*Joint Math Mtg. 2008:*
*Climate Change & GFD Session*

San Diego, Sunday,
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# Robust Climate Projections and

## *Stochastic Structural Stability of Dynamical Systems*


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*Please see these sites for further details:*

http://www.environnement.ens.fr/

http://www.atmos.ucla.edu/tcd/

# Motivation

- The ***climate system*** is highly ***nonlinear and*** quite ***complex***.
- Its ***major components*** — the atmosphere, oceans, ice sheets — ***flow*** on many time and space scales.
- Its ***predictive understanding*** has to rely on the system's physical, chemical and biological modeling,
  but also on the mathematical analysis of the models thus obtained.
- The ***hierarchical modeling*** approach allows one to give proper weight to the understanding provided by the models *vs*. their realism, respectively.
- This approach facilitates the evaluation of ***forecasts (pognostications?)*** based on these models.
- Back-and-forth between ***"toy"*** (conceptual) and ***detailed*** ("realistic") ***models***, and between ***models*** and ***data***.

# Outline

- The IPCC process: results and questions
- Natural climate variability: source of uncertainties
  - sensitivity to initial state => error growth
  - sensitivity to model formulation => see below!
- Uncertainties and how to fix them
  - structural instability
  - random dynamical systems
- Conclusions and references

# Global warming and its socio-economic impacts

Temperatures rise:

- What about impacts?
- How to adapt?

The answer, my friend, is blowing in the wind, *i.e.*, it depends on the accuracy and reliability of the forecast …

