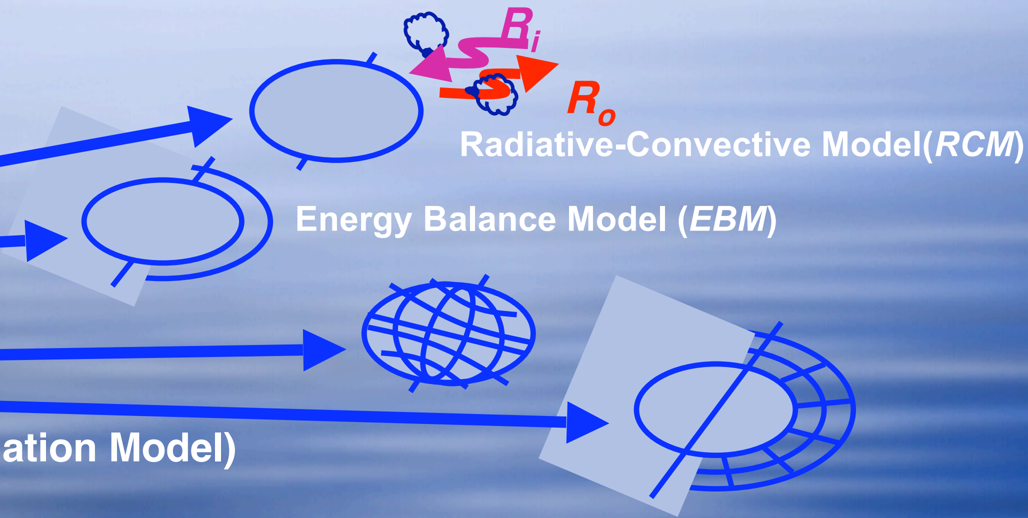


Courtesy of Bo Qiu, Jan. '09

Climate models (atmospheric & coupled) : A classification

- ***Temporal***
  - ♣ stationary, (quasi-)equilibrium
  - ♣ transient, climate variability

- ***Space***
  - ♣ 0-D (dimension 0)
  - ♣ 1-D
    - vertical
    - latitudinal
  - ♣ 2-D
    - horizontal
    - meridional plane
  - ♣ 3-D, GCMs (Général Circulation Model)
    - horizontal
    - meridional plane
  - ♣ Simple and intermediate 2-D & 3-D models



- ***Coupling***
  - ♣ Partial
    - unidirectional
    - asynchronous, hybrid
  - ♣ Full

Hierarchy: from the simplest to the most elaborate,
iterative comparison with the observational data

Modeling Hierarchy for the Oceans

Ocean models

- 0-D: box models – chemistry (BGC), paleo
- 1-D: vertical (mixed layer, thermocline)
- 2-D – meridional plane – THC
  → also 1.5-D: a little longitude dependence
  – horizontal – wind-driven
  → also 2.5-D: reduced-gravity models (n.5)
- 3-D: OGCMs - simplified
  - with bells & whistles ("kitchen sink")

Coupled 0-A models

- Idealized (0-D & 1-D): intermediate couple models (ICM)
- Hybrid (HCM) - diagnostic/statistical atmosphere
  - highly resolved ocean
- Coupled GCM (3-D): CGCM

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The double-gyre circulation and its low-frequency variability

An "intermediate" model of the mid-latitude, wind-driven ocean circulation: 20-km resolution, about 15 000 variables

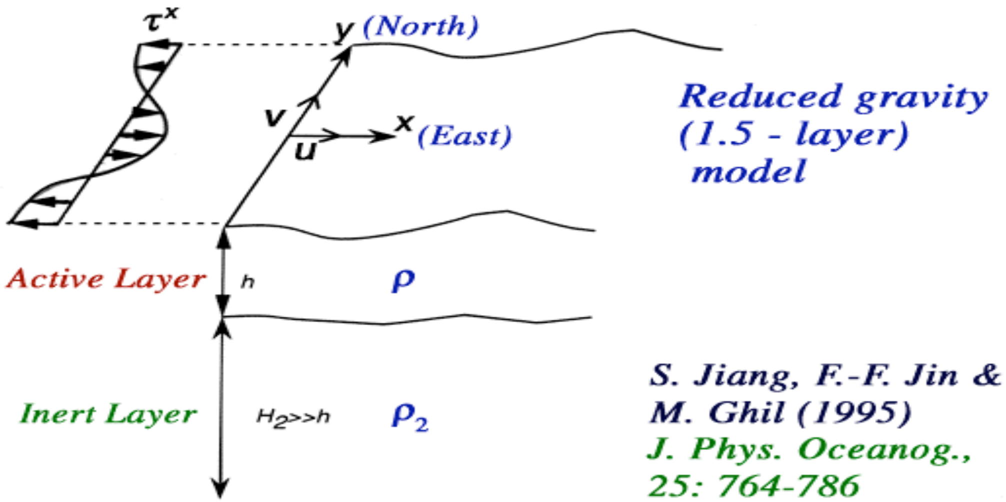
Shallow-water model

$$
\begin{aligned}
\frac{\partial U}{\partial t} + \nabla \cdot (\mathbf{u}U) &= -g'h\frac{\partial h}{\partial x} + fV + \alpha_A A\nabla^2 U - RU - \alpha_\tau \frac{\tau^x}{\rho} \\
\frac{\partial V}{\partial t} + \nabla \cdot (\mathbf{u}V) &= -g'h\frac{\partial h}{\partial y} - fU + \alpha_A A\nabla^2 V - RV \\
\frac{\partial h}{\partial t} &= -(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y})
\end{aligned}
$$

where

$$U\hat{e_x} + V\hat{e_y} = h\mathbf{u} = h(u\hat{e_x} + v\hat{e_y})$$

g':	reduced gravity	$(= g(\rho_2 - \rho)/\rho)$
A:	viscosity coefficient	$(= 300\ \mathrm{m^2 s^{-1}})$
R:	Rayleigh coefficient	$(= 1/200\ \mathrm{day^{-1}})$
τ^x:	wind stress $= \tau_o \cos 2\pi/L (\tau_o = 1\ \mathrm{dyn\ cm^{-2}}\ \& L = 2000\mathrm{km})$	



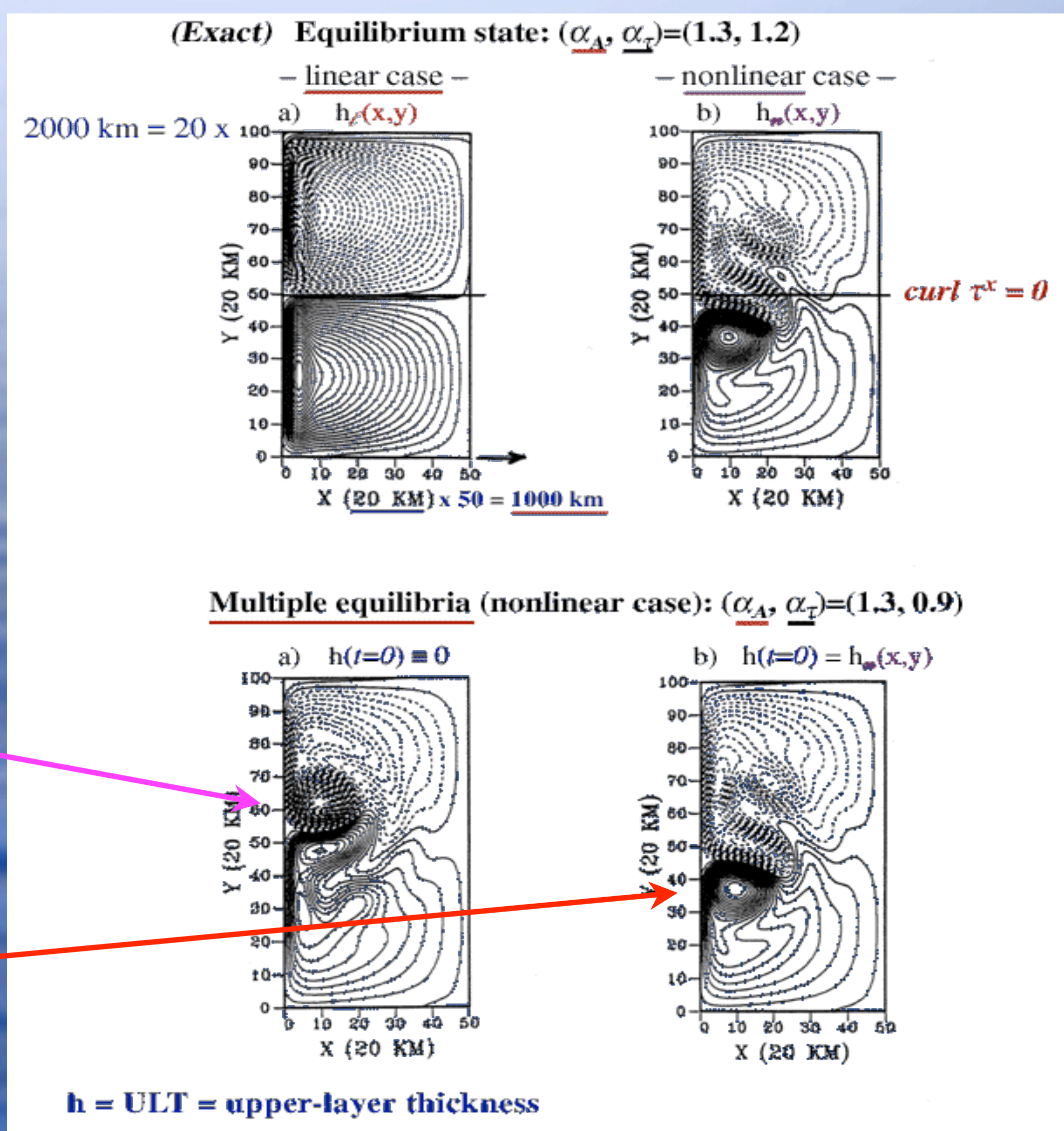
S. Jiang, F.-F. Jin & M. Ghil (1995) J. Phys. Oceanog., 25: 764-786

The JJG model's equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?

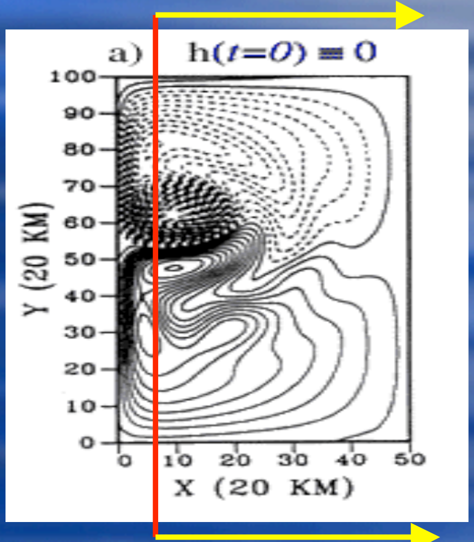
Subpolar gyre dominates

Subtropical gyre dominates



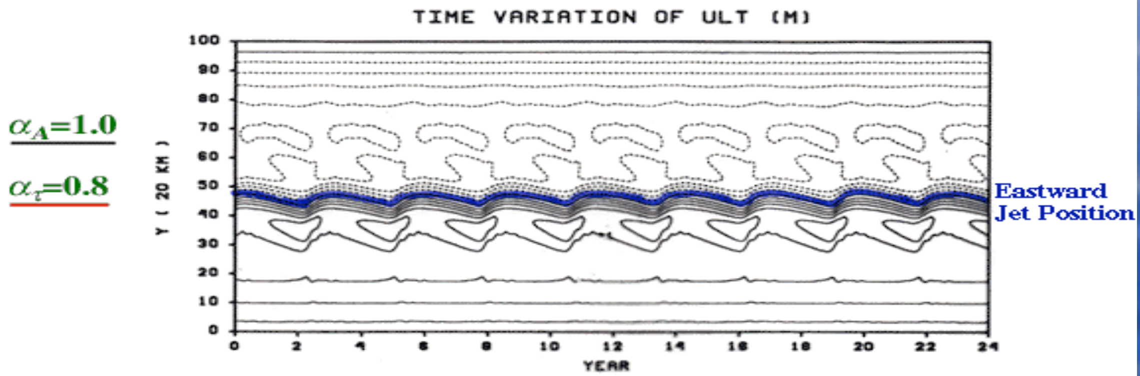
Time-dependent solutions: periodic and chaotic

