Empirical Modeling of Earth's Outer Radiation Belt

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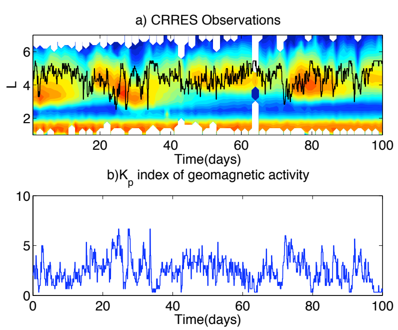
Abstract

We present application of a novel empirical mode reduction (EMR) methodology for constructing a hierarchy of empirical, stochastically forced models for the analysis and simulation of spatio-temporal variability in radiation belts. It is based on multiple polynomial regression to estimate deterministic propagator from the data and multi-level modeling of the stochastic forcing. This methodology encompasses both linear and nonlinear time-dependent models that aim to best describe the data set's statistics, and has been successfully applied before for ocean and atmosphere data sets. Here we apply this methodology to study acceleration of relativistic electrons during magnetic storms in the Earth's outer radiation belt. As a starting point, we consider a data set of 1MeV electron fluxes from integration of a radial diffusion model driven by Kp index. The data are projected onto leading empirical orthogonal functions (EOFs) which provide an optimal predictor set. With these predictors, our best reduced stochastic model forced by Kp time series, produces excellent fit to the statistics of underlying data set, as well as reproduces very well magnetic storms observed in the full model's integration.

Outer Radiation Belt

Daily 1MeV electron fluxes starting August 28, 1990.



Q: Why for similar level of geomagnetic activity there is a different response in radiation belt variability?

Empirical Mode Reduction(EMR)

Motivation:

- Sometimes we have spatio-temporal geophysical dataset but not a good model.

-We want models that are as simple as possible, but not any simpler.

Criteria for a good data-derived model:

- Capture statistics (histograms, correlations, spectra) and relevant dynamics: regimes, oscillations, etc.
- Deterministic dynamics easy to analyze analytically.
- Good noise estimates.
- Describes independent data.

Key Ideas:

Given spatio-temporal dataset **D(t,s)(t=1,N,s=1,M)** construct the reduced model in terms of **anomalies** using a few **K leading principal components (PCs)** - ***$x_i(t)$*** and **empirical orthogonal functions (EOFs)** - ***$e_i(s)$***, by diagonalizing MxM covariance matrix **C** of the field **D** :

$$\mathbf{C} = \frac{1}{N}(\mathbf{D} - <\mathbf{D}>)^{t}(\mathbf{D} - <\mathbf{D}>)$$
$$\mathbf{C}\lambda_i = \lambda_i e_i, x_i = (\mathbf{D} - <\mathbf{D}>)e_i$$

Consider the following system of stochastic ODEs:

$$\begin{aligned}
dx_i &= (\mathbf{x}^{\mathrm{T}}\mathbf{A}_i\mathbf{x} + \mathbf{b}_i^{(0)}\mathbf{x} + c_i^{(0)})\,dt + r_i^{(0)}\,dt,\\
dr_i^{(0)} &= \mathbf{b}_i^{(1)}[\mathbf{x}, \mathbf{r}^{(0)}]dt + r_i^{(1)}\,dt,\\
dr_i^{(1)} &= \mathbf{b}_i^{(2)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}]dt + r_i^{(2)}\,dt,\\
&\dots\\
dr_i^{(L)} &= \mathbf{b}_i^{(L)}[\mathbf{x}, \mathbf{r}^{(0)}, \mathbf{r}^{(1)}, \dots, \mathbf{r}^{(L)}]dt + dr_i^{(L+1)}
\end{aligned}$$

-**Multiple predictors**: **PCs** - x_i; **i=1÷K, K≤M**:
-**Predictant variables**: one-step time differences of predictors; step = sampling interval Δt.
- **matrices A_i, vectors b_i, and scalars c_i.** are fitted for **each *i*** independently by least squares.
-Multi-level modeling of noise r_i. to account serial correlations in the regression residuals.
- The number of levels is such that each of the last-level (L) regression residuals is "white" in time.
- Spatial (cross-channel) correlations of the last-level residuals are retained in subsequent regression-model simulations.
- **Number K of PCs** is chosen to optimize the EMR model performance (comparison with data).