*Source : IPCC* (2007), *AR4, WGI, SPM*

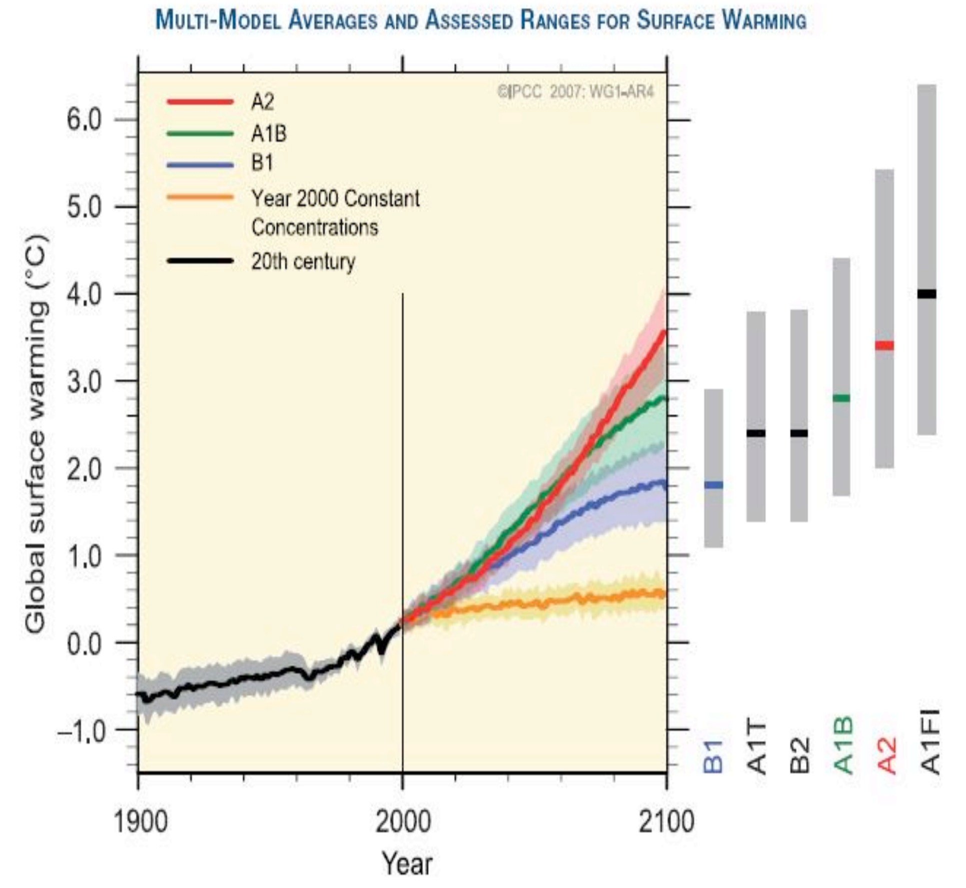
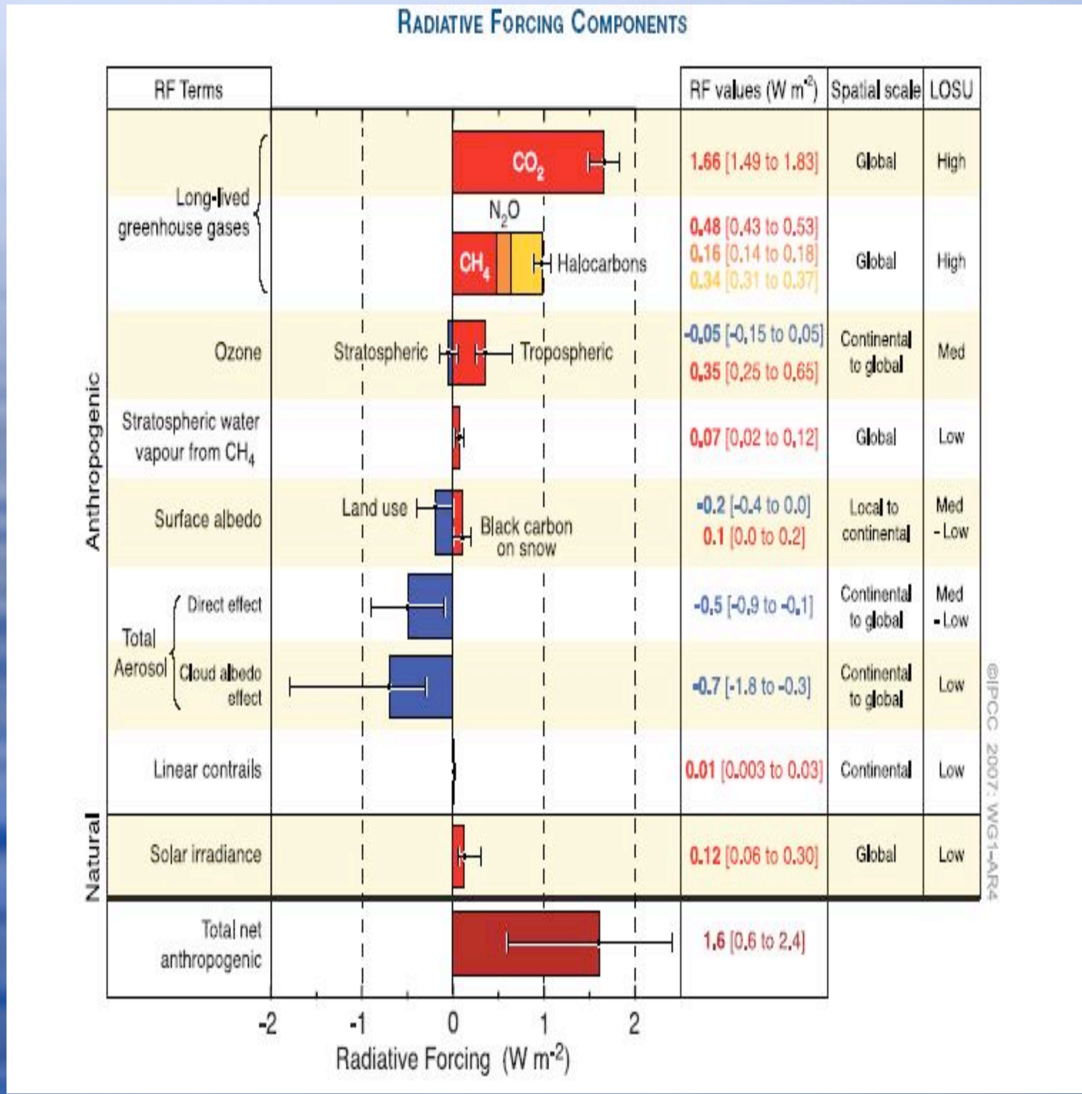