To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams

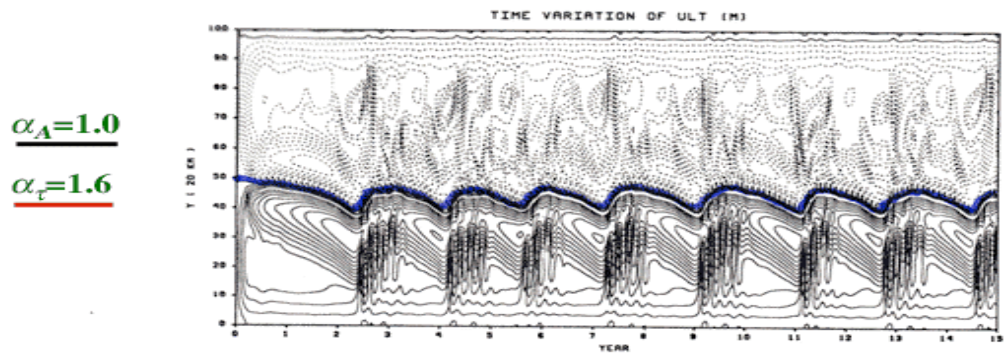


Time-dependent solutions

1. Periodic, w/ interannual period (2.8 years)



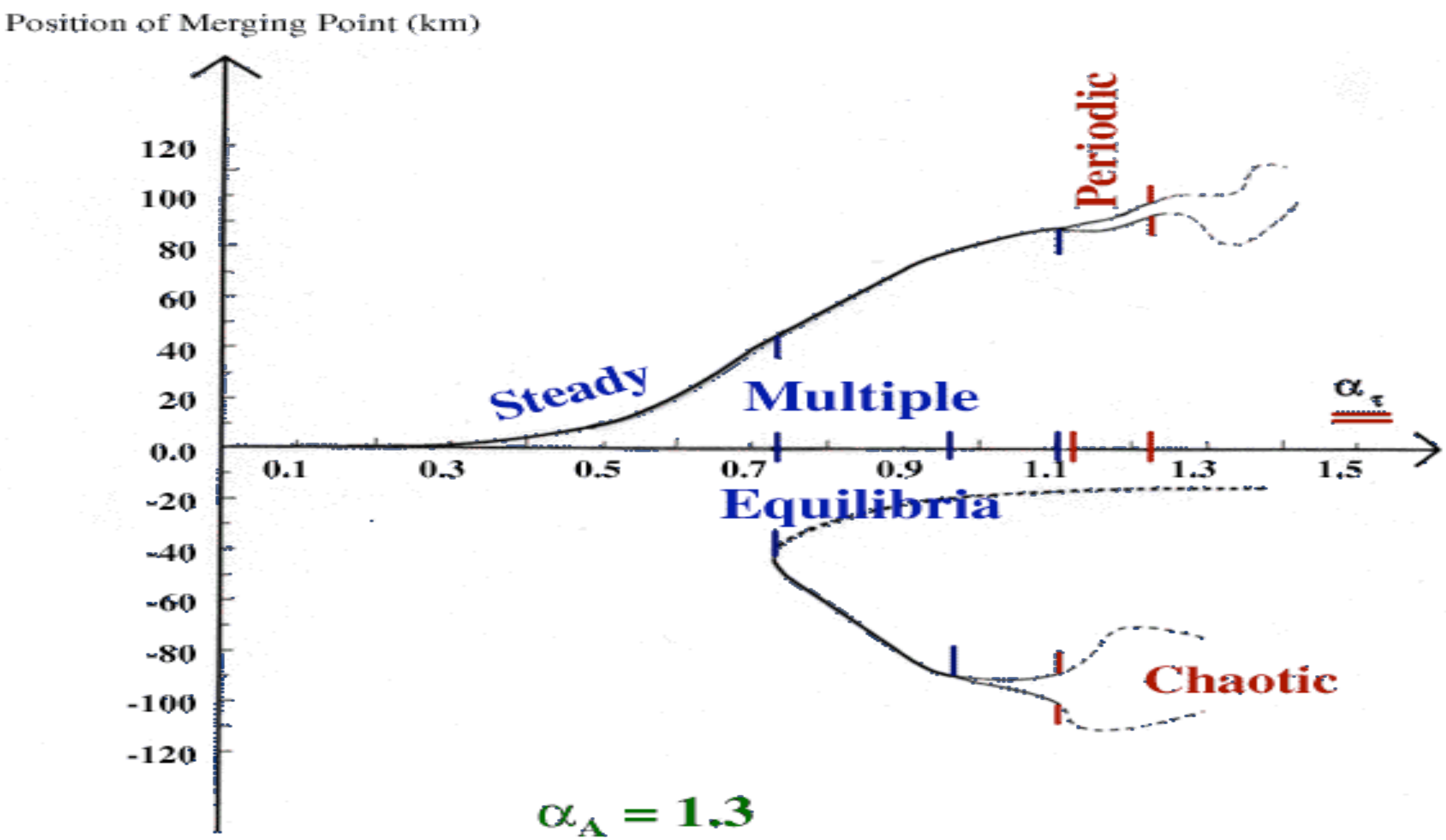
2. Aperiodic (weakly chaotic)



Poor man's continuation method

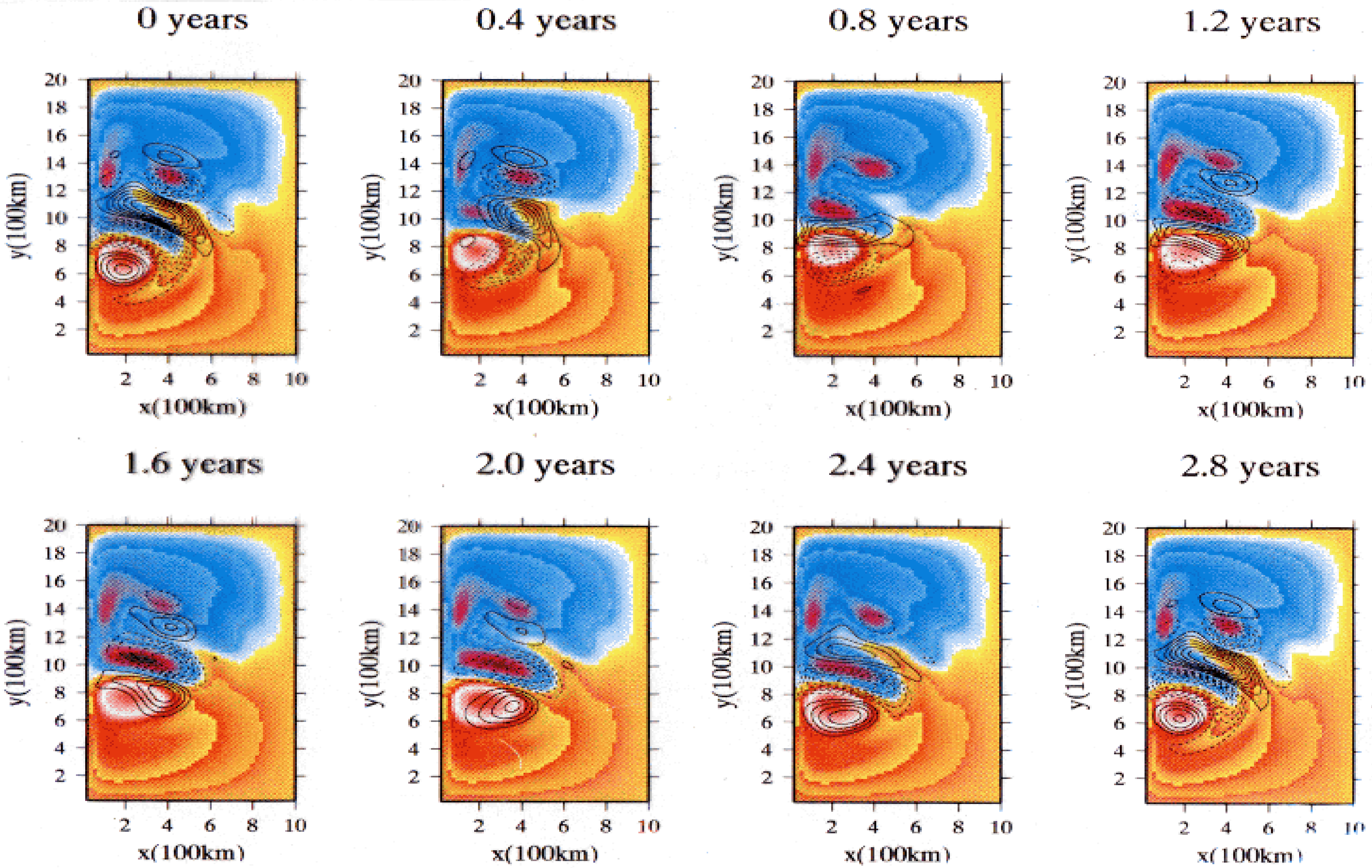
Bifurcation diagram

Perturbed pitchfork + Hopf + transition to chaos



$\alpha_A = 1.3$

Interannual variability: relaxation oscillation



Global bifurcations in "intermediate" models

Bifurcation tree in a QG, equivalent-barotropic, high-resolution (10 km) model: pitchfork, mode-merging, Hopf, and homoclinic

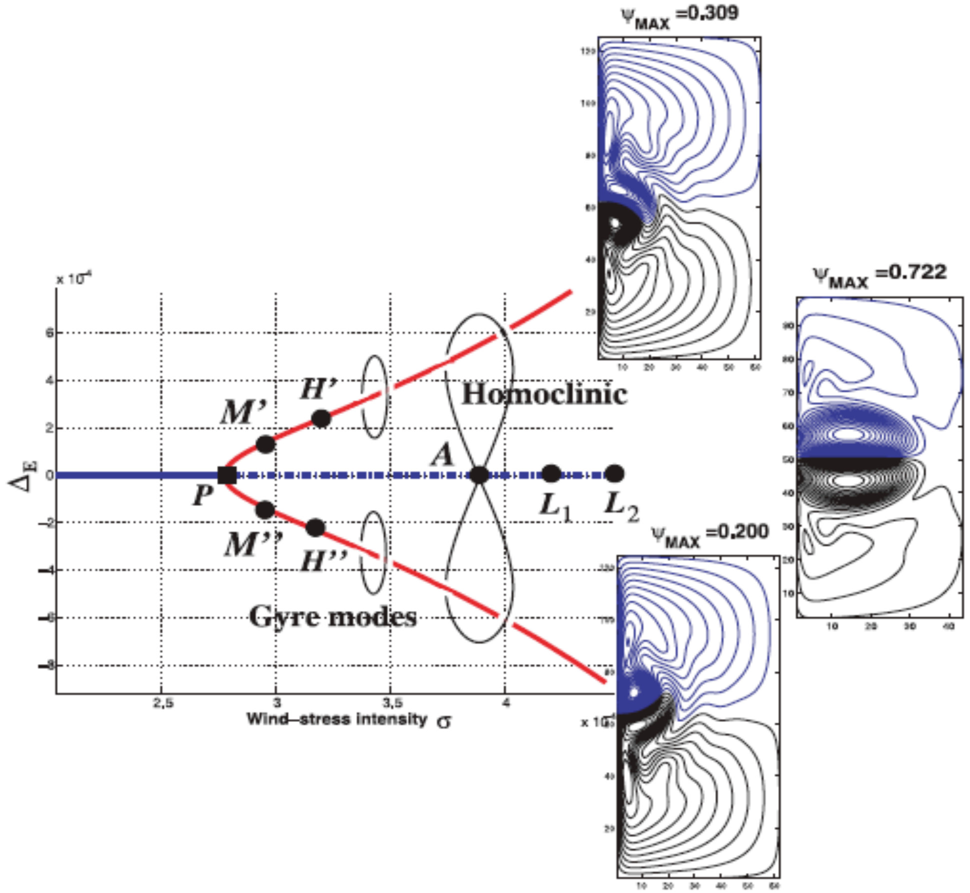


Figure 1. Schematic bifurcation diagram of an equivalent-barotropic QG model, plotted in terms of an asymmetry measure Δ_E (see Section 3a further below) vs. wind-stress intensity. The limit cycles are schematically drawn for illustrative purpose and the streamfunction patterns corresponding to the three steady-state branches—subtropical, antisymmetric, and subpolar (from top to

