Geophysical flows as dynamical systems: the influence of Hide's experiments

Michael Ghil, Peter L Read and Leonard A Smith recount the many and various ways that Raymond Hide has influenced their life and work in geophysical fluid dynamics, meteorology, climatology and planetary sciences, as well as in developing the study of dynamical systems in general.

Raymond Hide in his own words – and ours

"There have been some previous theoretical and experimental studies of thermal convection in a rotating fluid, most of which were concerned with the hydrodynamics of the general circulation of the Earth's atmosphere. The theoretical difficulties are great and the experimental work was spasmodic, unconnected and mainly qualitative.

"The author's experiments were stimulated by a desire to contribute to the subject of geomagnetism and the perspective of the work was unexpectedly changed when it was discovered that they have some bearing on meteorology." Abstract of Hide's PhD thesis at the University of Cambridge, UK (1953).

Raymond Hide's PhD thesis was a truly remarkable contribution to our understanding of nonlinear phenomena, not only in experimental work on rotating flows but also across a wide spectrum of fluid and continuum mechanics. It preceded the seminal contributions to the contemporary theory of differentiable dynamical systems and chaos (Lorenz 1963a,b, Smale 1967, Ruelle and Takens 1971a,b) by one or two decades. It certainly attracted considerable and timely attention in the meteorological literature (Lorenz 1967) and inspired substantial work along several lines of research in geophysical fluid dynamics. It is only considerably later that mature researchers from other branches of experimental physics conducted similar experiments to outline the parameter dependence of nonlinear flows, via regime diagrams and the successive bifurcations that explain them (Krishnamurti 1970a,b, Ahlers and Walden 1980, Gollub and Benson 1980, Libchaber 1985).

The authors of this paper, and many of their colleagues, would like Hide's experimental work in the early 1950s to be recognized at last as a true path breaker for the dynamical systems view, not only in the geosciences but in continuum physics in general.

Today it is well understood that the theory of dynamical systems, both finite- and infinite-dimensional (Constantin *et al.* 1989, Temam 1997), provides a powerful way of looking at the nonlinear systems of equations that govern geophysical and other flows and phenomena, from biology to society. But in the early 1950s, when Raymond Hide set out to explain the geodynamo, dynamical systems theory was not well known outside specialized circles of mathematicians. The vision of explaining complex phenomena by "climbing the bifurcation tree", i.e. proceeding systematically from the simple to the complex, via sudden changes in system behaviour, was hardly a glimmer in the eye of those in other fields. This was to change.

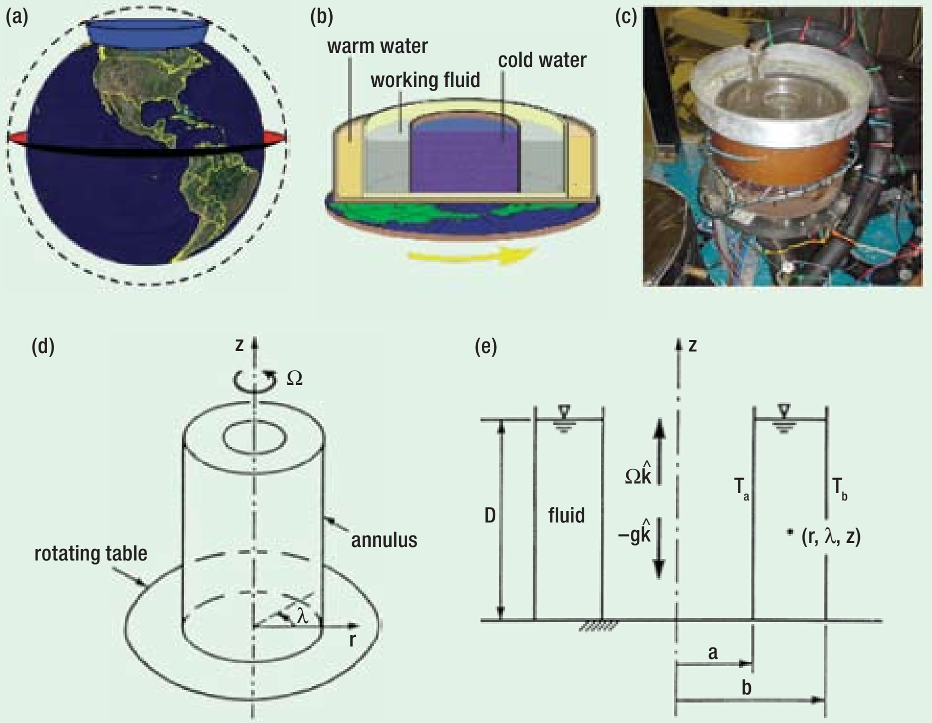
Geophysical fluid dynamics

In the beginning there was the experiment. Lorenz (1967) devotes chapter six, out of a total of eight, of his book on the general circulation of the atmosphere to "Laboratory models of the atmosphere". There are four basic approaches to the understanding of the general circulation: observational, theoretical, experimental and numerical. At present, with the advent of satellites on the one hand and computers on the other, the observational and numerical approaches have gained in importance; evermore human and material resources are invested in these two approaches. But a deeper understanding of the phenomena cannot be obtained without theoretical and experimental work, which benefit from concentrated efforts by individuals or smaller groups.

> **"A deeper understanding of the phenomena cannot be obtained without theoretical and experimental work"**

F Vettin (1857) carried out the first known laboratory experiments on the general circulation, using air as the working fluid and ice in the centre of a cylinder to create a temperature contrast. He obtained a Hadley-type circulation, visualizing the flow using smoke from a cigar. His contemporaries did not appreciate the relevance of these experiments to an understanding of meteorological phenomena, although J Thomson (1892) proposed similar experiments using water instead, but did not carry them out.