**Figure SPM.5.** *Solid lines are multi-model global averages of surface warming (relative to 1980–1999) for the scenarios A2, A1B and B1, shown as continuations of the 20th century simulations. Shading denotes the ±1 standard deviation range of individual model annual averages. The orange line is for the experiment where concentrations were held constant at year 2000 values. The grey bars at right indicate the best estimate (solid line within each bar) and the* **likely** *range assessed for the six SRES marker scenarios. The assessment of the best estimate and* **likely** *ranges in the grey bars includes the AOGCMs in the left part of the figure, as well as results from a hierarchy of independent models and observational constraints. {Figures 10.4 and 10.29}*

# GHGs rise

It's gotta do with us, at least a bit, ain't it?

But just how much?




*IPCC* (2007)

**Unfortunately, things aren't all that easy!**

***What to do?***

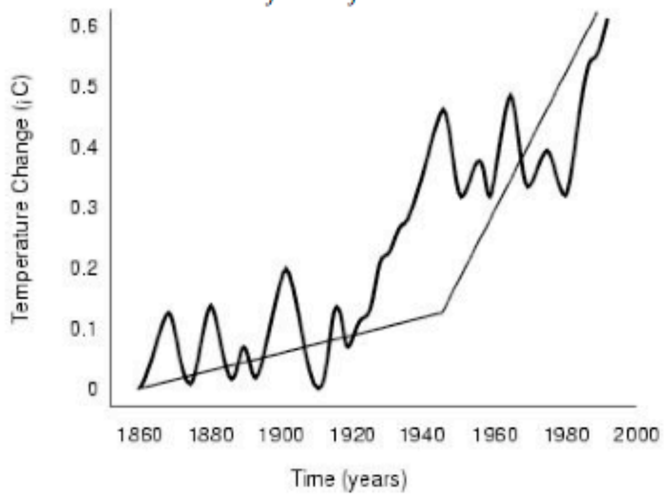
***Try to achieve better interpretation of, and agreement between, models …***

**Ghil, M.**, 2002: Natural climate variability, in *Encyclopedia of Global Environmental Change*, T. Munn (Ed.), Vol. 1, Wiley

Natural variability introduces additional complexity into the anthropogenic climate change problem

The most common interpretation of observations and GCM simulations of climate change is still in terms of a scalar, linear Ordinary Differential Equation (ODE)

$$c\frac{dT}{dt} = -kT + Q \qquad k = \sum k_i - \text{feedbacks (+ve and -ve)}$$
$$Q = \sum Q_j - \text{sources \& sinks}$$
$$Q_j = Q_j(t)$$




Linear response to $CO_2$ vs. observed change in T

Hence, we need to consider instead a system of nonlinear Partial Differential Equations (PDEs), with parameters and multiplicative, as well as additive forcing (deterministic + stochastic)

$$\frac{d\boldsymbol{X}}{dt} = \boldsymbol{N}(\boldsymbol{X}, t, \mu, \beta)$$

# So what's it gonna be like, by 2100?

**Table SPM.2.** *Recent trends, assessment of human influence on the trend and projections for extreme weather events for which there is an observed late-20th century trend. {Tables 3.7, 3.8, 9.4; Sections 3.8, 5.5, 9.7, 11.2–11.9}*