Homoclinic orbit: numerical and analytical

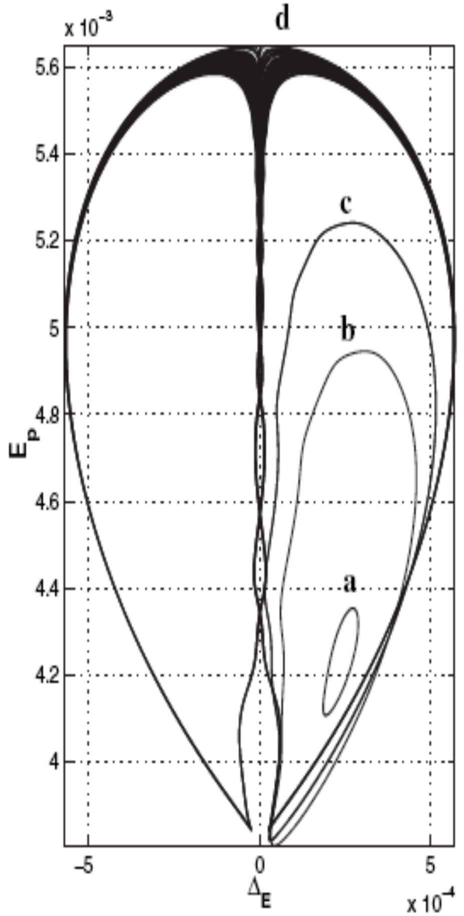


Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution E_p and the difference Δ_E between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are

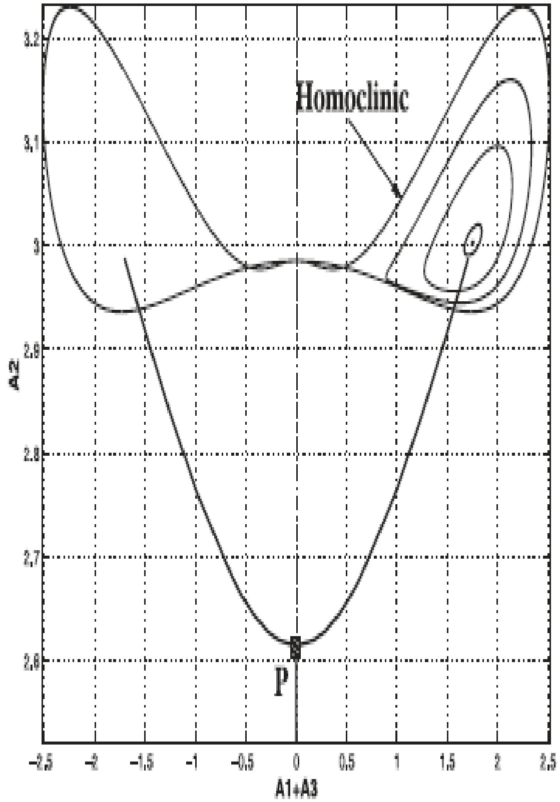


Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_3, A_2)$ plane for $\mu = 1$ and $s = 2$; P stands for pitchfork bifurcation at $\sigma = \sigma_p = 7.61$, while $\sigma = \sigma_{hc} \simeq 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.

The double-gyre circulation: A different rung of the hierarchy

Another "intermediate" model of the double-gyre circulation: slightly different physics, higher resolution – down to 10 km in the horizontal and more layers in the vertical, much larger domain, …

Quasi - geostrophic model

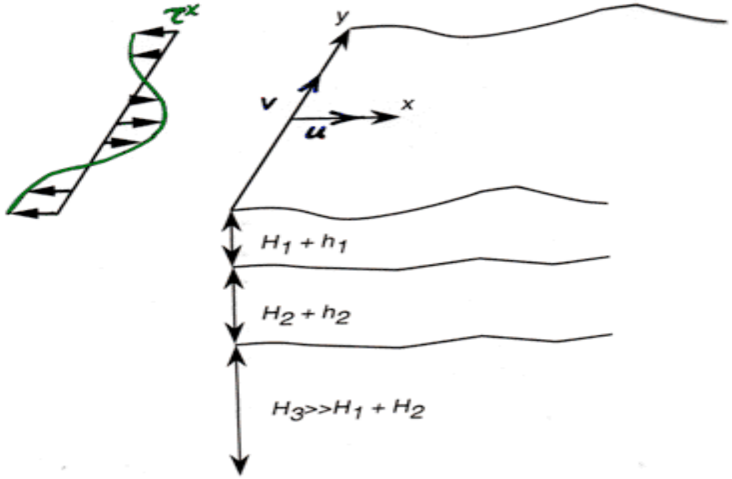
2.5-layer model

$$\frac{\partial}{\partial t}(\nabla^2 h_1 - \lambda_1^2(h_1 - h_2)) + \beta\frac{\partial h_1}{\partial x} = -\frac{g'}{f_0}J[h_1, \nabla^2 h_1 - F_1(h_1 - h_2)] + A_h\nabla^4 h_1 - C\nabla^2(h_1 - h_2) + \frac{f_0}{\rho_0 g' H_1}curl\vec{\tau}$$

$$\frac{\partial}{\partial t}(\nabla^2 h_2 - \lambda_2^2(h_2 - h_1)) + \beta\frac{\partial h_2}{\partial x} = -\frac{g'}{f_0}J[h_2, \nabla^2 h_2 - F_2(h_2 - h_1)] + A_h\nabla^4 h_2 - C\nabla^2(h_2 - h_1) - R\nabla^2 h_2$$

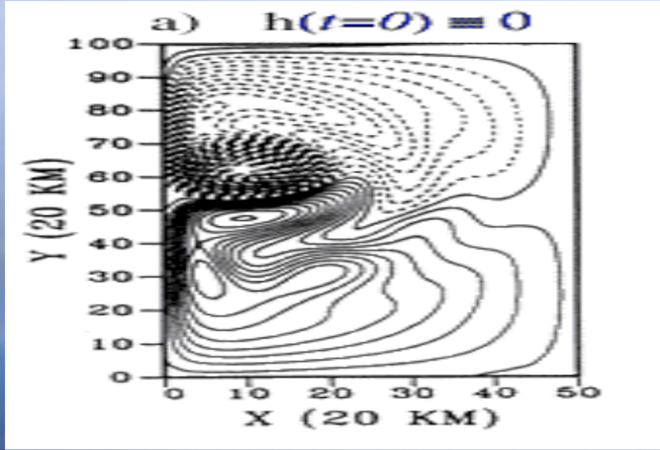
where

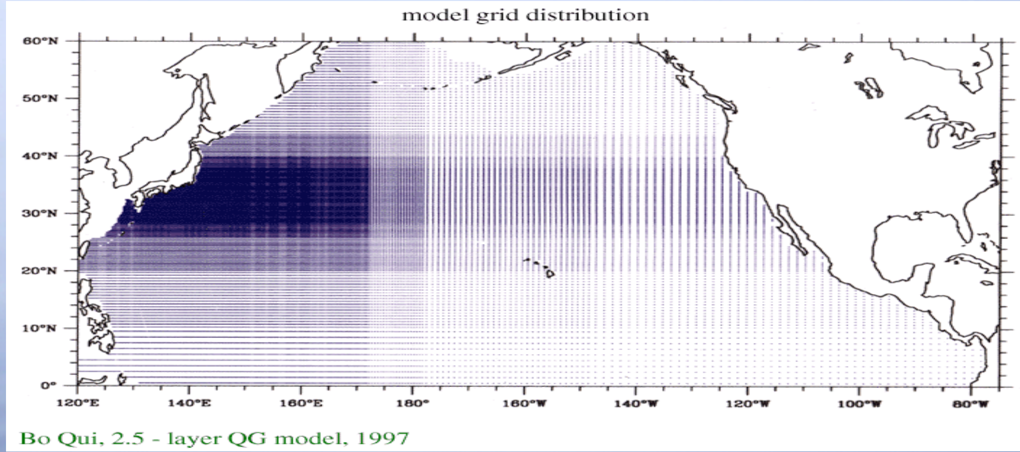
h_1, h_2:	height anomaly for upper and lower layer	(stream functions)
H_1, H_2:	mean height for upper and lower layer	
λ_1, λ_2:	Rossby radius of deformation	$\equiv \sqrt{h'H_1/f_0^2}, \sqrt{h'H_2/f_0^2}$
$\vec{\tau}$:	wind stress	
A_h:	viscosity coefficient	
C, R:	Rayleigh coefficient for interface and lower layer	
f_0, β:	Coriolis and beta parameters	
ρ_0, g':	mean density and reduced gravity	

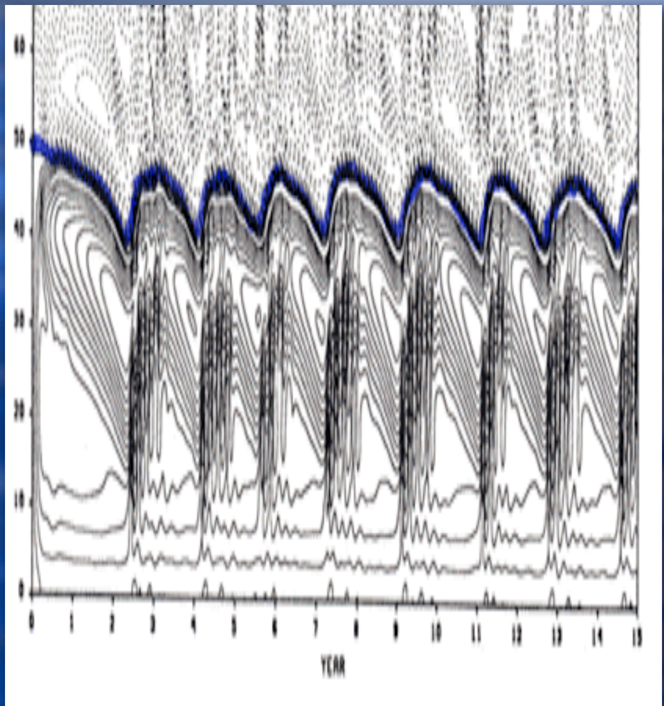


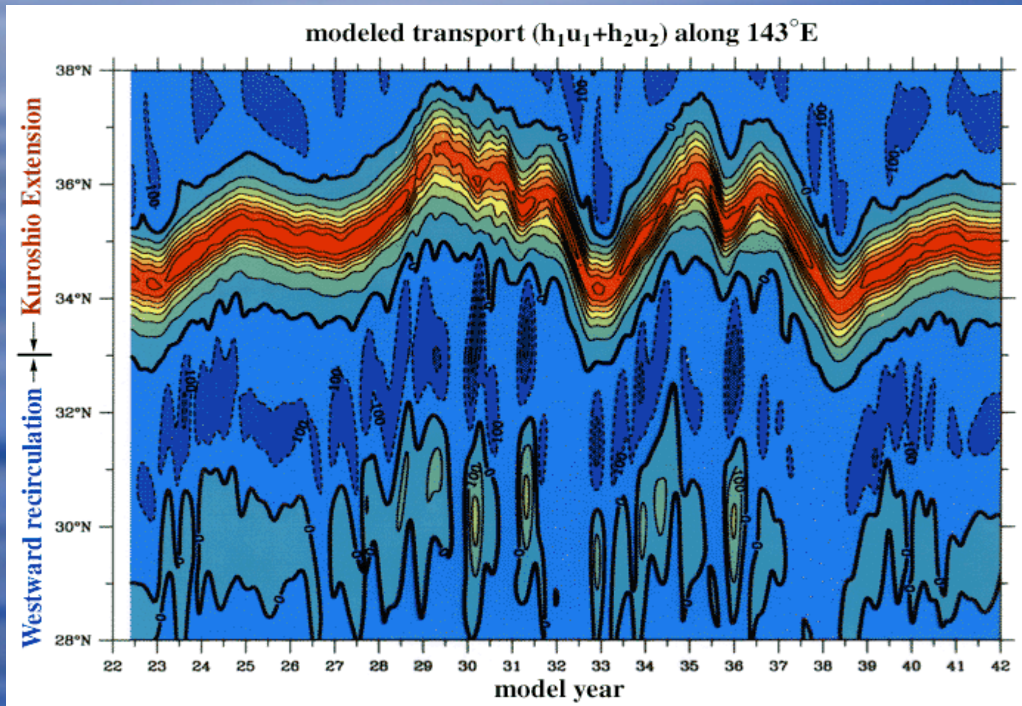
Bo Qiu, U. of Hawaii, pers. commun., 1997