After a few more experiments in the first half of the 20th century, D Fultz's group set out to perform a more systematic series of experiments, with the US Air Force as the sponsor, at the turn of the first into the second half of the century. These



1: The rotating, differentially heated annulus and its connection to the Earth's atmosphere. (a): Idealized sketch of the Earth, with a "cold rim" near the North Pole (blue crown) and a warm one near the equator (red). (b): Schematic diagram of the annulus, with cold water in an inner cylinder, warm water at the outer rim of the annular gap, and the working fluid between the two. (c): The actual apparatus. (d): Perspective sketch of the apparatus in panel (b), showing the cylindrical coordinates and the speed of rotation Ω. (e): Cross-section through the annulus in panel (d), with the inner and outer radii of the annulus, *a* and *b*, as well as the inner and outer temperatures, T_a and T_b. There are about 15 distinct parameters that characterize a given experiment, but the two nondimensional numbers that determine primarily the flow pattern are the Hide number $H=\alpha gD(\Delta T)/\Omega^2L^2$, also called the "thermal Rossby number", and the Taylor number $T=4\Omega^2L^5/\nu^2D$; here α is the thermal expansion coefficient of the fluid, *g* the acceleration of gravity, *D* the height of the fluid in the annular gap, $\Delta T=T_b-T_a$ the degree of differential heating that drives the fluid motion, $L=b-a$ the width of the gap, and ν the kinematic viscosity of the fluid.

experiments at the University of Chicago used water in a dishpan, mounted on a rotating table. A heat source was provided near the rim, and in some experiments there was a cold source in the centre. The idea was that the rim simulated the equator and the centre was the North Pole.

Two flow regimes were observed, one that was nearly axisymmetric, called the Hadley (1735) regime, the other wave-like, called the Rossby regime (Fultz 1951, Fultz *et al.* 1959). But the dishpan experiments were hard to control precisely and thus to replicate.

It was a young graduate student in Cambridge, who was interested in explaining the mechanism of the geodynamo, who set up an experimental apparatus that overcame the difficulties of the Fultz team and was destined to be the main progenitor of modern experimentation on the general circulation to this day. This path-breaking 1950–1953 episode is partially and briefly retold in Hide (2010). The apparatus is sketched in figure 1, whose caption also defines the two main parameters that help characterize the flow regime, namely the Hide number *H* and the Taylor number *T*.

Raymond Hide's unusual PhD thesis made at least three fundamental contributions:

- (i) the first documentation of periodic, regular Rossby (1939) waves in a fluid;
- (ii) the discovery of the quasi-periodic vacillation phenomenon; and
- (iii) one of the first, or maybe *the* first, study of what we call today bifurcation and regime diagrams in a fluid dynamical context.

The first of these three contributions solidly established the pertinence of simple theoretical studies of rotating, stratified flows. The second involved, eventually, the study of amplitude and shape – also called tilted-trough – vacillation. The third is an entirely different, and much broader story, discussed in later sections.

The first contribution is much less trivial than it might appear today, when Rossby waves are taken for granted and widely taught. At the time though, given the observed irregularity of atmospheric flows, the possibility of regular waves in a rotating fluid was far less than obvious (Lorenz 1967). Furthermore, the fact that the transition from the Hadley to the Rossby regime occurred by the recently discovered, truly 3-D baroclinic instability of Charney (1947) and Eady (1949), was ~~a further~~ deep insight for geophysical fluid dynamics (GFD). We shall treat the second and third contributions below.

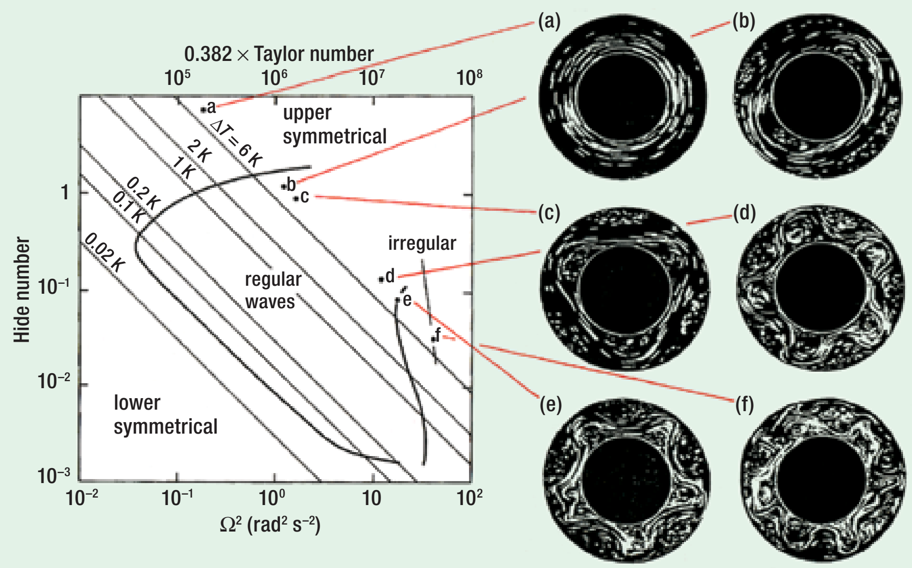
Vortices, radio astronomy and stars

Michael Ghil recalls doing military service (1967–1971) as an officer in the Israeli Navy, at that time headquartered in Haifa, while also serving as an instructor at the Technion-Israel Institute of Technology. In the latter capacity he worked as an assistant for the last graduate course that Sydney Goldstein taught after his retirement from Harvard; the course was based on Goldstein (1960). Ghil was completing an MSc degree in the Faculty of Mechanical Engineering, under Alex Solan, on heat transfer through a Rankine vortex as a model of the vortices in a Karman vortex street behind a cylinder (Goldstein 1966, Ghil and Solan 1973).

When he could get away from his naval duties, Ghil ran to the Technion library to see whether anybody had solved the MSc problem of his choice in the meantime. It was in the process of rummaging for results on vortices – any vortices – that he first stumbled on the work of Raymond Hide. This geophysical fluid dynamics connection provided later on – while Ghil was a PhD student at the Courant Institute of Mathematical Sciences in New York City, under Peter Lax – part of the motivation for taking a summer job at NASA's Goddard Institute for Space Studies and losing himself forever in the atmospheric and oceanic sciences.

Shortly after completing his PhD thesis, Ghil met Hide in person at the Summer School on Rotating Fluids in Newcastle upon Tyne (Hide 1977/1978, Roberts and Soward 1978). Hide kindly invited him right after that to the lab he was running at the time at the UK Meteorological Office in Bracknell, to observe some of the experiments (Ghil 1978) and obtain invaluable advice for his research and the life built around that research. As a result, Ghil invited Hide to lecture at a Summer School on Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics in Varenna, Italy (Ghil *et al.* 1985, Hide 1985) and subsequently dedicated chapter V of his next book (Ghil and Childress 1987) to Hide's experiments and Ed Lorenz's analysis thereof (Lorenz 1962, 1963b).

