Optimal estimation of electron lifetimes in the outer radiation belt

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Introduction

In this study, we demonstrate how data assimilation can help to estimate unknown parameters in a radial diffusion model in the Earth's radiation belts. We use `state augmentation` method in Kalman filter (KF) to estimate the model's lifetime parameter of relativistic electrons by assimilating observations from CRRES satellite.

EKF for parameter estimation

In the **"state augmentation"** method the parameters are treated as **additional state variables**. For simplicity, let us assume that there is only one unknown parameter $\boldsymbol{\mu}$ in the discrete numerical model **F** for the sate vector **x**. The underlying **"true"** natural system for **x** is then model **F** and in the simplest case persistence equation for $\boldsymbol{\mu}$, perturbed by noise $\boldsymbol{\varepsilon}$ **and** $\boldsymbol{\varepsilon^\mu}$ with given covariances $\mathbf{Q}=<\boldsymbol{\varepsilon}(t)\,\boldsymbol{\varepsilon}(t)^T>$, $\mathbf{Q}_\mu=<\boldsymbol{\varepsilon}^\mu(t)\,\boldsymbol{\varepsilon}^\mu(t)^T>$.

$$\bar{x}_k = \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} = \begin{pmatrix} F(x_{k-1}, \mu_{k-1}) \\ \mu_{k-1} \end{pmatrix} + \begin{pmatrix} \epsilon_k \\ \epsilon_k^\mu \end{pmatrix}$$

$$\bar{x}_k = \bar{M}_k \bar{x}_{k-1} + \bar{\epsilon}_k; \bar{M} = \frac{\partial \bar{F}}{\partial \bar{x}}$$

"Augmented" numerical model advances forecast ("f") and propagates it's error covariance matrix *P*:

$$\bar{x}_k^f = \bar{M}_k \bar{x}_{k-1}^f; \bar{P}_k^f = \bar{M}_k^T \bar{P}_{k-1}^f \bar{M}_k + \bar{Q}$$

The Kalman filter obtains analysis ("a") and reduces error by assimilating state observations with specified error covariance $\mathbf{R}=<\boldsymbol{\varepsilon}^o(t)\,\boldsymbol{\varepsilon}^o(t)^T>$:

$$y_k^o = (H\ 0) \begin{pmatrix} x_k \\ \mu_k \end{pmatrix} + \epsilon^0 = \bar{H}\bar{x}_k + \epsilon^0$$

$$\bar{x}_k^a = \bar{x}_k^f + \bar{K}(y_k^o - \bar{H}\bar{x}_k^f); \bar{P}_k^a = (I - \bar{K}\bar{H})\bar{P}_k^f$$

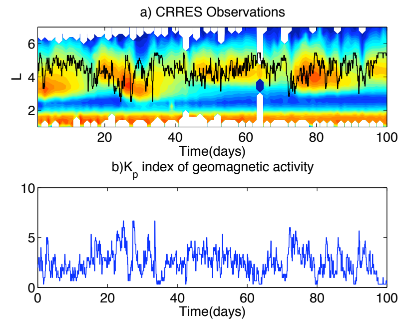
$$\bar{K} = \bar{P}^f \bar{H}^T (\bar{H}\bar{P}^f \bar{H}^T + R)^{-1}$$

- Parameters are non-observable, BUT the **cross-covariances** drive parameter changes from innovations of the state:

$$\bar{P}^f = \begin{pmatrix} P_{xx}^f & P_{x\mu}^f \\ P_{\mu x}^f & P_{\mu\mu}^f \end{pmatrix}; \bar{K} = \begin{pmatrix} P_{xx}^f H^T \\ P_{\mu x}^f H^T \end{pmatrix} (HP_{xx}^f H^T + R)^{-1}$$

- Parameter estimation is always **a nonlinear problem** even if the model is linear in terms of the model state, so Extended Kalman Filter (EKF) is needed.