Model-to-model, qualitative comparison



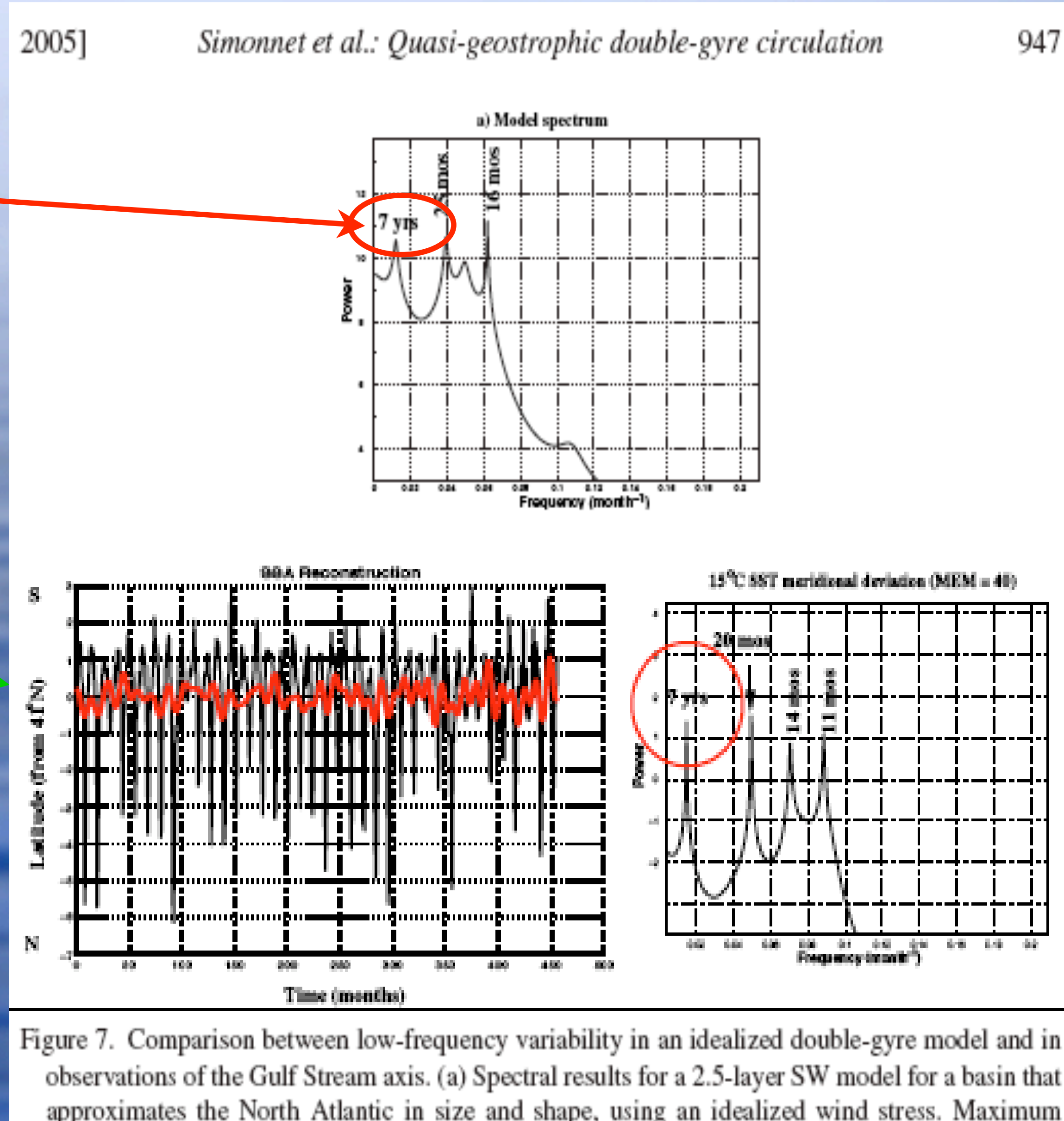






Model-and-observations, quantitative comparison

Spectra of
(a) kinetic energy of 2.5-layer shallow-water model in North-Atlantic–shaped basin; and
(b) Cooperative Ocean-Atmosphere Data Set (COADS) Gulf-Stream axis data



2005] *Simonnet et al.: Quasi-geostrophic double-gyre circulation* 947

Figure 7. Comparison between low-frequency variability in an idealized double-gyre model and in observations of the Gulf Stream axis. (a) Spectral results for a 2.5-layer SW model for a basin that approximates the North Atlantic in size and shape, using an idealized wind stress. Maximum

More spatio-temporal data

Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year "gyre mode" of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

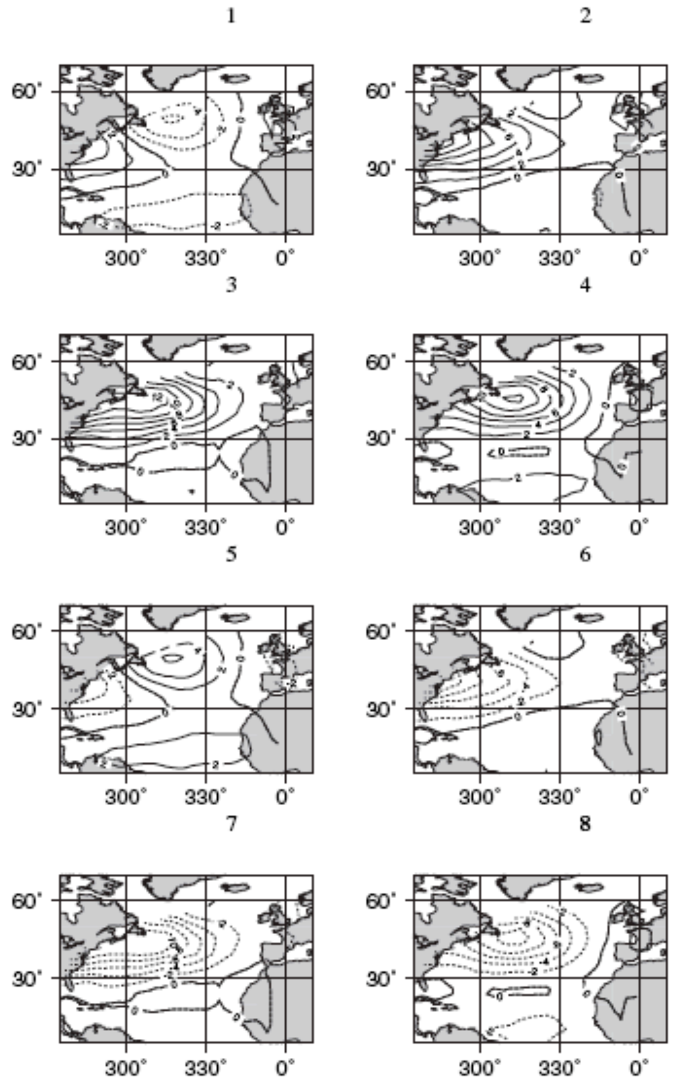


Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.

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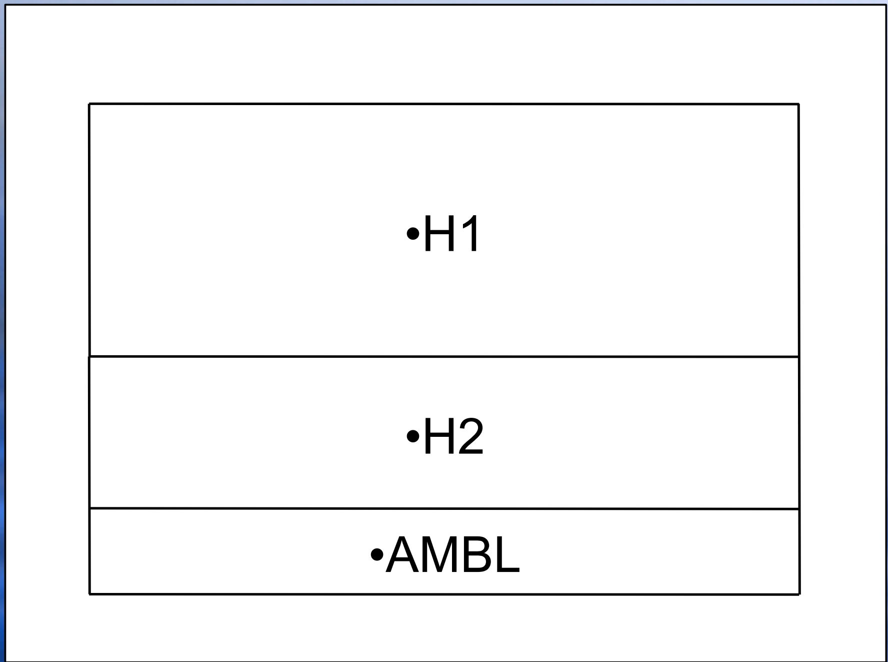
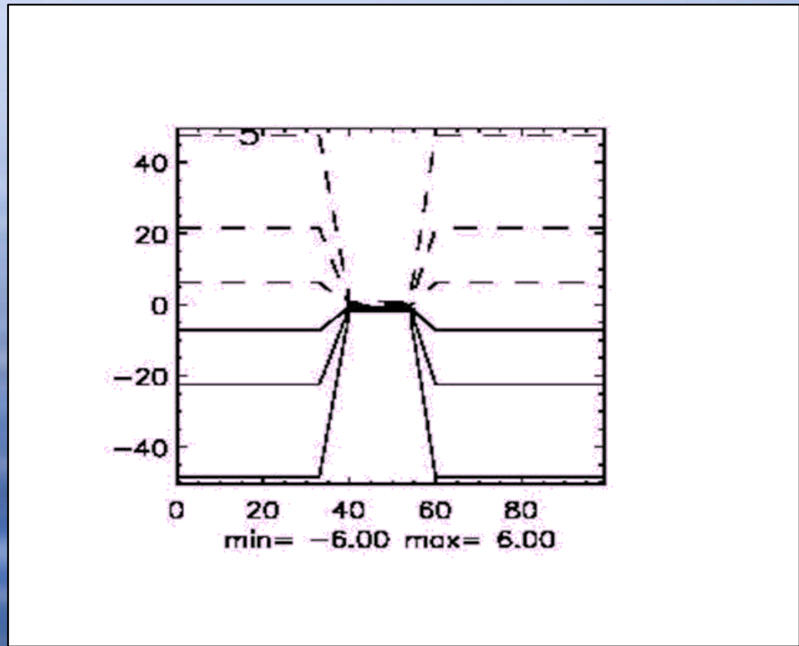
Atmospheric impact of mid-latitude SST anomalies: A highly contentious issue





- A quasi-geostrophic (QG) atmospheric model in a periodic β-channel, first barotropic (Feliks *et al*., *JAS*, 2004; FGS'04), then baroclinic (FGS'07).
- Marine atmospheric boundary layer (ABL), analytical solution.
- Forcing by idealized oceanic SST front.