Peter Read joined Raymond Hide's research group at the Met. Office in 1980, shortly after completing a PhD in radioastronomy with Martin Ryle's group at Cambridge. This was a particularly interesting and exciting time, very soon after the Voyager spacecraft had flown past Jupiter and returned astonishing images and measurements of Jupiter's Great Red Spot, its equatorial jet and other features about which Hide had theorized since the 1960s. It was also just at the time when theoretical ideas relating to autonomous dynamical systems were making a transition from the arcane world of pure



2: Flow patterns and their localization in the plane of the most important dimensionless control parameters (*H*, *T*). (Left): Experimental regime diagram, in which *T* is replaced by ~~a stability~~ Ω^2. (Right): Collage of upper-level flow patterns, visualized as streak trajectories, from panel (a) to panel (f).

mathematics and starting to make a significant impact on a number of applied sciences, notably experimental physics and meteorology.

One of Read's first recollections of Hide's group was making the trip from Bracknell in Berkshire, as a newly arrived post-doctoral scientist, to the Mathematics Institute at Warwick University in early 1980. This was to attend an informal workshop in which leading pure mathematicians from Warwick, such as Christopher Zeeman, Ian Stewart and David Rand, sought to educate curious, though (at the time) somewhat ignorant, physicists and applied mathematicians in the mysteries of low-dimensional attractors and chaos, and to convince them of their significance for complex nonlinear phenomena in the "real world". Though well-meaning, communication between these disparate groups was not easy, because it seemed to many of the physicists that the mathematicians were expending inordinate amounts of effort proving aspects such as the existence and uniqueness (or not!) of phenomena and enumerating the roles of symmetries that seemed "bl...ding obvious" to many applied scientists! But despite this clash of cultures, many came away with a determination to explore the ideas further in individual research areas.

In Read's case, this led to a programme of laboratory experiments in the following decade in which, under Hide's tutelage, he began to apply some of the ideas and methods discussed at the Warwick workshop to analyse measurements from various rotating annulus experiments explicitly in the context of finite-dimensional attractors and bifurcations. On Raymond Hide's retirement from the Met. Office in 1990, Read inherited much of the laboratory he had nurtured during his time there, and continues the tradition today in the Department of Physics at Oxford University.

Leonard Smith arrived in Oxford in 1992, shortly after the annulus laboratory itself. His first direct encounter with data from the annulus had occurred a few years before, and had been presented in a second conference in Warwick in 1991, alongside new results presented by Peter Read (Read 1992, Smith 1992). He had already learned of the rotating annulus, and Hide by reputation, while completing his PhD under Ed Spiegel at Columbia. In the early 1960s, Spiegel, along with Derek Moore, had concocted a system of three ordinary differential equations that model a horizontally stratified, ionized stellar atmosphere, subject to magnetic and thermal forces (Moore and Spiegel 1965).

Like Lorenz's (1963a) problem, the Moore–Spiegel system also admitted chaotic dynamics for some range of parameter values. Hide's (1953) observations of physical "rapid and complicated fluctuations" relate to Lorenz's (1962) "irregular nonperiodic", Lorenz's (1963a) "deterministic nonperiodic", and Moore–Spiegel's (1965) "aperiodic or irregular" motions in ordinary differential equations; and to chaos with a focus on nonlinearity and instability more generally. Spiegel had worked alongside Hide in Chicago in the 1950s, and suggests that it was Hide who introduced him to some of the more Earthy fluids – gin in particular! In the early 60s, there was real concern that the "interesting" (i.e. chaotic) behaviour in newfangled numerical model integrations might result from numerical errors in finite differencing and hence might not be a real feature of the equations of interest (see, for example, Lorenz 1962).

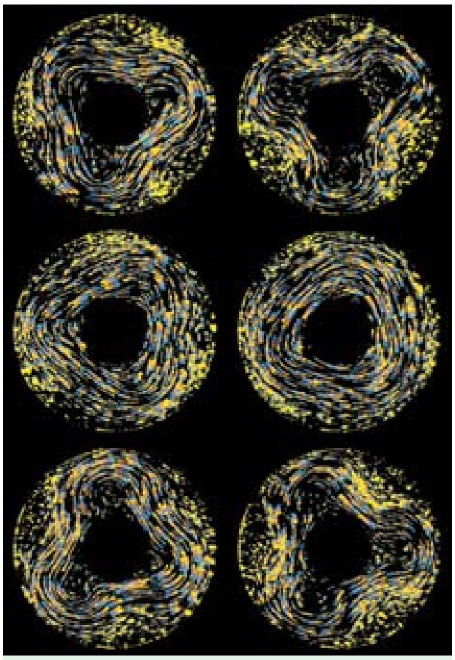
In a sense, there is nothing very exciting about detecting properties that mathematicians consider "universal", especially when the necessary symmetries, as noted above, appear "bl...ding obvious". Smith's interest in the annulus was not only in generic chaos, but also in our understanding of the particular properties of *this* particular annulus, viewed as a dynamical system. At the time, there was much interest in obtaining, from observational or experimental data, a statistical "proof" that a "low-dimensional description" of the dynamics existed; such a description would require only a small number of independent degrees of freedom, a number called the "dimension" d of the system (Grassberger and Procaccia 1983).

Smith conjectured that it required more data to establish that such a description existed than it did to actually provide it (Smith 1997). Indeed, faithful, low-dimensional reconstructions of the dynamics of the rotating annulus were found, and as well as illustrating less generic, but perhaps equally interesting, aspects of nonlinear dynamical systems (e.g. Read *et al.* 1992, Mullin 1993), the annulus inspired many discussions of the advantages and disadvantages of viewing the dynamics in this way. There are significant theoretical and practical advantages in building low-dimensional models for the evolution of annulus flows, in a phase space with $d \sim 10$ dimensions, and considering model trajectories in this phase space; still, there is also much information that is lost with respect to working with much higher-dimensional simulations, based on truncated partial differential equations (Ghil 1978, Ghil *et al.* 1985).