| Phenomenon[a] and direction of trend | Likelihood that trend occurred in late 20th century (typically post 1960) | Likelihood of a human contribution to observed trend[b] | Likelihood of future trends based on projections for 21st century using SRES scenarios |
|---|---|---|---|
| Warmer and fewer cold days and nights over most land areas | *Very likely*[c] | *Likely*[d] | *Virtually certain*[d] |
| Warmer and more frequent hot days and nights over most land areas | *Very likely*[e] | *Likely (nights)*[d] | *Virtually certain*[d] |
| Warm spells/heat waves. Frequency increases over most land areas | *Likely* | *More likely than not*[f] | *Very likely* |
| Heavy precipitation events. Frequency (or proportion of total rainfall from heavy falls) increases over most areas | *Likely* | *More likely than not*[f] | *Very likely* |
| Area affected by droughts increases | *Likely* in many regions since 1970s | *More likely than not* | *Likely* |
| Intense tropical cyclone activity increases | *Likely* in some regions since 1970 | *More likely than not*[f] | *Likely* |
| Increased incidence of extreme high sea level (excludes tsunamis)[g] | *Likely* | *More likely than not*[f,h] | *Likely*[i] |

# Can we, nonlinear dynamicists, help?

The uncertainties
might be ***intrinsic***,
rather than mere
"tuning problems"

If so, maybe
***stochastic structural stability*** could help!

Might fit in nicely with
recent taste for
"stochastic
parameterizations"

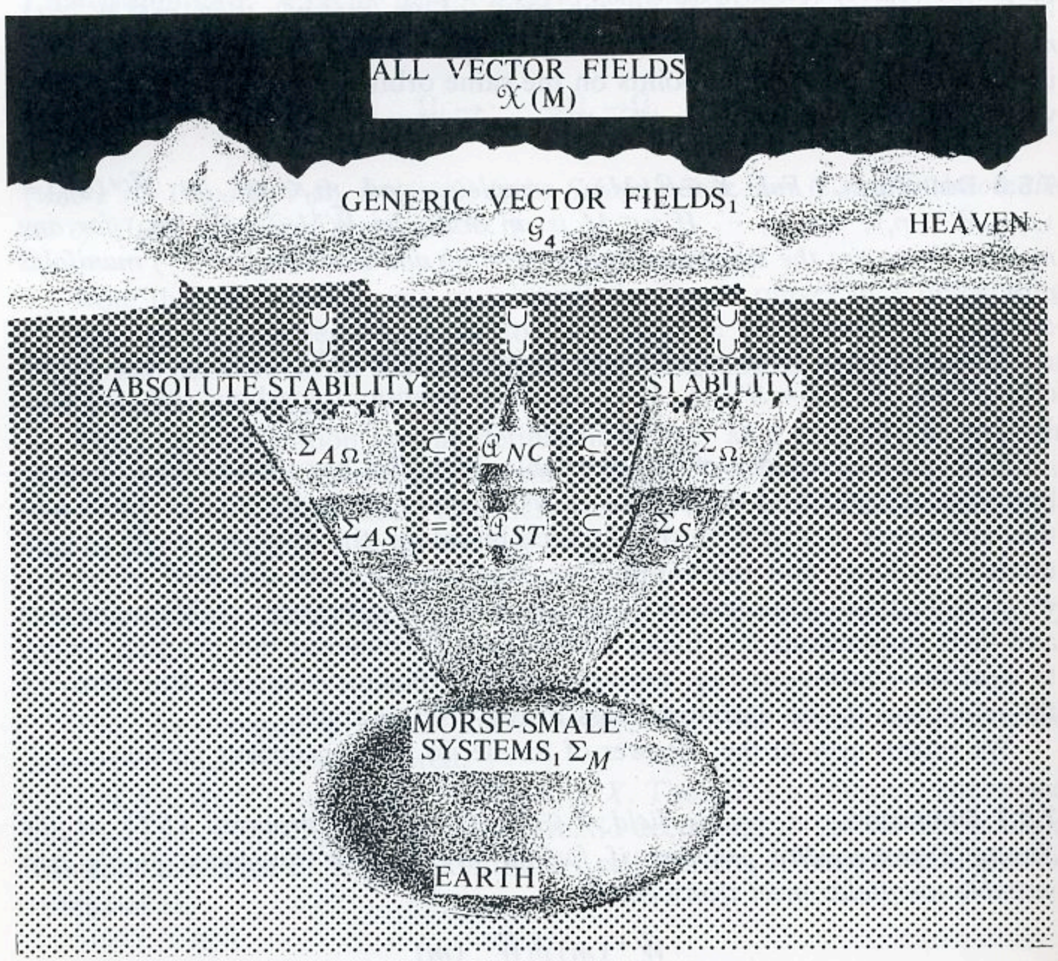



**Figure 7.5-1.** The three towers of differentiable dynamics.

*The DDS dream of structural stability* (from *Abraham & Marsden*, 1978)