Ocean-atmosphere coupling mechanism (II)

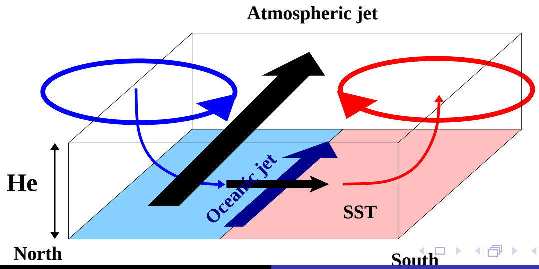
Vertical velocity at the top of the marine ABL

- The nondimensional $w(H_e)$ is given by

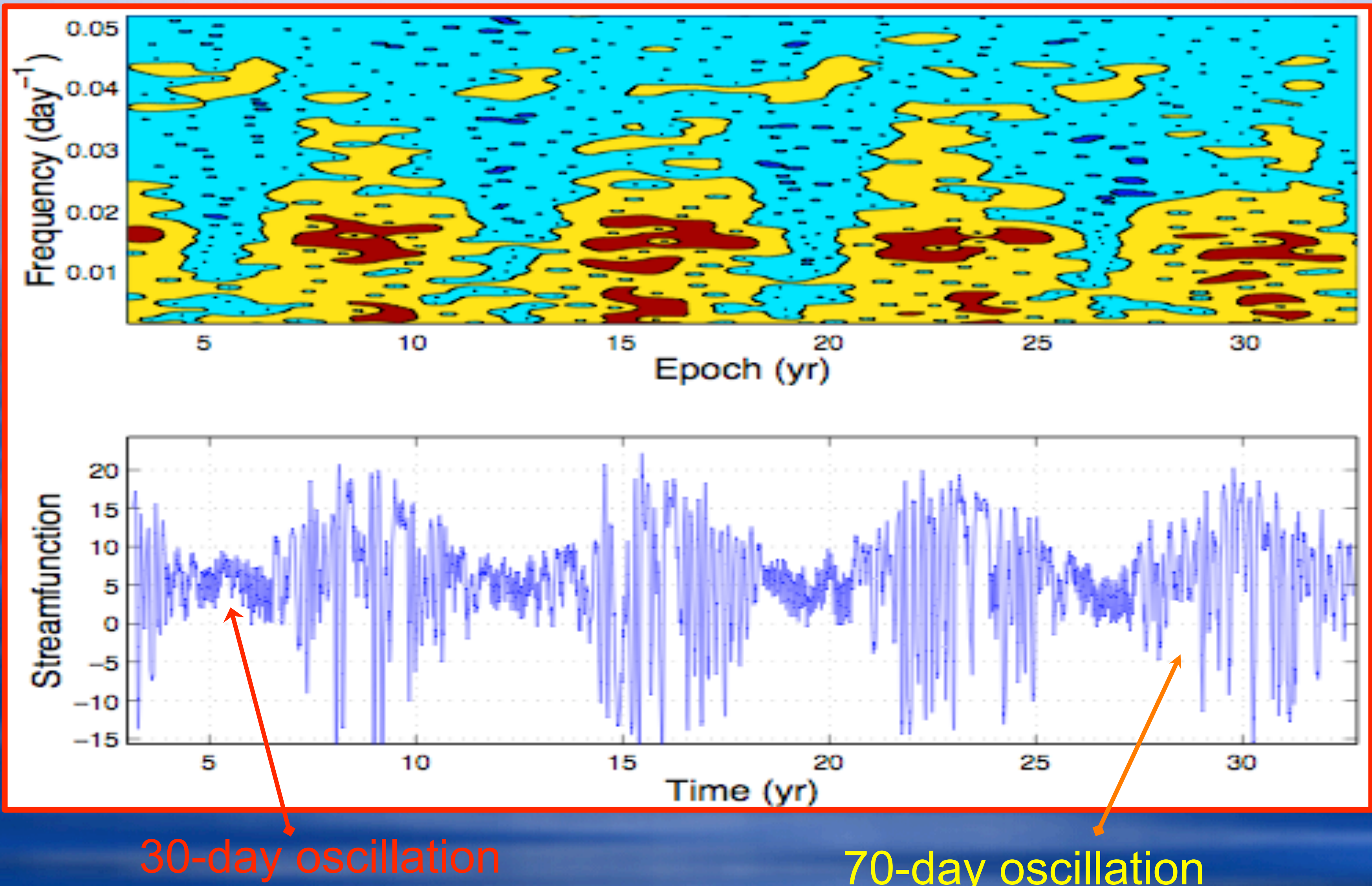
$$w(H_e) = \left[\gamma\zeta_g - \alpha\nabla^2 T\right],$$

with $\gamma = c_1(f_0 L/U)(H_e/H_a)$ and $\alpha = c_2(g/T_0 U^2)(H_e^2/H_a)$, where H_a is the layer depth of the free atmosphere ($\sim$ 10 km), and ζ_g the atmospheric geostrophic vorticity.

- Two components: one **mechanical**, due to the geostrophic flow ζ_g above the marine ABL and one **thermal**, induced by the SST front.



Evolutive spectral analysis



IPCC-class GCM:
LMD-Z has zooming capability

Model set-up

– **19 levels, 3°x3°** outside the zoomed area and **0.5°x0.5°** inside it;

– zoomed area of (20° lat. x 40° long.), centered at (65°W,40°N);

– **perpetual forcing**, corresponding to February 15.

3 simulations, 800-day long:

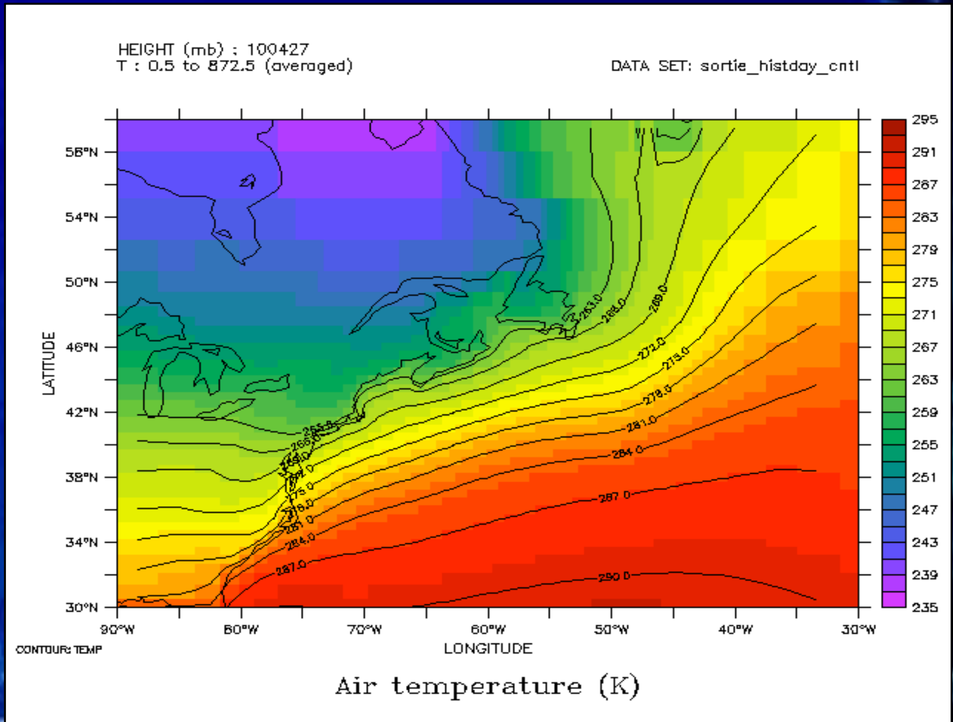
– a control simulation with the climatological SST field and no zoom;

– one with zoom and the climatological SST field still; and

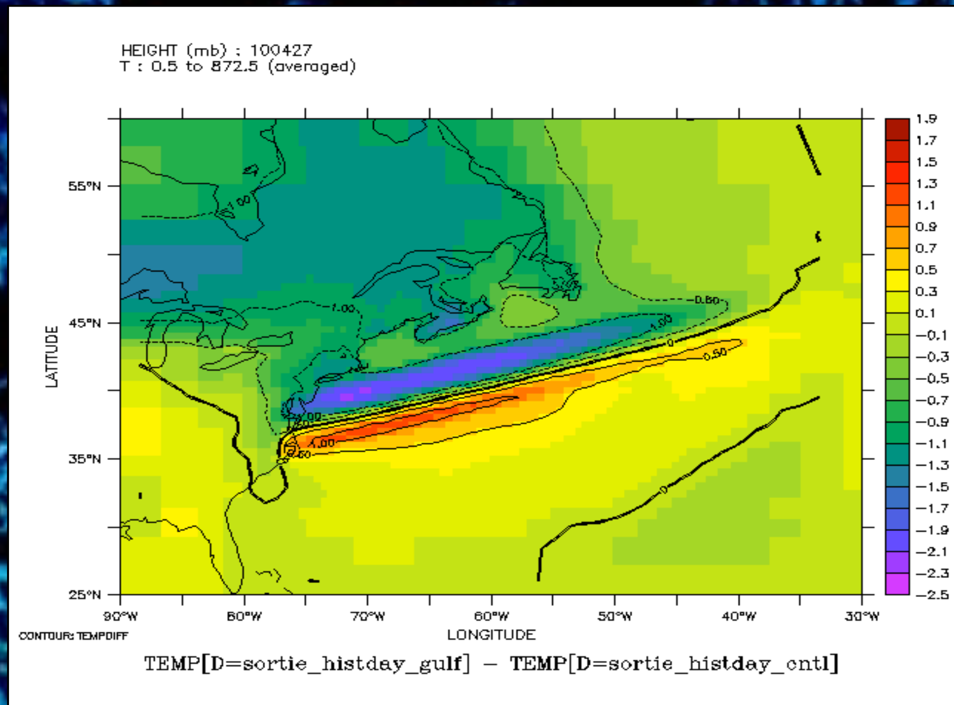
– and one with zoom and a sharper SST front.

IPCC-class GCM: LMD-Z

Climatology



Superimpose $f(T) = 2\cos(x)*(-8)\sin(y)$, for a Gulf Stream that has an axis inclination of 25° to zonal.

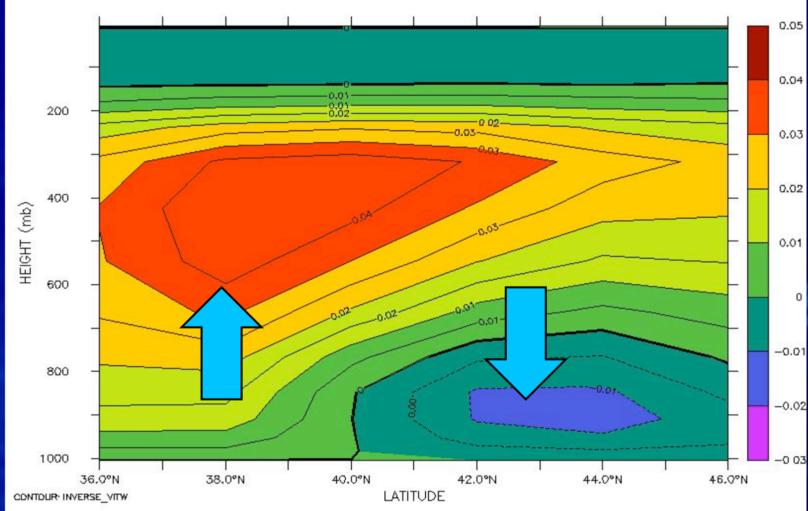


Results

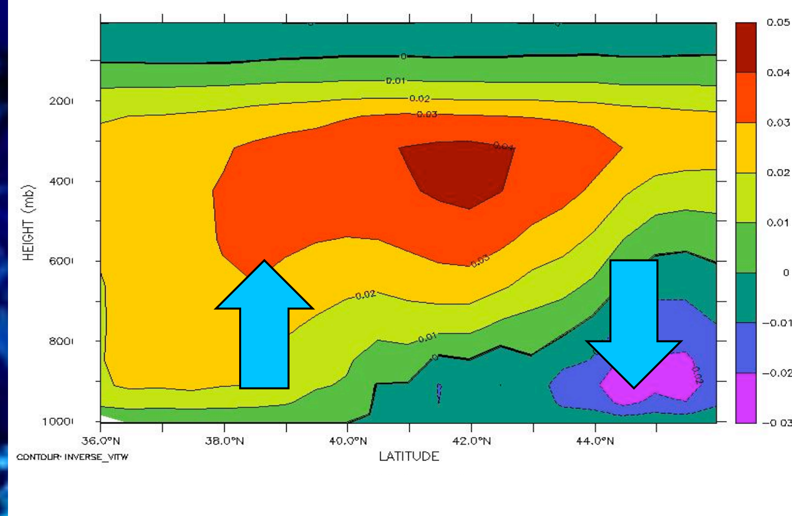
Mean *w* averaged from 70°W to 40°W.

Height *vs.* latitude cross-section; red/blue means +ve/–ve upward velocity.