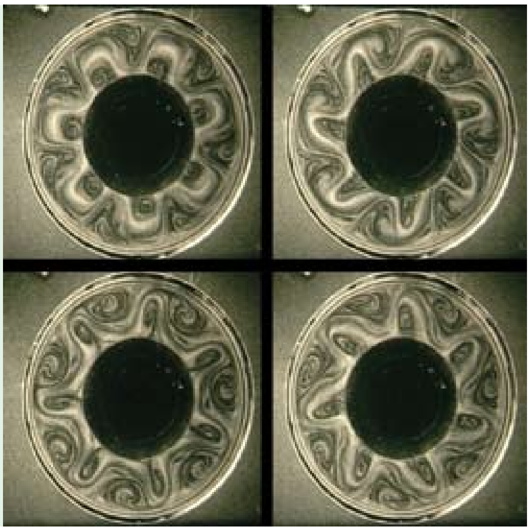
Bifurcations, dynamical systems and chaos

The origins of the modern theory of differentiable dynamical systems lie in the pioneering work of Henri Poincaré in the late 19th century, while in the first half of the 20th century G D Birkhoff in the United States and A M Lyapunov, A A Andronov and colleagues in Russia made many key contributions. Several references can be found, for instance, in Guckenheimer and Holmes (1983). But the interest of physicists for differentiable dynamical systems theory had to wait for connections with some striking experiments to be established.

The authors of this tribute are convinced that Raymond Hide's experiments were certainly among the first and most insightful, and we shall now elaborate. Figure 2a shows a detailed regime diagram, in which the various flow types are separated by sharp boundaries. For a fixed apparatus of height D and gap width $L = b - a$, and a given fluid with expansion coefficient α and viscosity ν, one can change the rotation rate Ω and the temperature difference $\Delta T = T_b - T_a$. The latter involves the two heat baths, outside and inside the fluid gap (see figure 1e), and their



3: Sequence of "streak" visualizations of the horizontal flow near the top boundary in a rotating annulus experiment in the "amplitude vacillation" regime. Flow is visualized from the trajectories of small tracer particles suspended in the flow and illuminated from the side. Streaks are colour-coded with the leading end of the streak coloured yellow and the trailing end blue. The panels are separated by a time interval of 75s, going from left to right and top to bottom. They show the successive quasi-periodic growth, decay and regrowth of the amplitude of the dominant baroclinic (Rossby) wave, while the whole pattern slowly drifts anticlockwise around the apparatus (in the rotating frame).



4: Sequence of visualizations of the horizontal flow near the top (free) surface in a rotating annulus experiment in the "tilted-trough vacillation" regime. Flows are visualized using a rheoscopic suspension of tiny platelet particles that align themselves with the shear in the flow, so that when illuminated from the side they pick out regions of strong and weak shear. The panels are separated by a time interval of around 20s, in the same order as in figure 3. They show the successive quasi-periodic forwards and backwards tilt of the dominant wave, while the whole pattern slowly drifts anticlockwise around the apparatus (in the rotating frame). (Adapted from a movie taken by R Pfeffer, G Buzyna and R Kung, Florida State University, USA)

temperatures T_b and T_a require a relatively long time to stabilize, while the former only depends on the motor that makes the apparatus rotate. Because we have $H \sim \Delta T/\Omega^2$, while $T \sim \Omega^2/\nu^2$, setting ΔT as constant and gradually increasing Ω corresponds to moving along a given straight, downward-slanting diagonal, to the right and down, in a log–log diagram of H vs T, such as that in figure 2a.

Figure 2b illustrates the main flow regimes found in the rotating, differentially heated annulus of figure 1. There are essentially five distinct qualitative regimes, which are further described in figure 5b below: (H) the axisymmetric Hadley regime (figure 2a), in which the fluid rises close to the outer rim, sinks close to the inner rim, and is deflected by the Coriolis force as it flows near the surface from the outer to the inner rim; (R_a) the purely periodic Rossby regime (figure 2b and figure 2c, with azimuthal wave number $k=2$ and $k=3$, respectively), in which waves characterized by a low wavenumber k (roughly $2 \leq k \leq 6$) and by a fixed shape and amplitude travel around the annulus; (R_b) the quasi-periodic Rossby regime (figure 2d), in which either the amplitude or the shape of the waves changes in a periodic fashion (see figures 3 and 4 below); (T_r) a transitional regime (figure 2e), in which the principal wavenumber is no longer an integer or even constant in time and the motion is somewhat irregular; and (T_{qg}) a turbulent regime (figure 2f), in which waves are still recognizable, but the flow is quite irregular. The last two regimes look more strikingly like large-scale atmospheric flows in the Earth's atmosphere, though we now know that the second and third regimes bear a strong resemblance to similar large-scale flows in the atmosphere of Mars.

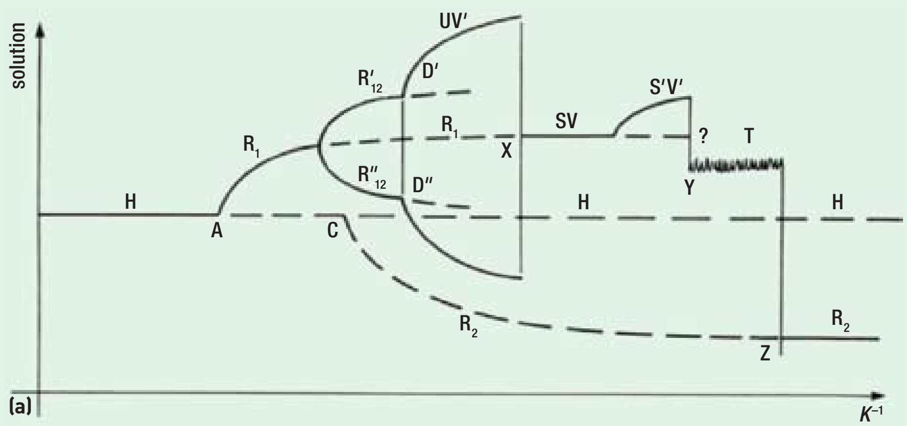
Figure 3 shows a sequence of flow patterns that illustrate the so-called "amplitude vacillation" regime, in which the flow is dominated by a single wave that drifts around the annulus and whose amplitude grows and decays periodically in regular cycles. The period of the amplitude cycle is distinct from that of the drift cycle. When the two periods are rationally unrelated, the motion is termed quasi-periodic in differentiable dynamical systems (DDS) theory.