EMR for Radiation Belt

Synthetic Data:

Radial diffusion equation for phase space density:

$$\frac{\partial f}{\partial t} = L^2 \frac{\partial}{\partial L}\left(L^{-2} D_{LL} \frac{\partial f}{\partial L}\right) - \frac{f}{\tau_L} \qquad (1)$$

τ_L is the electron lifetime, D_{LL} is the radial diffusion coefficient with nonlinear dependence on Kp. Different parameterizations are used outside/inside plasmasphere L_{pp}=5.6-0.46*Kp(t):
$\tau_{LO} = \zeta/Kp(t)$, $\zeta=3$; τ_{LI}=10 days.

Main Idea: test EMR on the model dataset for which we know the origin ("truth") and learn something new about PDE or/and dynamics.

Basic steps:

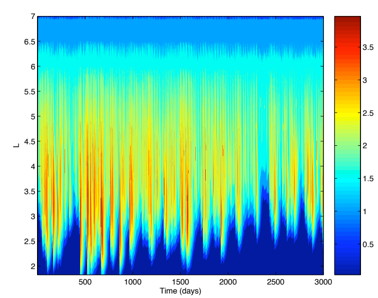
- Obtain long time integration of the PDE model (1) forced by historic Kp to obtain dataset for analysis.
-Calculate PCs of log(fluxes) and fit EMR.
- Obtain simulated data from the integration of reduced model and compare with the original dataset.

EMR for externally forced system:

- modify linear and constant terms on **the main ("0")** level to account for explicit forcing:

$$\mathbf{b}_i^{(0)} = \mathbf{b}_{i0}^{(0)} + \mathbf{b}_{ik}^{(0)} K_p(t), c_i^{(0)} = c_{i0}^{(0)} + c_{ik}^{(0)} K_p(t)$$

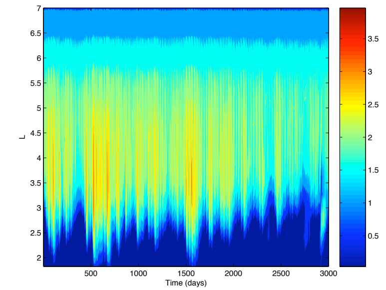
- Consider **anomalies** with respect to the "quite" state: in this study steady solution of (1) for Kp=3.



Model 1MeV electron fluxes , 1990-1998

- 24000x26 dataset (3hr resolution)
- Six leading PCs (account for 90% of the variance).
- **Best** EMR model **is linear** with **3 levels**
- 6 spatial degrees of freedom (instead of 26).

EMR Simulated data



Random realization from **continuos** integration of EMR model forced by Kp.

- EMR model is **constant** over the whole time interval.
- because of stochastic component, the particular **EMR** integration **WILL NOT** reproduce the original dataset in details.
- preliminary analysis of the deterministic part of EMR model shows unstable eigenmodes which may explain the origin of storms in terms of optimal perturbations.

Conclusions

- EMR is a promising tool for constructing hierarchy of empirical models of radiation belt variability with potential for predictability, data assimilation, etc.

Future Work:

- Cross-validation for testing the model.
- Statistical analysis.
- Dynamical analysis: stability, optimal perturbations.
- Consider observational dataset with alternative external drivers (solar wind parameters).

References

1. Kondrashov, D., S. Kravtsov, A. W. Robertson, and M. Ghil, 2005: A hierarchy of data-based ENSO models. J. Climate, 18, 4425–4444.
2. Kondrashov, D., S. Kravtsov, and M. Ghil, 2006: Empirical mode reduction in a model of extratropical low-frequency variability. J. Atmos. Sci., (63), 1859–1877.

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