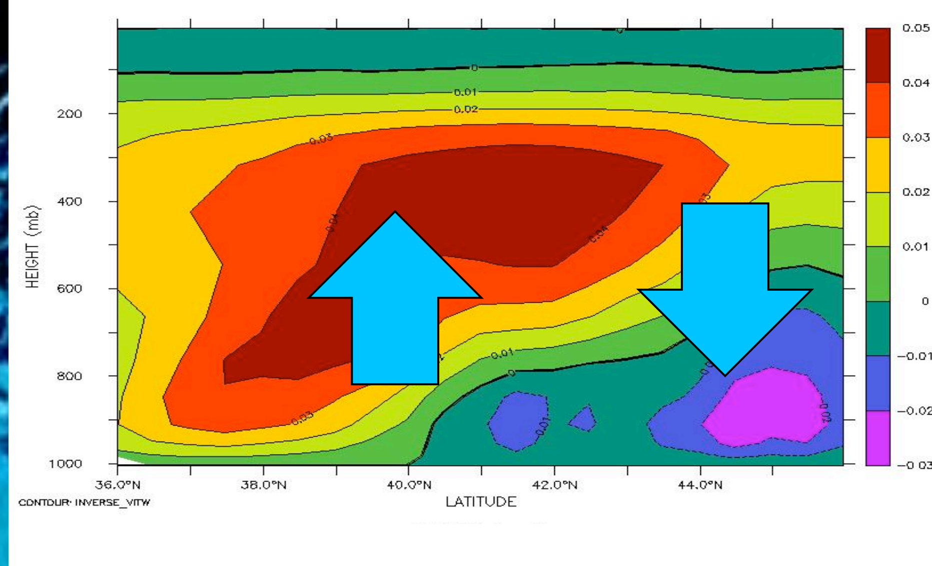
Without Zoom & without SST front



With Zoom & without SST front



With Zoom & with SST front



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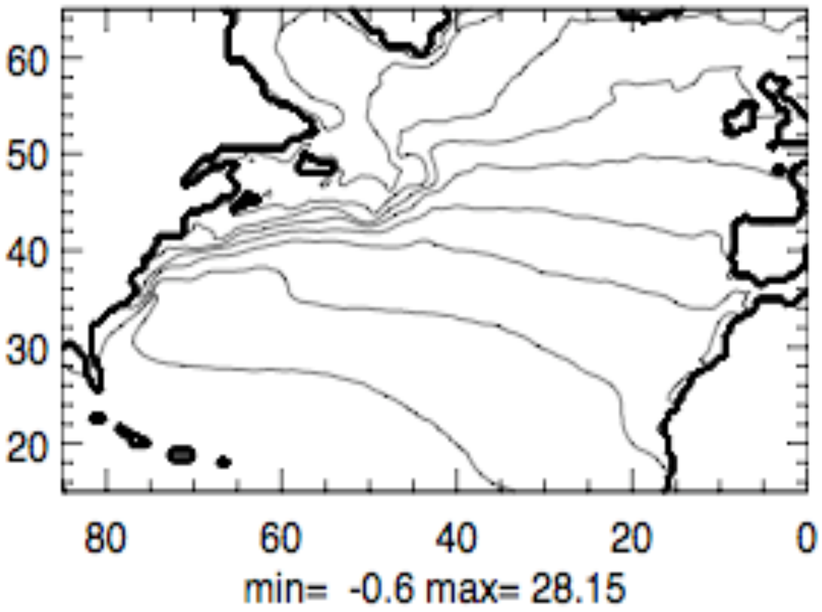
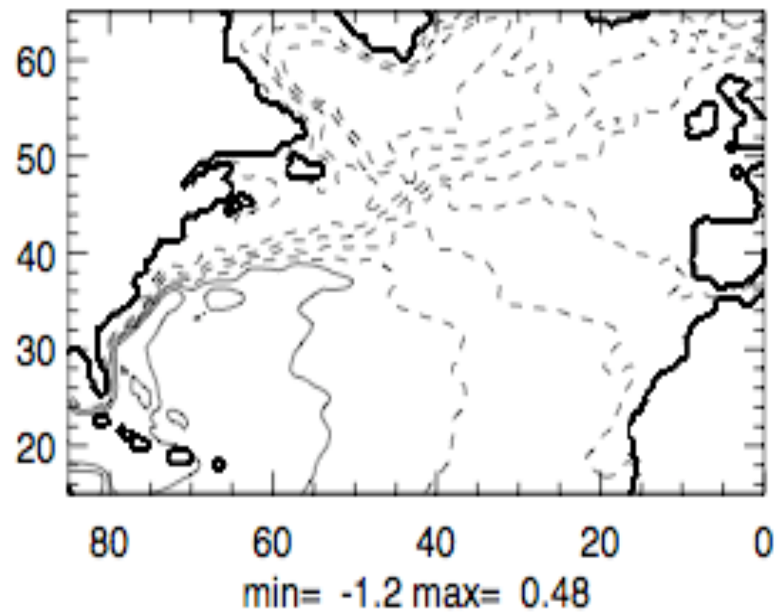
The 7–8-yr mode in oceanic data – I: A still contentious issue

Mean SST field



Mean SSH field

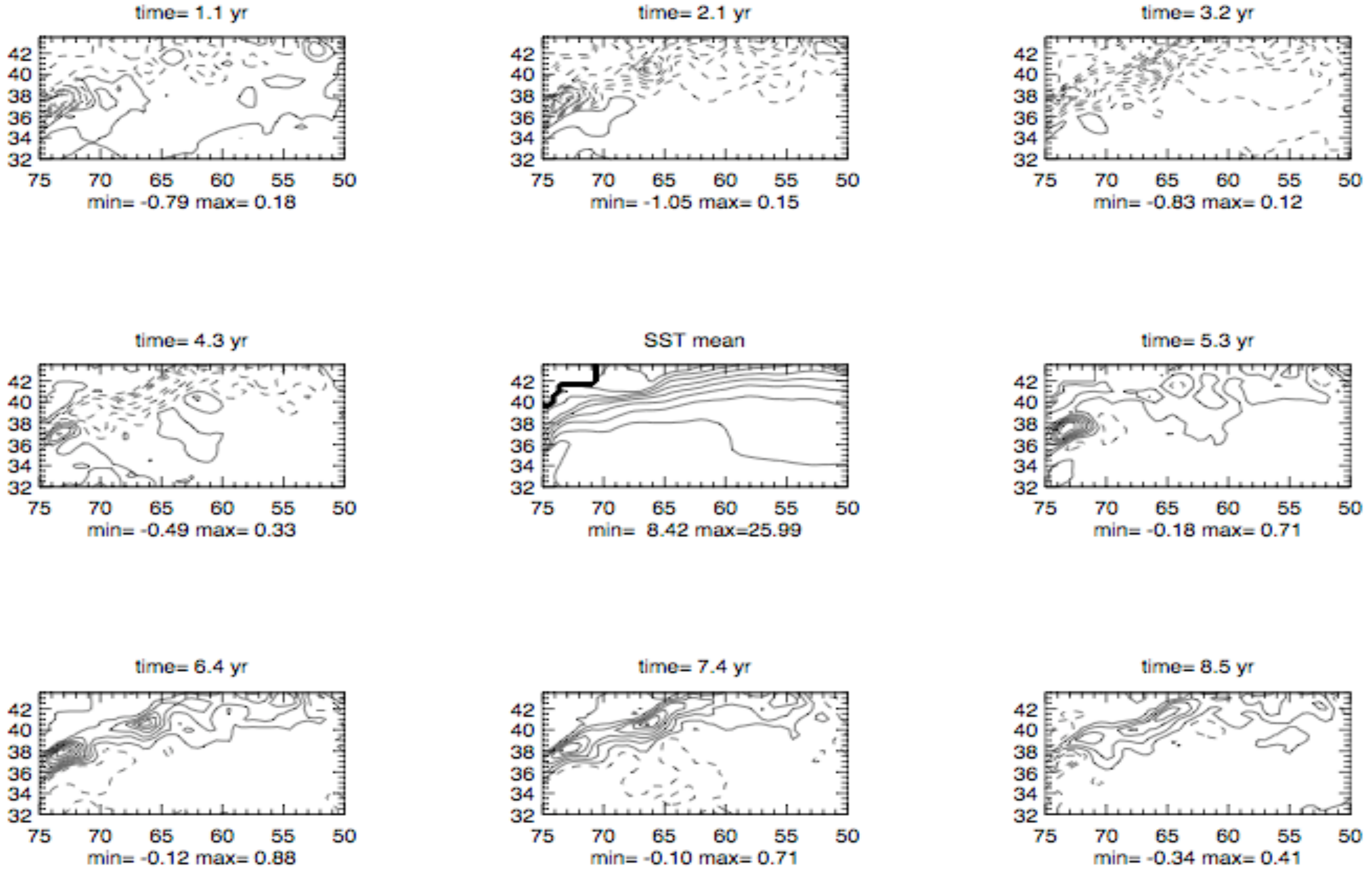


Simple Ocean Data Assimilation (SODA) reanalysis:

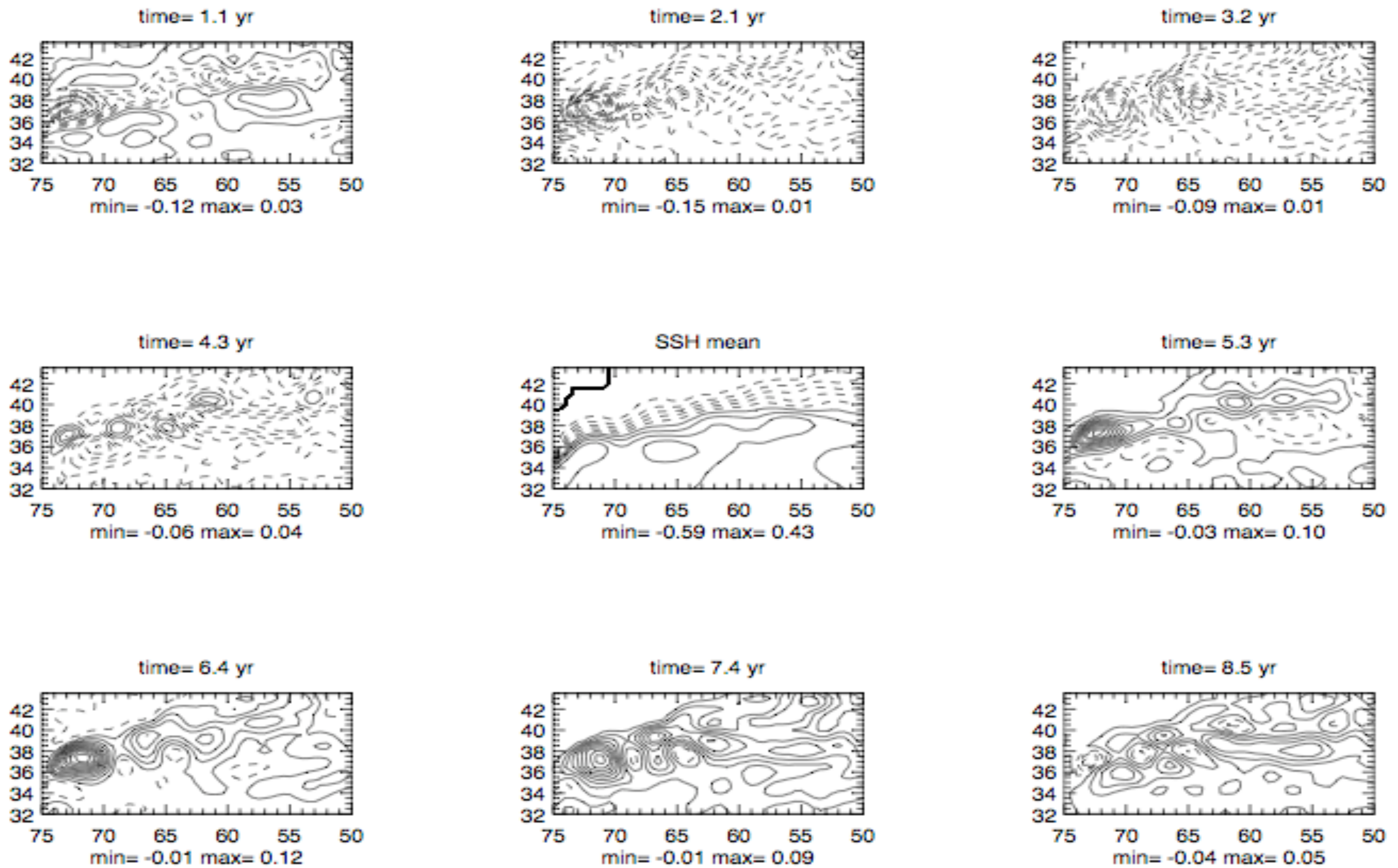
- Western North-Atlantic “rectangle” (28 N–42.5 N, 80 W-67.5 W);
- 50 years = Jan. 1958–Dec. 2007 (Carton and Giese, *MWR*, 2008).