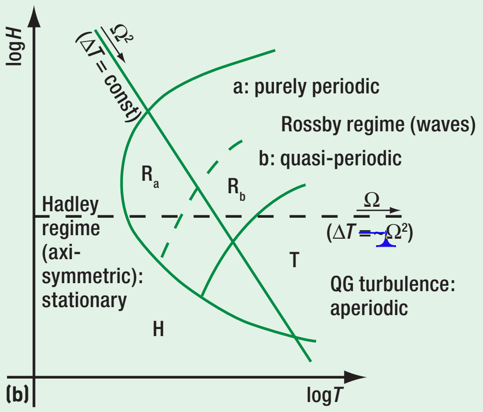
The other main kind of "vacillation" is illustrated in figure 4, which shows a sequence of flow pattens during a "tilted trough" or "structural" vacillation. In this case, the amplitude of the main wave doesn't change much, but its structure – in particular its tilt in the radial direction – changes back and forth with time. As indicated before, the discoveries of the amplitude and tilted-trough vacillation associated with the quasi-periodic Rossby regime (R_b) were very important discoveries in GFD, and they had major implications for the theory of the general circulation of atmospheres and oceans. The former was shown to be largely due to baroclinic phenomena associated with the modulation of the transport of heat by the waves, and hence the rate of release of potential energy; the latter is mainly due to the nonlinear interaction of barotropic waves and thus entails direct exchanges of kinetic energy. Both changes in amplitude and tilt of atmospheric waves play a key role in the eddy transport of heat and momentum across latitudes, as well as in the establishment of "persistent anomalies" or "blocking" in the mid-latitudes.

Lorenz (1962, 1963b) chose a highly simplified model of Hide's experiment, in which a periodic channel in Cartesian coordinates replaced the annular gap, and the flow was governed by the two-layer quasi-geostrophic equations. Furthermore, these partial differential equations were projected onto an orthonormal basis of sine and cosine functions. Lorenz made the ingenious choice of limiting the truncated set of basis functions to a total of six components: zonal flow and a single wave (represented by a sine and a cosine function, in quadrature), of arbitrary wavenumber n in the zonal direction x, and two wave numbers, called modes, with $m=1, 2$, in the meridional direction. His analysis is reviewed, in the now widely accepted language of successive bifurcations, in chapter V of Ghil and Childress (1987), and the results are sketched in figure 5a here.

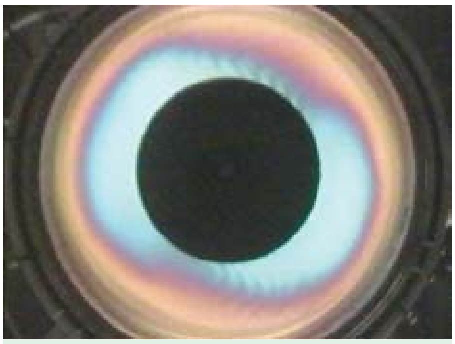
The sharp transitions in flow regime obtained by Hide in his thesis and the many follow-up experiments with similar apparatus can indeed be explained in this language. The transition from the steady, axisymmetric Hadley regime (H) to the purely periodic, steady-wave regime (R_a) corresponds to a Hopf bifurcation, in which a stationary, equilibrium solution transfers its stability to a periodic one, while the transition from (R_a) to the vacillation regime (R_b) is a secondary Hopf bifurcation, from a simply to a doubly periodic solution. Finally, the transitional regime (T_r) corresponds to a form of what we now call deterministic chaos or weak turbulence, while (T_{qg}) is fully developed, albeit quasi-geostrophic, turbulence.

The first two bifurcations are so-called local bifurcations, whose analysis only requires suitable linearization about the equilibrium – called a fixed point in DDS theory – or the simply periodic solution, called a limit cycle, and they are well captured by Lorenz's (1963b) model. The





5: Lorenz (1962, 1963b) model solutions for the rotating annulus experiments. (a): Schematic bifurcation diagram: solution behaviour as a function of model parameter $K^{-1} \sim T$, showing local bifurcations, such as the transition from the steady-state Hadley regime (H) to a single-wave Rossby regime (R_1, stable, R_2, unstable), and on to a mixed-mode, singly-periodic regime (R'_{12} or R''_{12}). (b): Schematic regime diagram in the plane (log$H$, log$T$) plane, showing the regimes H, R_a, R_b, T_r and T_{qg}. (Both adapted from chapter five in Ghil and Childress 1987)



6: Visualization of interfacial baroclinic Rossby and gravity waves in a rotating annulus experiment with two immiscible fluid layers. Yellow represents upwards, and blue downwards, displacements of the interface. The large-scale two-lobed wave pattern is the quasi-geostrophic, baroclinic Rossby wave, while the shortwave ripples at 12 o'clock and 6 o'clock represent gravity waves generated spontaneously within the flow (see Williams *et al.* 2005 for details).

latter two transitions, though, are not so well captured, for two reasons. First, they are nonlocal bifurcations, which involve the presence of so-called homoclinic or heteroclinic orbits; such orbits are structurally unstable, i.e. small perturbations of the differentiable dynamical systems destroy them. Second, they require an increase in the number of basis functions – i.e. in the model's dimension – in order to capture the more complex spatio-temporal behaviour. Based on the theoretical considerations above, we sketch in figure 5b a simplified regime diagram of flow in the rotating, differentially heated annulus.

Variations on the rotating annulus

Ed Lorenz emphasized – in his well-known monograph on the general circulation of the atmosphere (Lorenz 1967) – that a key contribution of laboratory experiments such as the rotating annulus was to enable one to distinguish the most fundamental factors determining the properties and behaviour of the atmosphere and climate from factors that are of lesser significance. Hide (2010) further highlights this contribution in the present issue. The simple experiments first studied in detail by Raymond Hide and Dave Fultz in the 1940s and 1950s clearly show that cyclones, anticyclones, jet streams and fronts do not require for their existence the presence of continents and oceans, moisture, mountains or even the spherical curvature of the Earth: only the basic ingredients of an equator-to-pole contrast in heating and cooling and sufficiently rapid background rotation of the planet are truly of the essence.