This theory is a combination of measure (probability) theory and dynamical systems initiated by the “Bremen group" (L. Arnold, 1998). It allows one to treat Stochastic Differential Equations (**SDEs**), and more general systems driven by some “noise," as **flows**.

**Setting:**

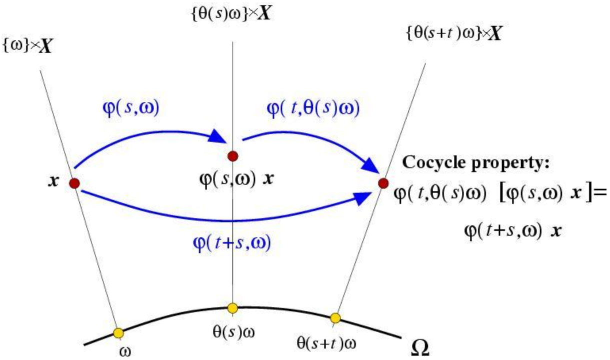
(i) A phase space $X$. **Example**: $\mathbb{R}^n$.

(ii) A probability space $(\Omega, \mathcal{F}, \mathbb{P})$. **Example**: The Wiener space $\Omega = C_0(\mathbb{R}; \mathbb{R}^n)$ with Wiener measure $\mathbb{P} = \gamma$.

(iii) A model of the noise $\theta(t) : \Omega \to \Omega$ that preserves the measure $\mathbb{P}$, i.e. $\theta(t)\mathbb{P} = \mathbb{P}$; $\theta$ is called the driving system. **Example**: $W(t, \theta(s)\omega) = W(t + s, \omega) - W(s, \omega)$; it starts the noise at $s$ instead of $t = 0$.

(iv) A mapping $\varphi : \mathbb{R} \times \Omega \times X \to X$ with the cocycle property. **Example**: The solution of an SDE.

# Random Dynamical Systems - A geometric view of SDEs

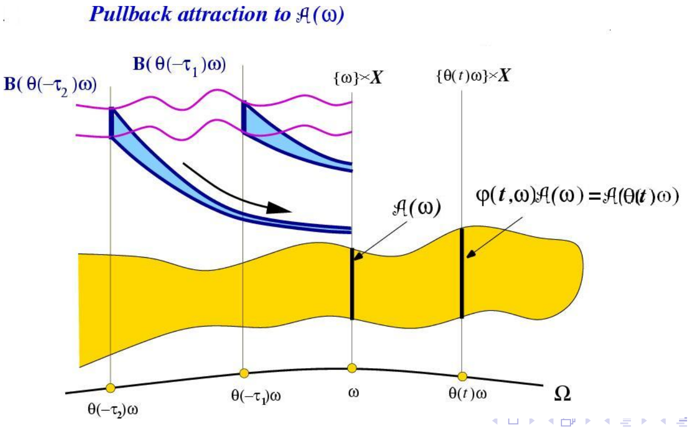



- $\varphi$ **is a random dynamical system (RDS)**
- $\Theta(t)(x, \omega) = (\theta(t)\omega, \varphi(t, \omega)x)$ **is a flow on the bundle**

**A random attractor** $\mathcal{A}(\omega)$ is both *invariant* and “pullback" *attracting*:

(a) **Invariant**: $\varphi(t,\omega)\mathcal{A}(\omega) = \mathcal{A}(\theta(t)\omega)$.

(b) **Attracting**: $\forall B \subset X$, $\lim_{t\to\infty} \mathrm{dist}(\varphi(t,\theta(-t)\omega)B, \mathcal{A}(\omega)) = 0$ a.s.