The 7–8-yr mode in oceanic data – II:

The SST field



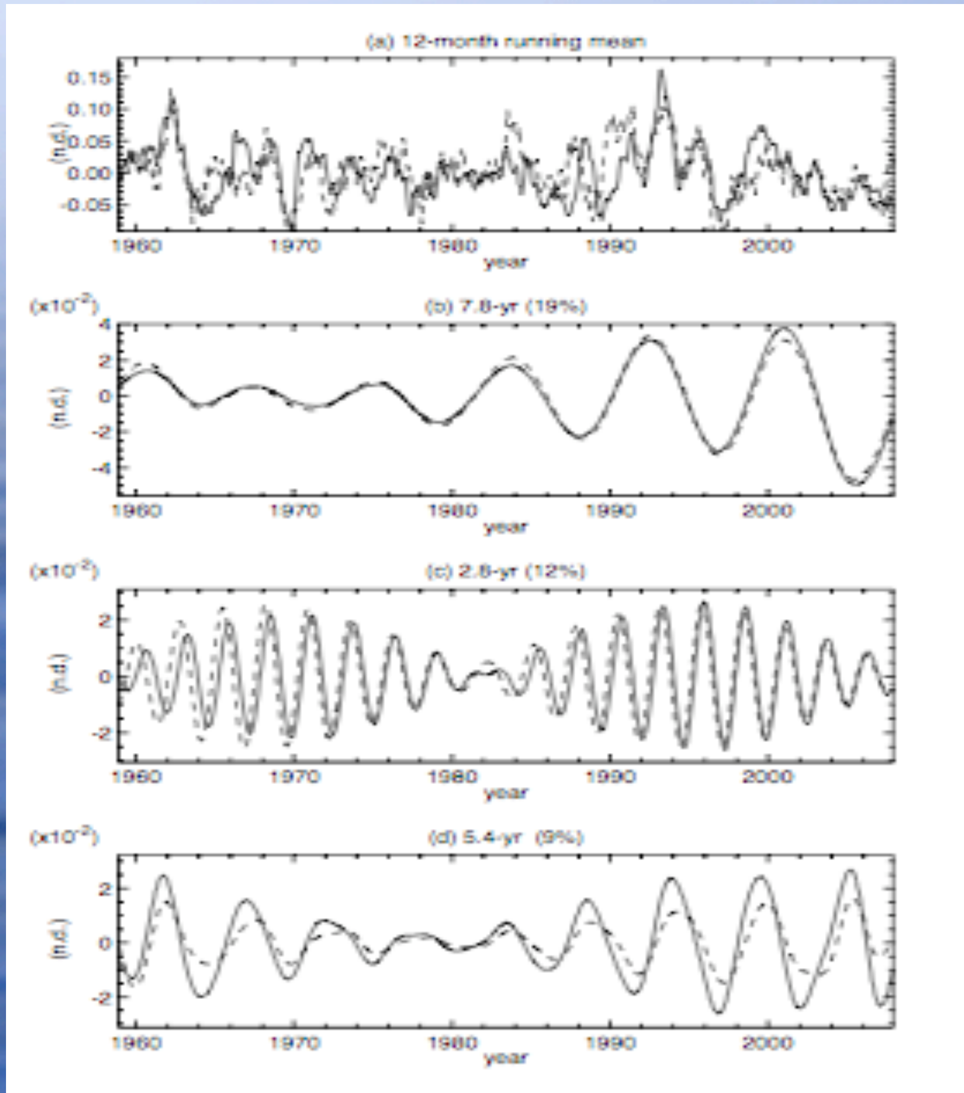
The 7–8-yr mode in oceanic data – III:
The SSH field



The 7–8-yr mode in atmospheric data
Likewise a contentious issue

Simulate atmospheric response to SODA data over the Gulf Stream region

- Use SST (–5 m) data from the SODA reanalysis (50 years)
- Use the FGS'07 QG model in periodic β-channel
  – baroclinic + marine ABL
- Figure shows NAO index:
  – simulated (solid)
  – observed (dashed)



Outline

- Introduction: the NAO and the oceans' wind-driven circulation
- The low-frequency variability of the double-gyre circulation
  - – bifurcations in a toy model
    - => multiple equilibria, periodic and chaotic solutions
  - – some intermediate model results
- Atmospheric impacts
  - – simple and intermediate models + GCMs
- Some data analysis – atmospheric and oceanic
- A very promising coupled O–A model
- Conclusions
  - – The coupled climate system: is it the tail or the dog?
  - – Natural climate variability: a source of decadal predictability
- Some references

The next step in the modeling hierarchy

- Realistic East Coast contour, at -200 m isobath.
- An oceanic QG baroclinic model with four layers and internal Rossby radii from observational dataset (Mercier et al., JPO, 1993).
- Climatological, annual-mean COADS wind-stress forcing (1 deg).
- Realistic bathymetry.
- Transport equation for the SST relaxed to the climatological SST field.
- Full coupling with a QG barotropic atmosphere in a periodic β-channel, with vorticity feedback to the ocean.
- No-slip B.C.s for the ocean at the coasts parametrized following Verron and Blayo (JPO, 1996); free-slip B.C.s elsewhere.
- Neuman B.C.s for SST field, thus ensuring that $\int_\Omega \nabla^2 T \, dx = 0$.
- Free-slip and periodic B.C.s for the atmosphere.

A model of the North Atlantic basin (II)

Gulf Stream (GS) separation and WBC instabilities: issues

- Correct no-slip oceanic B.C.s crucial to obtain separation at Cape Hateras (well-known) $\Rightarrow$ positive vorticity advected into Florida Current.
- Strong inertial flow is necessary to obtain correct GS path (see Chassignet et al., etc.); trade-off between viscosity and wind-stress intensity

  $\Rightarrow$ sufficiently high resolution is necessary!
- Model is sensitive to stratification parameters: too strong baroclinic and/or bathymetric instabilities destroy GS path along Florida coast

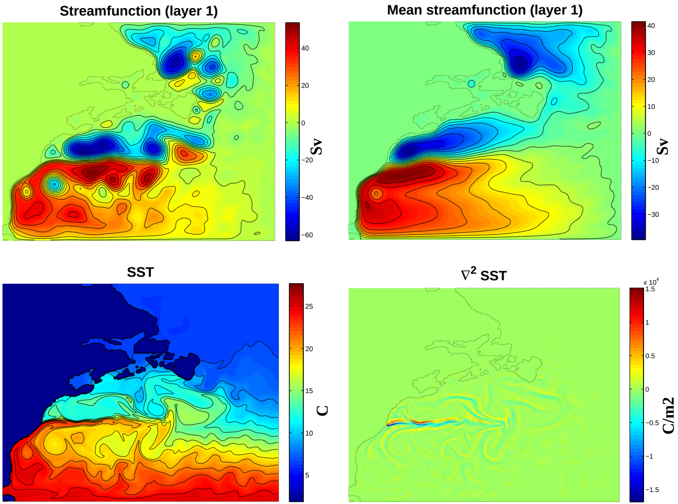
  $\Rightarrow$ barotropization of the GS.
- Occurrence of GS retroflection: true bimodality or model artifact?
- **Correct stratification parameters enhance GS penetration into the ocean interior!**
- Thermal diffusivity is important to insure smoothness of the SST front w.r. to spatial resolution. It also controls the atmospheric jet strength.

QG modeling is far more difficult than in rectangular basins

but it works!

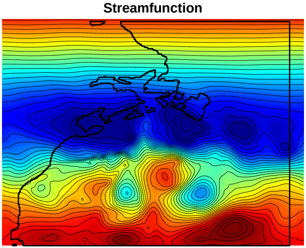
Coupled model results, at (1/9) deg resolution (I)

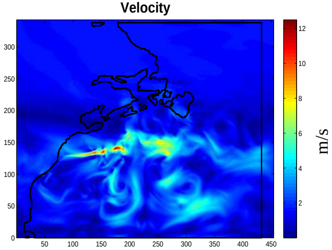
$A_h|_{\text{ocean}} = 200$ m^2/s, $\kappa|_{\text{SST}} = 1200$ m^2/s

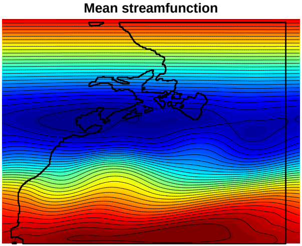


Coupled model results, at (1/9) deg resolution (II)

$H_e = 800$ m, $A_h|_{\mathrm{atmos}} = 400$ m^2/s







Concluding remarks

Summary

- We have a realistic coupled ocean-atmosphere QG model of the North Atlantic basin; 700 000 grid-point variables (ocean + SST + atmos.).
- Coupling mechanism is through Ekman pumping in the marine ABL.
- Persistent, eastward atmospheric jet $\sim 10 m/s$ in the troposphere.
- Atmospheric oscillations with periods of 80 days and 11 months.
- Interannual oscillations in the ocean and atmosphere.

Ongoing work

- Robustness of intraseasonal and interannual oscillations in the model.
- Spatio-temporal structure of the 80–day intraseasonal oscillation.
- Interannual variability in the coupled ocean-atmosphere $\simeq$ NAO?
- Bimodality of the Gulf Stream?
- Baroclinic atmosphere.
- Finer spatial resolution: effects on the Gulf Stream and troposphere.

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Concluding remarks

What do we know?

- There's an NAO, & it's important.
- It has decadal variability (7–8 yr).
- An oscillatory mode, albeit weak, can help prediction.

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What do we know less well?

- How does the climate system really work?
- Is it the tail that wags the dog —

  i.e., weather noise that drives a passive ocean?

•Or does the dog bite its tail –

  i.e., coupled O–A modes of decadal variability?

•Or does the old dog ocean plain wag its tail, the atmosphere?

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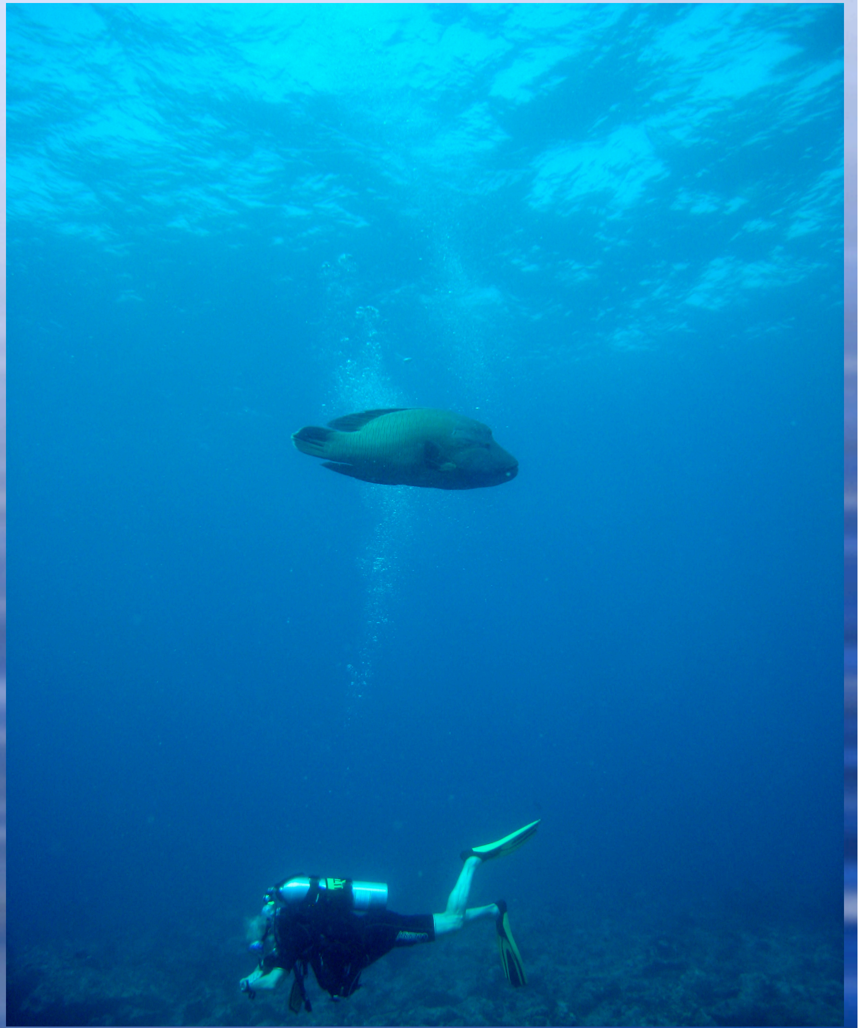
What to do?

- Work the model hierarchy, and the data!