Outer Radiation Belt Model



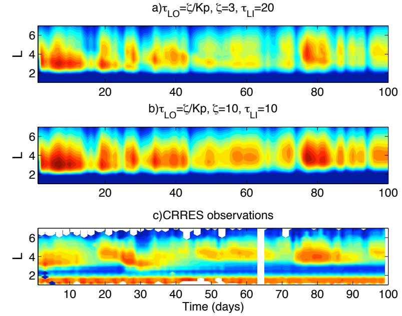
Daily 1MeV electron fluxes starting August 28, 1990.

Radial diffusion equation for phase space density:

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) - \frac{f}{\tau_L} \qquad (1)$$

where τ_L is the electron lifetime, and D_{LL} is the radial diffusion coefficient. Different parameterizations are used outside/inside plasmasphere L_{pp}=5.6-0.46*Kp(t): $\tau_{LO} = \zeta/Kp(t)$ and τ_{LI}=constant.

What are the "optimal" τ?

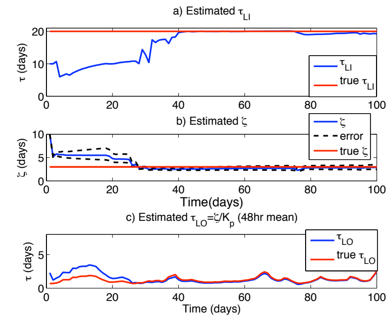


Various features are best captured with different parameter values, perhaps also varying with time:

- temporal persistence with ζ=10?
- spatial position with ζ=3?
-We will use EKF to estimate ζ and τ_{LI} in a highly nonlinear system via dependence on L_{pp} and K_p!

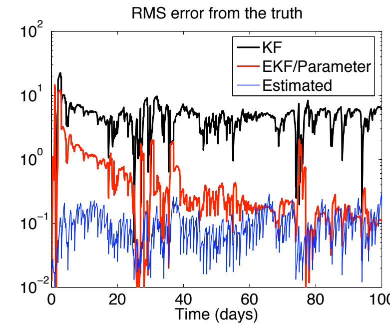
Identical Twins (synthetic data)

- obtain daily flux observations from the model run with **"truth"** parameter values: **ζ= 3**, $\boldsymbol{\tau_{LI}}$ **=20**
- start with **"wrong"** model parameters: **ζ= 10**, $\boldsymbol{\tau_{LI}}$**=10** and assimilate "truth" observations for **L<5**
- goal: recover model parameters of **"truth"**!
- simple (diagonal) error covariances **Q** and **R**



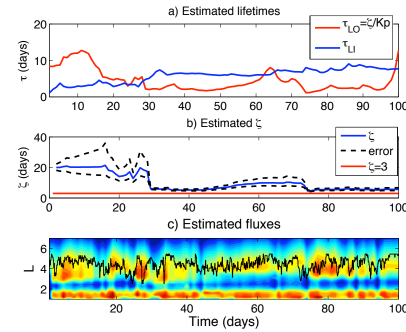
-Convergence depends on the relative position of plasmapause relative to the assimilation region.

-Once converged, parameters remain locked to **"truth"**.



-Parameter estimation helps optimality: **estimated** error [tr(P^f)] ≈ **actual** (known for synthetic data)

CRRES (observational data)



-start with **"unreasonable"** model parameters: **ζ**= 20, $\boldsymbol{\tau_{LI}}$=1 days.

-Convergence rate is similar to identical twins

- Different $\boldsymbol{\tau_{LO}}$ and ζ in CIR (50≤T≤70) and CME (75≤T≤100) storms: ζ≈10 for CIR, ζ≈5 for CME.

Conclusions

- Confirmed range of lifetimes for electrons in outer radiation belt suggested theoretically: $\tau_{LI} > \tau_{LO}$; τ_{LI}≈6-10 days (Lyons et al. 1972, Abel and Thorne, 1998); τ_{LO}≈2-3 days (Thorne et al. 2005).

- Suggests different physical mechanisms for CIR and CME dominated storms (local source?)

Future Work:

- Incorporate parameterizations of local loss and acceleration from quasi-linier theory of wave-particle interactions into parameter estimation.

- Analysis of the Phase Space Density (PSD) derived from CRRES data for various values of the MLT and pitch-angles.

- Increase time resolution for parameter estimation

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