## A tool for classification: stochastic equivalence

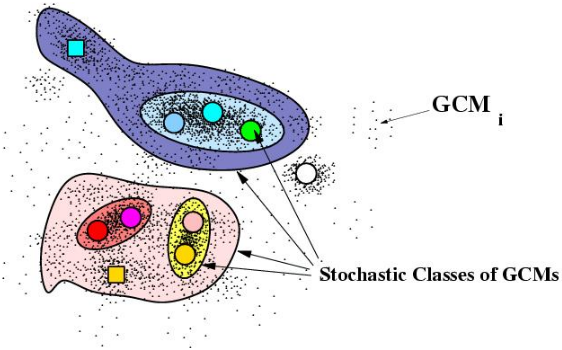
- Stochastic equivalence: two cocycles $\varphi_1(t,\omega)$ and $\varphi_2(t,\omega)$ are conjugated iff there exists a **random homeomorphism** $h \in \mathrm{Homeo}(X)$ and an invariant set $\tilde{\Omega}$ of full $\mathbb{P}$-measure (w.r.t. $\theta$) such that $h(\omega)(0) = 0$ and:

$$\varphi_1(t,\omega) = h(\theta(t)\omega)^{-1} \circ \varphi_2(t,\omega) \circ h(\omega); \qquad (1)$$

  $h$ is also called cohomology of $\varphi_1$ and $\varphi_2$. It is a **random change of variables**!

- Motivation: We would like to measure quantitatively as well as quantitatively the difference between climate models.

# Stochastic equivalence - Could noise help the classification?



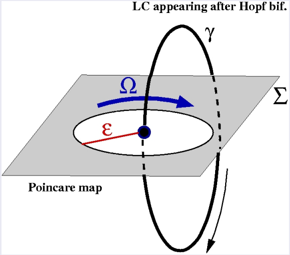

As the noise variance tends to zero and/or the parametrizations are switched off, one recovers the structural instability, as a “granularity" of model space. For nonzero variance, the random attractor $\{\mathcal{A}(\omega)\}$ associated with several GCMs might fall into larger and larger classes as the noise level increases.

# Investigation of these ideas on a family of dynamical *toy* systems - Theoretical and numerical results

## V. Arnold's family of diffeomorphisms

- We want to perform a *classification* in terms of stochastic equivalence.
- Our first theoretical laboratory is **Arnold's family of diffeomorphisms of the circle:**

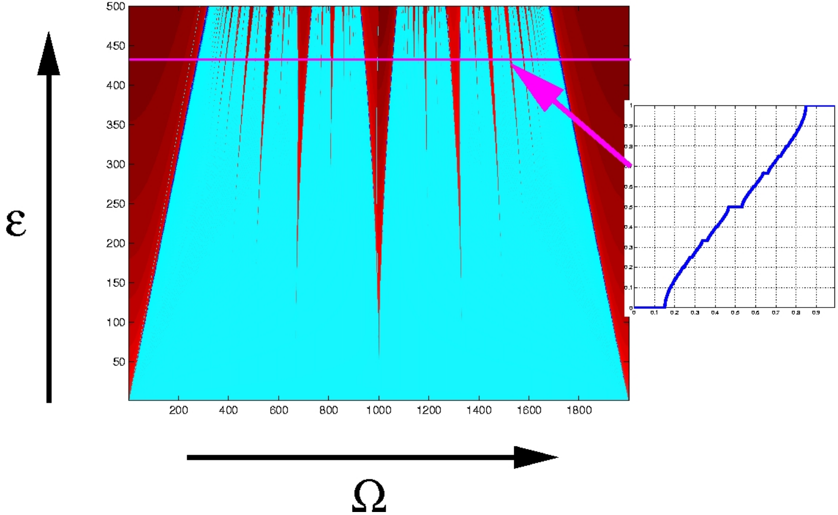
$$x_{n+1} = F_{\Omega,\varepsilon}(x_n) := x_n + \Omega - \varepsilon \sin(2\pi x_n) \quad \text{mod } 1$$