With many thanks to numerous collaborators on related work over 15 years:



and to the 4 most recent ones:

S. Brachet, Y. Feliks,

A. W. Robertson, and

E. Simonnet!

Some general references

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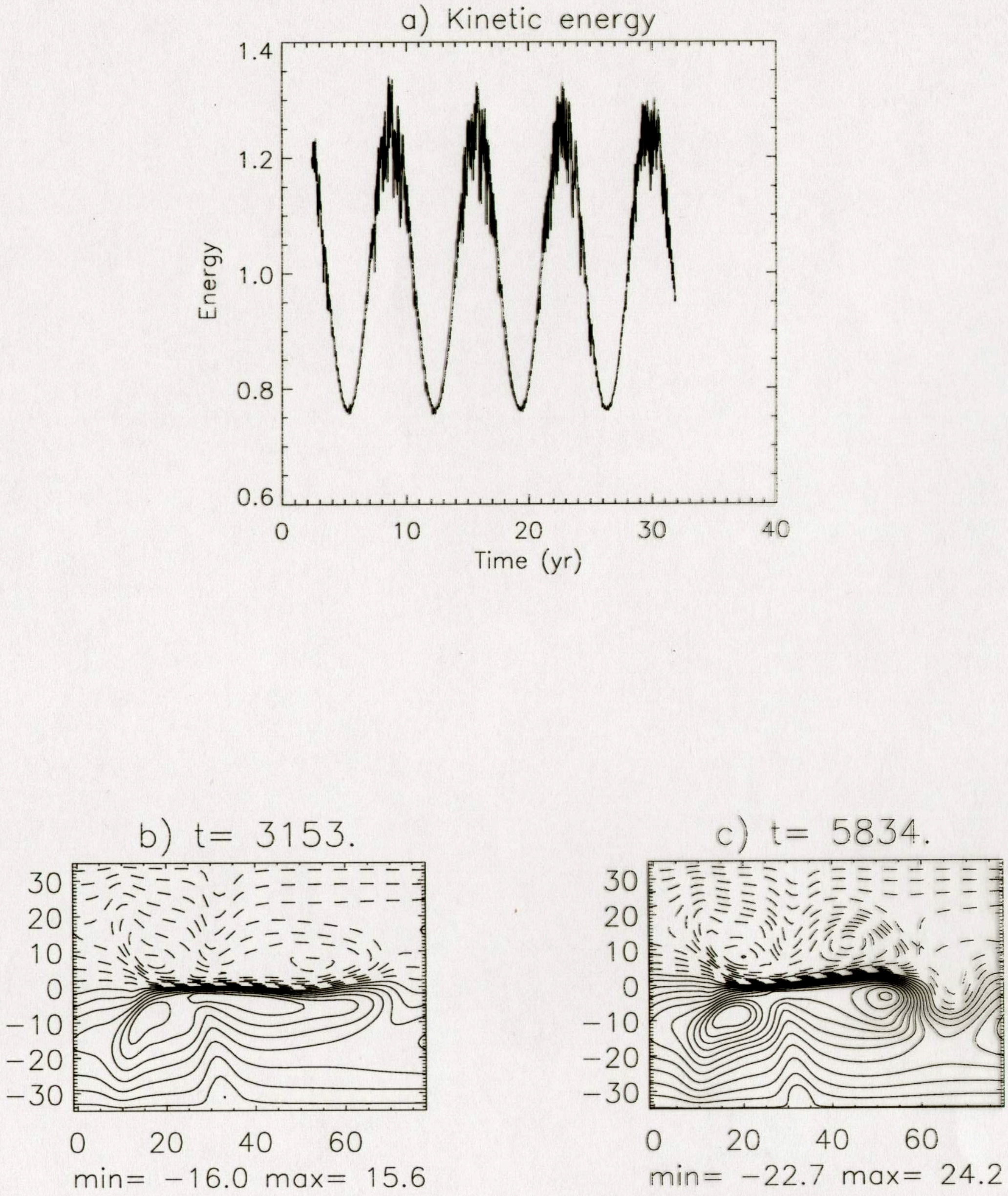
Reserve slides

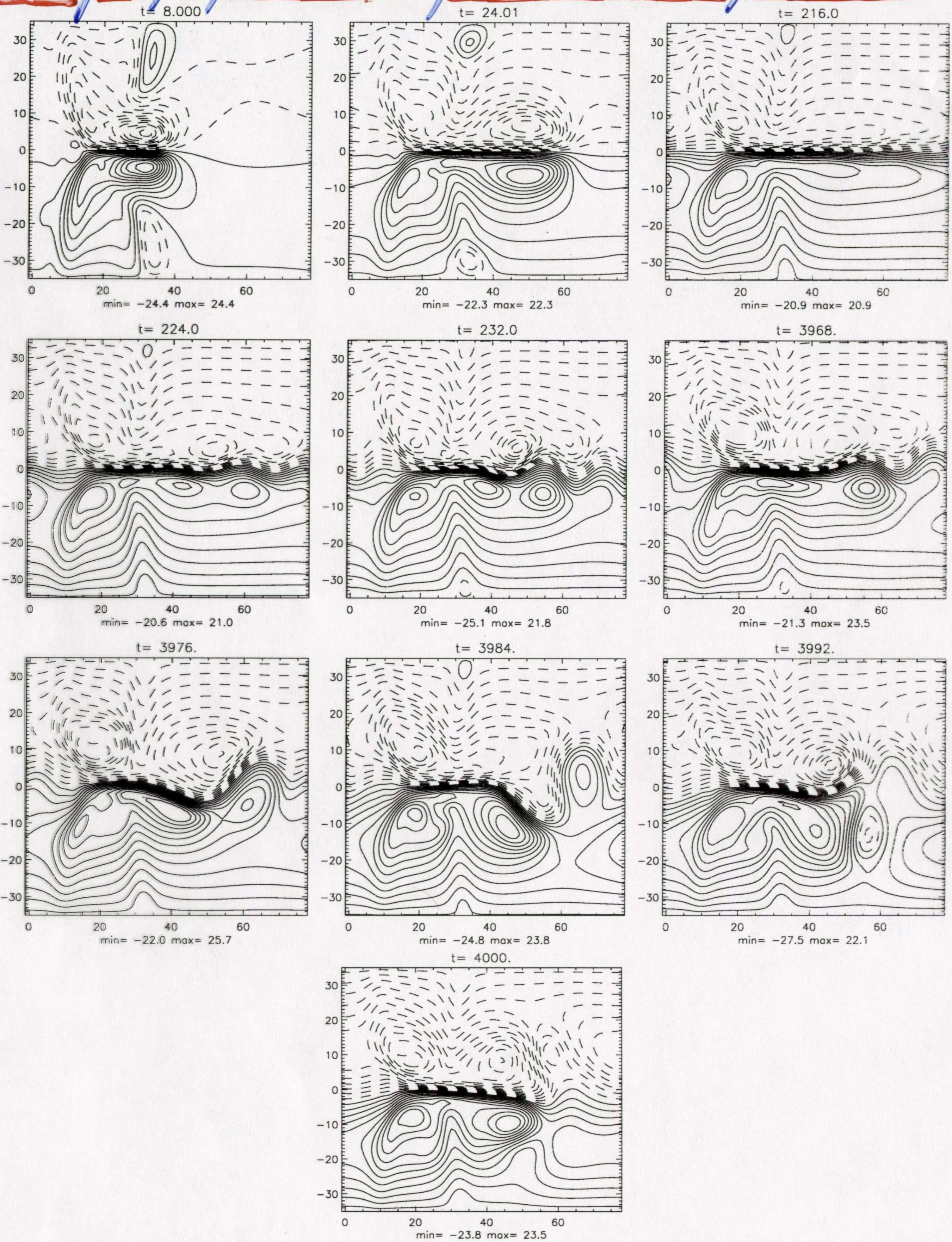
Forced 7-yr cycle



Low-energy phase

High-energy phase

Spin-up of free-atmospheric streamfunction



Appendix. Model equations

Oceanic QG equations ($i = 1, 4$)

$$\partial_t q_i + J(\psi_i, q_i) + \beta \partial_x \psi_i = \nu_i \nabla^4 \psi_i + \delta_{i1} \left(\sigma \gamma \nabla^2 \psi_a + \nabla \times \mathcal{H}(x, y) \right),$$

$$q_i = \nabla^2 \psi_i - S_{ii+1}(\psi_i - \psi_{i+1}) - S_{ii-1}(\psi_i - \psi_{i-1}) + \delta_{i4} c_b \mathcal{B}(x, y)$$

SST equation

$$\partial_t T + J(\psi_1, T) = \kappa \nabla^2 T + \chi(\bar{T} - T)$$

Atmospheric QG equation

$$\partial_t q_a + J(\psi_a, q_a) + \beta \partial_x \psi_a = \nu_a \nabla^4 \psi_a - \gamma \nabla^2 \psi_a + \alpha \nabla^2 T.$$

Can we, nonlinear people, help?

The uncertainties might be ***intrinsic***, rather than mere "tuning problems"

If so, maybe ***stochastic structural stability*** *could help*!

Might fit in nicely with recent taste for "stochastic parameterizations"

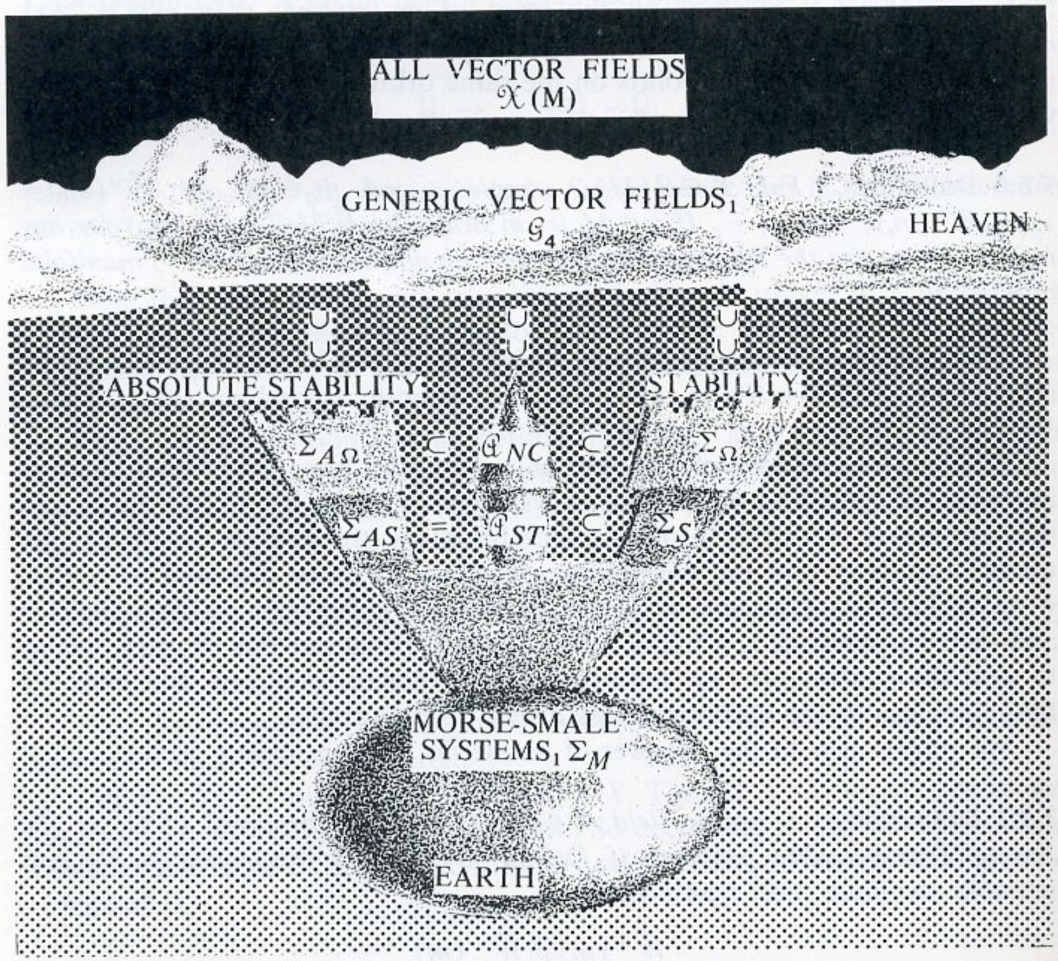


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from *Abraham & Marsden*, 1978)