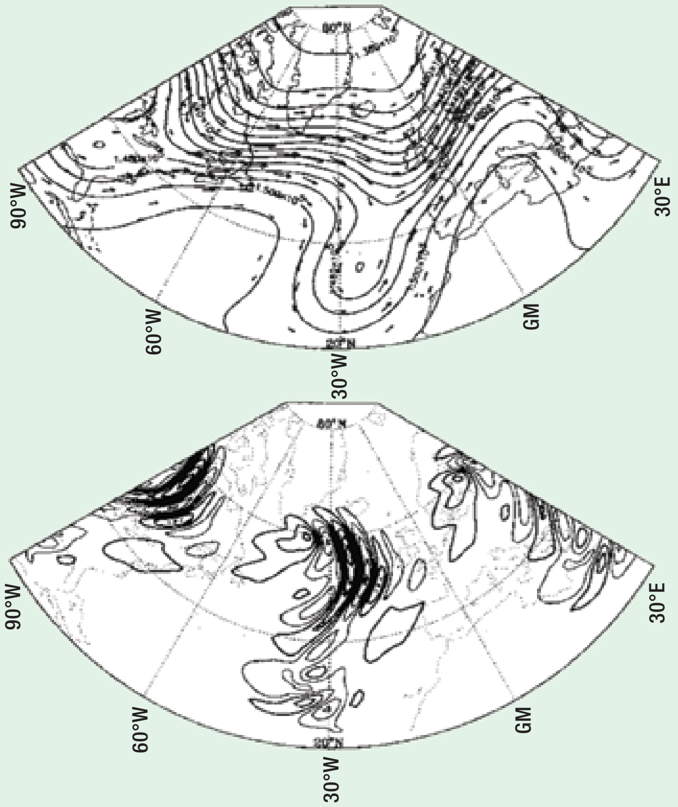
In later research following on Hide's pioneering experiments, however, much work has been devoted to investigating the impact of re-introducing at least some of these less essential factors – in particular those that can be emulated in the laboratory fairly easily – as well as unravelling some of the quantitative details of the bifurcations and flow regimes of the original experiments.

> **"It is remarkable that this simple lab analogue of large-scale atmospheric motion still challenges and surprises us"**

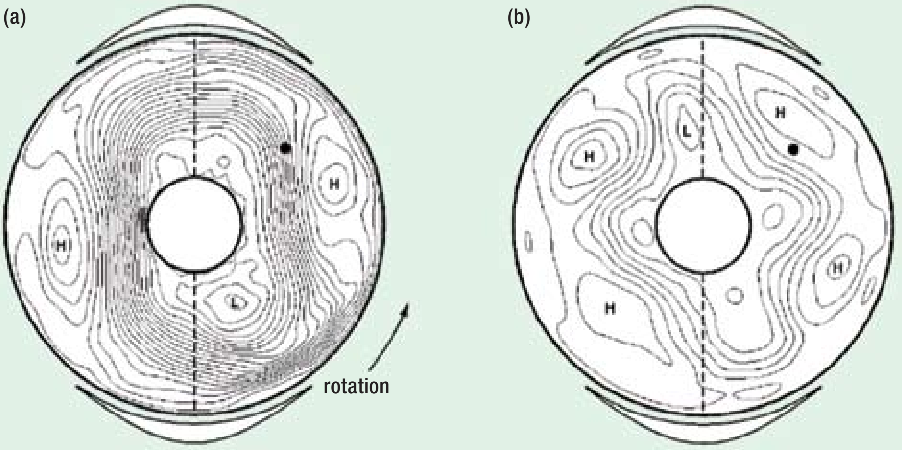
Some of the low-dimensional flow regimes predicted by simple models, such as those of Lorenz (1962, 1963b), turn out to be reproducible in small areas of the parameter space in the laboratory, though not all. The latter shortcoming is because the simple models expand the spatial structure of the flow into a set of orthonormal basis functions, and then truncate this set fairly aggressively to result in a tractable set of ordinary differential equations. The retention of only a small subset of modes implicitly assumes that the rest of the spectrum will either be damped out by friction or that the behaviour of the heuristically neglected modes will be dominated by (or "enslaved to") the behaviour of the dominant, retained modes. Laboratory experiments have shown that this will happen only if the flow is not too strongly baroclinically unstable; still, it is of of major interest that low-dimensional, quasi-periodic and chaotic flows do occur in a real fluid, thus validating the modeling approach of B Saltzman (1962), Lorenz and others. For more unstable flows, however, in the transitional region (T_r), the behaviour is no longer low-dimensional but energy begins to spread across many more spatial modes, and thus the transition to (quasi-geostrophic) turbulence eventually energizes a wider and wider range of spatial scales to produce a highly complex flow in the fully irregular (T_{qg}) regime.

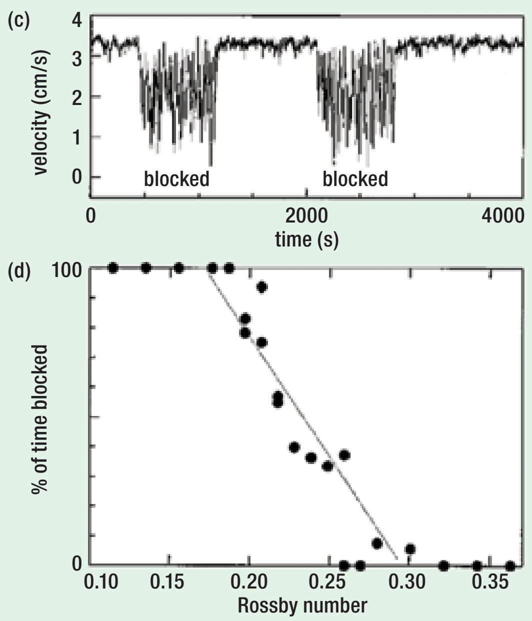
Several other, less essential, factors have been successively introduced in recent variations on the theme of the rotating, differentially heated annulus experiment (e.g. Hide and Mason 1975). These factors include the addition of radially sloping lower or upper boundaries, to emulate some of the effects of the spherical curvature of a real planet, variations in the radial and vertical distributions of heating and cooling, and the addition of non-axisymmetric topography; the latter includes full and partial radial barriers, to emulate the effects of mountain ranges, valleys and even coastlines on large-scale atmosphere and ocean circulation systems. These experiments have confirmed that such additional effects often only modify the details of the main baroclinic waves, though in extreme cases they can change the precise parameter values under which the Rossby wave regimes occur in the experiment.

