Empirical Mode Reduction and non-Gaussian Signatures of Planetary Low-Frequency Atmospheric Modes

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Abstract

We apply empirical mode reduction (EMR) methodology (Kravtsov et al. 2005; Kondrashov et al. 2006) to the output of a long simulation of a global baroclinic, quasigeostrophic, three-level T21 model (QG3) with topography (Marshall and Molteni 1993), to obtain a reduced nonlinear stochastic model of extratropical low-frequency variability. We revisit the question of origin of the nonlinear signatures in model's phase space, by looking at the **mean phase space tendencies** and "important" interactions detected by EMR that contribute to observed nonlinear behavior.

Empirical Model Reduction

The EMR models are nonlinear multi-level generalizations of the linear inverse models (LIMs: Penland 1989, 1996; Penland and Ghil 1993) to include quadratic (and higher-order polynomial, if necessary) combinations of predicted variables in the dynamical operator of the main model level. Additional model levels are included to simulate the main-level time-dependent stochastic forcing. The number of model levels is chosen to ensure that the forcing at the last level can be well approximated by a vector-valued white-noise process.

Goal: Capture statistics (histograms, correlations, spectra) and important dynamical aspects of **linear (oscillations)** and **nonlinear (regimes)** of the original dataset's **"resolved"** behavior. The stochastically forced simulations of EMR model can be exploited to analyze various dynamical aspects of **the observed evolution** (when no good physical model is available), or **high-end model** generated **integration** by using a reduced model with much fewer d.o.f, as well as used for prediction (ENSO, Kondrashov et al. 2005).

$$
\begin{aligned}
dx_i &= (N_{ijk}x_jx_k + L^{(0)}_{ij}x_j + F_i)\,dt + r_{0,i}\,dt,\\
dr_{0,i} &= L^{(1)}_{ij}[\mathbf{x},\mathbf{r}_0]_j dt + r_{1,i}\,dt,\\
dr_{1,i} &= L^{(2)}_{ij}[\mathbf{x},\mathbf{r}_0,\mathbf{r}_1]_j dt + r_{2,i}\,dt,\\
&\dots\\
dr_{L-1,i} &= L^{(L)}_{ij}[\mathbf{x},\,\mathbf{r}_0,\,\mathbf{r}_1,\,\dots,\,\mathbf{r}_{L-1}]_j dt + d\xi_i
\end{aligned}
$$

x_i time series (can be PCs), $i = 1,2,...M$, are predictors.

Computed tendencies dx_i are predictants.

Multiple linear regression to estimate L_{ij}, N_{ijk} and F_i for $i,j,k = 1,2,...M$ and $l=0, 2, ...L$.

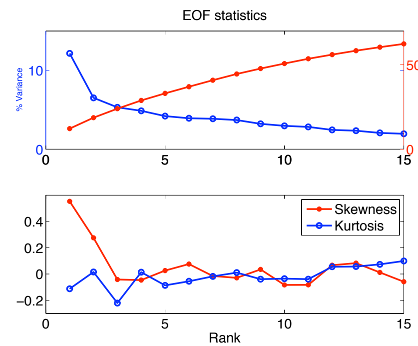
Multi-level **noise** modeling for regression residuals $r_{l,i}$.

$d\xi_i \sim N(0,\mathbf{Q})$, $\mathbf{Q}$ = sample cov($\mathbf{r_L}$).

Regularized regression fitting of EMR coefficients.

QG3 atmospheric model

The QG3 model's (with ~10^3 d.o.f.) low-frequency variability (LFV) is characterized by the existence of a few persistent and recurrent flow patterns, or **weather regimes**, as well as by **intraseasonal oscillations** (Kondrashov et al. 2004, 2006). We use 10 leading EOFs of daily 500-hPa streamfunction from $5 \cdot 10^4$ days of integration to construct 3-level quadratic EMR with O(100) independent coefficients.



Mean phase space tendencies

Recent studies have used the mean phase space tendencies in the subspace of leading EOFs to identify distinctive signatures of nonlinear processes in both the intermediate QG3 model (Selten and Branstator, 2004; Franzke et al. 2007) and more detailed GCMs (Branstator and Berner, 2005). Of particular interest is to establish the relative contributions of **"resolved"** and **"unresolved"** modes that may lead to observed deviations from Gaussianity, e.g. to double-swirls.

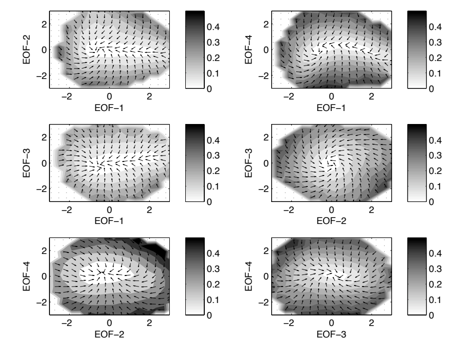
- We estimate the tendencies $<(dx_j,dx_k)>=F(x_j,x_k)$ in a given plane of the EOF pair (*j, k*) from QG3 and EMR simulation data.

- The **"resolved"** vs. **"unresolved"** split depends on assumptions about **"signal"** and **"noise"**. With no pronounced time-scale separation between individual EOFs, we consider EOFs x_i ($i \le 4$) as **"resolved"** because:

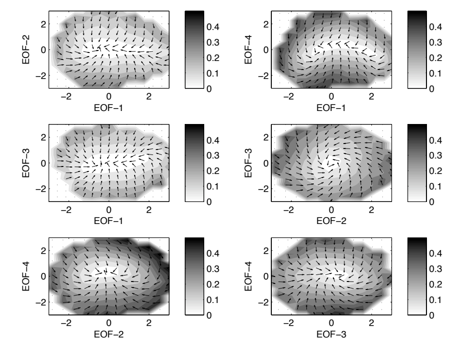
1) these EOFs have the most pronounced deviations from the Gaussianity in terms of skewness and kurtosis.

2) they determine the most interesting dynamical aspects of **LFV; linear (intraseasonal oscillations)** as well as **nonlinear (regimes)** (Kondrashov et al. 2004, 2006).

QG3 tendencies

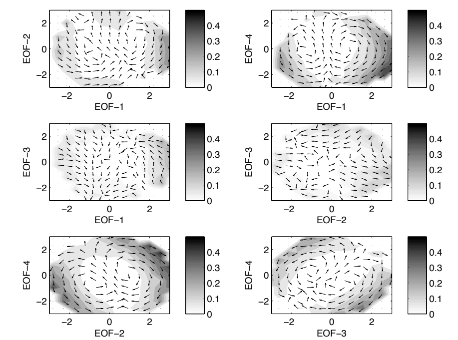


EMR tendencies

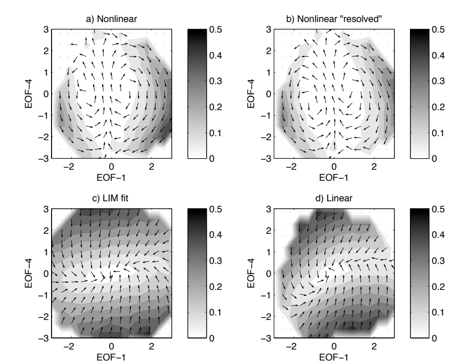


***Linear features for EOF pairs (1-3), (2-3) only*: antisymmetric for reflections through the origin and constant speed along ellipsoids (Branstator and Berner, 2005). Excellent agreement between EMR and QG3!** *(shading indicates the magnitude in 1 std dev day–1, and arrows are normalized to have the same length).*

EMR nonlinear tendencies



EMR tendencies budget



Pronounced nonlinear double swirls for EOF pairs (1-2), (1-4), (2-4) and (3-4).

For a given x_i ($i \le 4$), we split **nonlinear** interaction x_jx_k as **"resolved" T_R (set Ω of (*j,k*); *j,k* ≤4)**:

$$T_R = N_{ijk}\,x_j,x_k - R_i,$$
$$R_i = < N_{ijk}\,x_j,x_k >$$

and **"unresolved" T_U** for **(*j*,k) ∉ Ω**:

$$T_U = N_{ijk}\,x_j,x_k + R_i + F_i$$

Since F_i ensures $< dx_i > = 0$: $F_i = - < N_{ijk}\,x_j,x_k > \ \forall$ j,k we have $<T_R> = 0$, $<T_U> = 0$, and $<T_R + T_U> = 0$!

Conclusions

The multiplicative-noise explanations of **non-Gaussian** atmospheric behavior depend on how the partition is made between **unresolved** and **resolved** variables. When they are reasonably defined, we find that the **nonlinear "double-swirl"** feature is mostly due to the **"resolved" nonlinear interactions**!

References

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