After more than 50 years, it is remarkable that this simple laboratory analogue of large-scale atmospheric motion continues to challenge and surprise us in ways that may be both topical and visually striking. As an example, figure 6 shows a remarkable visualization of a baroclinic



7: Snapshots from a numerical simulation of baroclinic cyclones and anticyclones in the Earth's atmosphere. The upper panel effectively shows the pressure field in the middle of the troposphere, around an altitude of 5000 m, while the lower panel shows the two-dimensional, horizontal divergence field ∇.u at the same time and level. The pressure field is dominated by the pattern of large-scale cyclones and anticyclones, whereas the divergence field shows the presence of shortwave gravity ripples, generated spontaneously within the baroclinic waves in the atmosphere. (Adapted from O'Sullivan and Dunkerton 1995)





8: Experiments in a barotropically driven rotating annulus (Weeks *et al.* 1997, Tian *et al.* 2001).
(a) and (b): Horizontal streamfunction fields for zonal and blocked flows, respectively; the ridges of the wavenumber-two topography on the bottom of the annulus (dashed lines) are aligned with the vertical.
(c): Azimuthal velocity measurements as a function of time, alternating between the blocked, low-velocity and zonal, high-velocity states.
(d): The fraction of time the flow remains in the blocked state as a function of the main control parameter *R* (the Rossby number). (After Weeks *et al.* 1997)

Rossby wave in a rotating annulus filled with two immiscible fluids: an organic oil in the lower layer and water in the upper. In this case, baroclinic instability shows up as large-scale waves (with $k = 2$) on the interface between the two layers. But the visualization also shows some much smaller-scale ripples in the troughs of the long "planetary" Rossby wave. Further investigation (Williams *et al.* 2005) demonstrates that these shortwave ripples have actually a completely different origin: they owe their existence to the gravitational restoring forces in the fluid and are hence known to meteorologists and oceanographers as "gravity waves".

Such short-wavelength, high-frequency waves do occur in the atmosphere and oceans, and are responsible for phenomena such as clear air turbulence, well known to air travelers, and lee wave clouds associated with mountains and mountain ranges. They are also being increasingly recognized as a significant source of uncertainty in weather forecasting, and may play a major role in the dissipation of energy in the oceans as well. It has been known for many years that atmospheric convection and flow over mountains and valleys can excite these waves in the atmosphere, but it is only recently that meteorologists have discovered that similar gravity-wave trains can be excited by complex nonlinear interactions within large-scale baroclinic cyclones and anticyclones.

These gravity waves are difficult to capture in numerical models because of the large difference in scale between them and the Rossby waves, but high-resolution models are now able to do this: figure 7 illustrates such an example. The resemblance between the "chevron-shaped" waves in the model simulation (figure 7, lower panel) and the annulus experiment (see again figure 6) demonstrates that such beautiful laboratory experiments can provide valuable sources of insight, even for the latest generation of meteorologists!

An important issue addressed by a slightly different type of annulus experiment is the interaction of the large-scale atmospheric flow with topography. Charney and DeVore (1979) formulated and analysed a low-order model for this interaction, in a mid-latitude channel; blocked and zonal flow patterns arise as two stable fixed points of their model. The S-shaped bifurcation diagram associated with this model can be seen in chapter 6 of Ghil and Childress (1987, figure 6.5). Legras and Ghil (1985) carried out a detailed study of a more highly resolved model on the sphere, in which – for reasonable values of the parameters – no stable fixed points were present. Instead, zonal and blocked flow patterns were organized into distinct flow regimes, surrounding an unstable fixed point and an unstable limit cycle, respectively.

The latter results were put to the test first in the traditional, "baroclinic" annulus (Bernardet *et al.* 1990) of Météo-France in Toulouse. Unfortunately, the very active baroclinic waves that dominated this experimental set-up did not allow a direct confrontation with either the theoretical results of Legras and Ghil's (1985) model or with atmospheric observations. Indeed, it was already well known in the early 1980s that so-called low-frequency atmospheric variability, in the frequency band of 10–100 days, is predominantly barotropic (Wallace and Blackmon 1983).

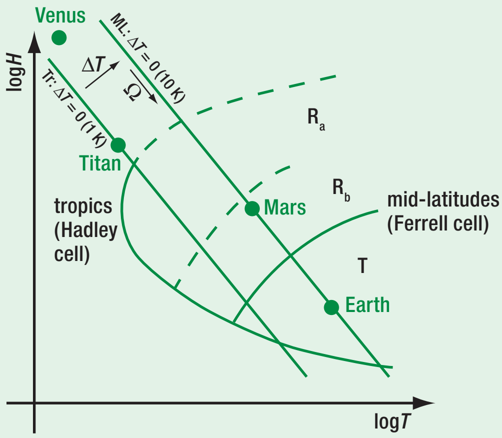
It seemed, therefore, more appropriate to use the "barotropic annulus" of Harry Swinney's research group at the University of Texas in Austin (Sommeria *et al.* 1989). In this annulus, the flow is driven by injection of fluid through a set of holes in the bottom, arranged equidistantly on an outer circle of radius r_1, with $L_a < r_1 < L_b$, and by the suction of the same fluid through another set of holes, lying on a concentric, inner circle of radius r_2, with $L_a < r_2 < r_1$; this pumping of fluid replaces the differential heating of the baroclinic annulus we have discussed so far.

First, some preliminary "shoelace-and-wax" experiments – to quote Sydney Goldstein's advice to Michael Ghil, years before – with clumps of clay on the bottom, did yield blocked and zonal flows in the Austin annulus. Next, a smooth wavenumber-two topography and a more powerful pump were installed, and a careful series of experiments were conducted to generate a regime diagram in the parameter plane of suitably defined Ekman and Rossby numbers, E and R. The results included the coexistence of zonal and blocked flow regimes (figures 8a and 8b, respectively), with residence times of the flow in either regime that changed with the nondimensional parameter values, as predicted by Legras and Ghil (1985). The changes in azimuthal velocity (at fixed E and R) and in residence times, as R changes at fixed E, are shown in figures 8c and 8d, respectively.

We have seen so far that the rotating annulus and its variants have enabled, and continue to enable, the discovery and understanding of a wide variety of dynamical phenomena that are reminiscent of flows in the atmosphere and oceans. Even more fascinating is the possibility of generalizing the results of such experiments to other planetary circulations. As Raymond Hide himself wrote in 1969: "The experiments have emphasized the necessity for truly quantitative considerations of planetary atmospheres. These considerations must, at the very least, be sufficient in the first instance to place the Earth's atmosphere in one of the free non-axisymmetric regimes of thermal convection discovered in laboratory work" (Hide 1969).

In fact it is entirely possible to define dimensionless parameters equivalent to H and T for a given planetary atmosphere, replacing ΔT with the equator–pole thermal contrast, ν with an appropriate "eddy viscosity" and using the planetary radius, a, as the principal length scale (e.g. Read 2010). On this basis, we would expect to find a regime diagram representing the style of atmospheric circulation as a function of H and T, similar to figures 2a and 5b. In such a regime diagram, the Earth's mid-latitude atmosphere might appear towards the bottom right of the diagram, as shown schematically in figure 9, while its tropical atmosphere would appear close to the top left, possibly close to Venus. Other planetary bodies of the solar system with substantial atmospheres or oceans – such as Mars and Titan – could also now be located on this diagram, perhps as indicated in figure 9.

9: Schematic regime diagram for the rotating, differentially heated annulus. The up-left to down-right slanting diagonals indicate isoclines similar to those in figures 2a and 5b, for $\Delta T \sim 10$ K, (characteristic of the meridional gradient in Earth's mid-latitudes) and $\Delta T \sim 1$ K (characteristic of Earth's tropics), respectively. The heavy dots indicate the approximate position of the following planetary circulations: Earth's atmosphere, tropical and mid-latitude, along with Mars, Venus and Titan.



One has been able to explore other atmospheric circulations in the solar system – subject to very different conditions from those on Earth – by using detailed numerical simulations, as well as spacecraft observations, for the past couple of decades. Still, the experimental set-up described so far is very helpful in providing much more concrete insights. Furthermore, with the burgeoning discovery of planets around other stars, the fluid laboratory can guide the exploration of the atmospheres of faraway bodies, for which the details required for numerical simulations are not yet available, and which are not yet attainable by spacecraft. Figure 9 here thus provides a conceptual framework for comparative planetary meteorology, not only for planets within the solar system but also elsewhere in the universe (Read 2010).

"The experiments have emphasized the necessity for truly quantitative considerations of planetary atmospheres"

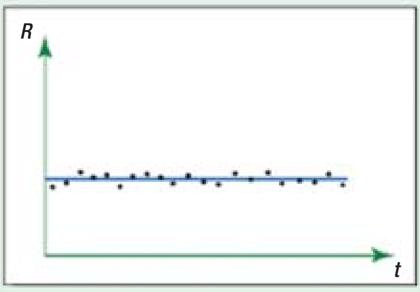
Chaos and predictability

One of the main promises of deterministic chaos is that it's more predictable than purely random phenomena. Figure 10 illustrates the increasing difficulty of predicting phenomena that are (a) constant in time, (b) purely periodic, (c) quasi-periodic, and (d) truly aperiodic. In the latter case, while true randomness may be hard to distinguish visually from deterministically chaotic phenomena, many statistical ways of telling the difference have been developed. Furthermore, the presence of coarse graining or of limit cycles that are only slightly unstable in the system's phase space can further increase the predictability (Ghil and Robertson 2002).

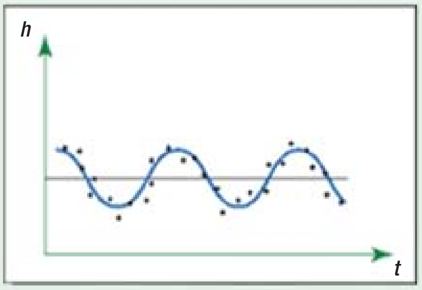
The early 1990s saw the advent of "ensemble forecasts" in operational weather forecasting on both sides of the Atlantic (Molteni *et al.* 1996, Toth and Kalnay 1993); distinct methodologies were used to generate the initial ensemble of weather states (Legras and Vautard 1996). Questions surrounding how to evaluate the skill of large, complex, nonlinear simulation models are still with us (e.g. McSharry and Smith 2004), but ensemble forecasts of flow in the annulus provided a "neutral" setting, as well as better statistics, which allows the comparison of competing operational forecast methods (e.g. Gilmour 1998). Shadowing of trajectories provides a valuable approach for evaluating the level of detail with which a given model can reflect an actual physical system; this approach is robust in the face of chaos or other nonlinear instabilities that disqualify the use of traditional prediction scores. Once again, the annulus is providing a test bed for comparisons between full-blown numerical simulations and laboratory observations (Young and Read 2010, Young *et al.* 2010). This work continues at the London School of Economics and Oxford and will be using the annulus to test methods of model evaluation that could help us determine the space and time scales on which full-scale general circulation models of the atmosphere and oceans might reliably inform economic and political decision making, and those on which they cannot.

As Raymond Hide said in 1953: "The possibility of solving problems of dynamical meteorology experimentally is an important one in view of the great theoretical difficulty involved"

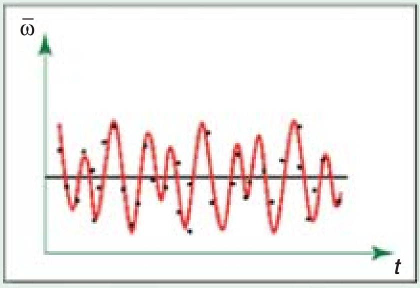
10: Prediction and predictability. (a)–(b) show a typical quantity on the ordinate vs time on the abscissa: black points are hypothetical data, while the curves show the theoretical form of the evolution that an observable of the given type would have.



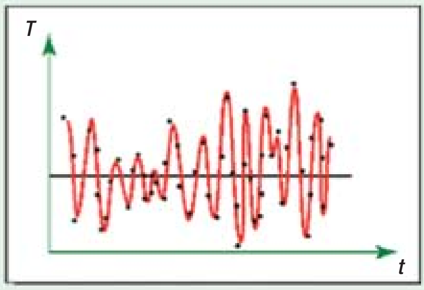
(a): Easiest to predict. Constant phenomena, e.g. the radius of the Earth, *R;* only need one number.



(b): Next easiest. Periodic phenomena, e.g. sunrise, tides; only need three numbers: period, amplitude, phase.



(c): Harder. Multi-periodic phenomena, e.g. celestial mechanics; need (finitely) many numbers.



(d): Hardest. Aperiodic phenomena, e.g. thermal convection, weather; need infinitely many numbers.

(Hide 1953). Contrasting models and observations of the rotating annulus make it possible to go beyond the classical problems of dynamic meteorology into the highly topical realm of problems that involve the entire climate and even full Earth system, with the atmosphere, ocean and other components interacting across space and time scales. This possibility has become much more significant in view of the great difficulties involved in observing and numerically simulating the climate system, let alone performing experiments on it. Some 50 years ago, the rotating annulus provided concrete insights into phenomena not then well-captured by incipient numerical models of the atmosphere. It still offers today an invaluable platform for critical evaluation of model errors, data assimilation and parameter estimation methodology, and probabilistic forecast interpretation. The multifaceted applications of the annulus experiments continue to provide new insights over a wide range of the Earth and planetary sciences, while also continuing to play a key role in the education of students in these fields (Illari *et al.* 2009). ●

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