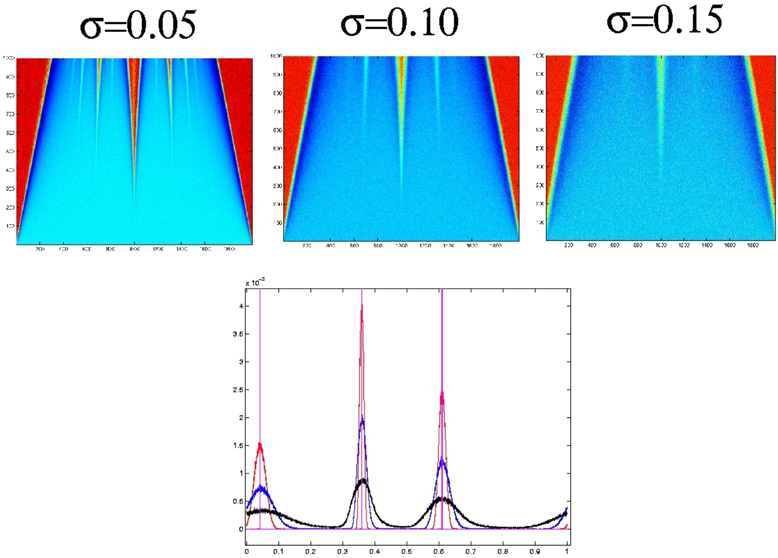
# Which paradigm is represented by this family? Why this family?

- Frequency-locking phenomena & Devil's staircase
- **Topological classification** of Arnold's family $\{F_{\Omega,\varepsilon}\}$:
  - **Countable** regions of structural stability,
  - **Uncountable** structurally unstable systems with **non-zero Lebesgue measure!**
- **Two types of attractors:**
  - Periodic orbits in the circle.
  - The whole circle.

# Arnold's tongues and Devil's staircase



# Effect of the noise on topological classification?




**Effect of the noise on the PDF of Arnold's tongue 1/3**

# Extension of the paradigm - Devil's quarry

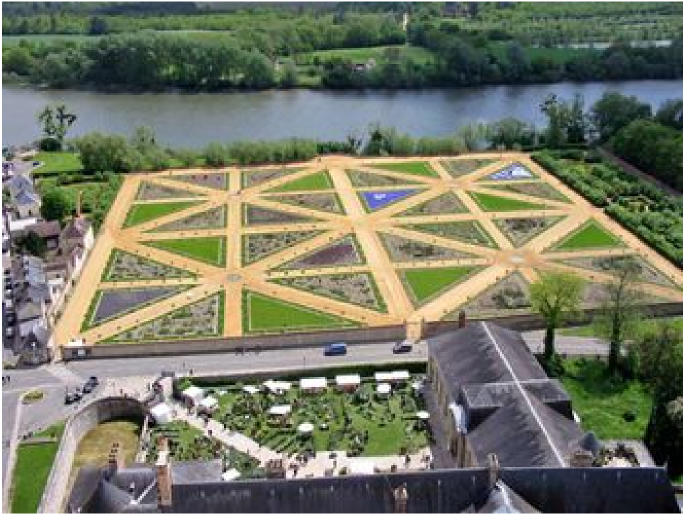
## Short description of the deterministic model

- Dynamics on a 2-D torus:

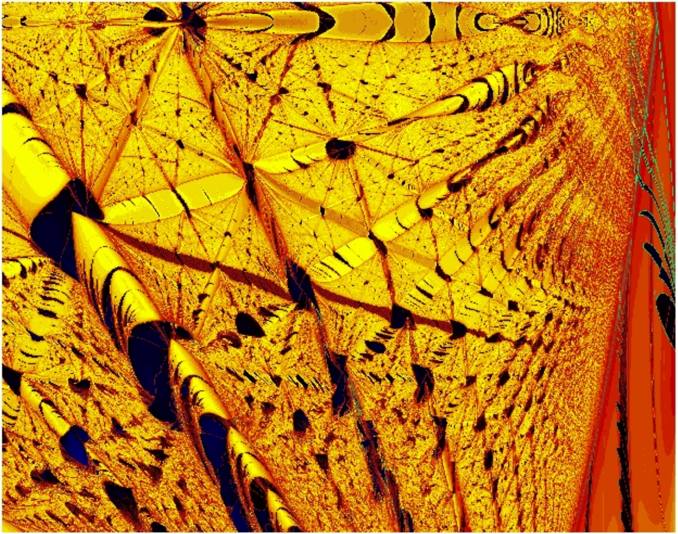
$$
\begin{aligned}
x_{n+1} &= x_n + \Omega_1 - \varepsilon \sin(2\pi y_n), \quad \text{mod } 1 \\
y_{n+1} &= y_n + \Omega_2 - \varepsilon \sin(2\pi x_n) \quad \text{mod } 1
\end{aligned}
$$

- Web of resonances & chaos:
  - Partial resonance ($\Omega_1, \Omega_2$ are rational and there is one rational relation $m_1\Omega_1 + m_2\Omega_2 = k \in \mathbb{Z}^*$ with $(m_1, m_2) \in \mathbb{Z}^* \times \mathbb{Z}^*$)
  - Full resonance
  - Chaos with possibly multiple attractors
- A more realistic paradigm of observed dynamics in the geosciences, and more...
- What is the effect of noise in such a context?

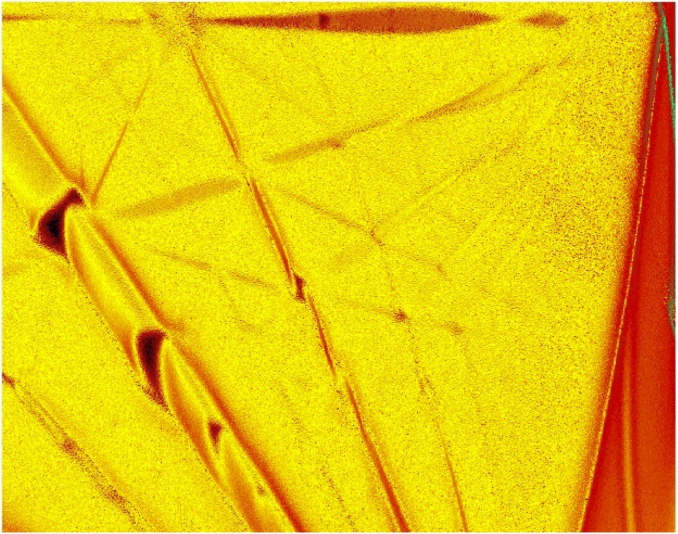
# A French garden near the castle of La Roche-Guyon



# Devil's quarry for a coupling parameter $\varepsilon = 0.15$: a web of resonances



# Effect of the noise on Devil's quarry



# Some conclusions &/or questions

***What do we know?***

- It's getting warmer.
- We do contribute to it.
- So, we should act as best we know and can!

***What do we know less well?***

- How does the climate system really work?
- How does natural variability interact with anthropogenic forcing?

***What to do?***

- Better understand the system and its forcings.
- Better understand the effects on economy and society, and vice-versa.
- Explore the models', and system's, stochastic structural stability.

# Some general references


Andronov, A.A., and L.S. Pontryagin, 1937: Systèmes grossiers. *Dokl. Akad. Nauk. SSSR*, **14**(5), 247–250.

Arnold, L., 1998: *Random Dynamical Systems,* Springer Monographs in Math., Springer, 625 pp.

Charney, J., *et al.*, 1979: *Carbon Dioxide and Climate: A Scientific Assesment.* National Academic Press, Washington, D.C.

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Ghil, M., R. Benzi, and G. Parisi (Eds.), 1985: *Turbulence and Predictability in Geophysical Fluid Dynamics and Climate Dynamics*, North-Holland,, 449 pp.

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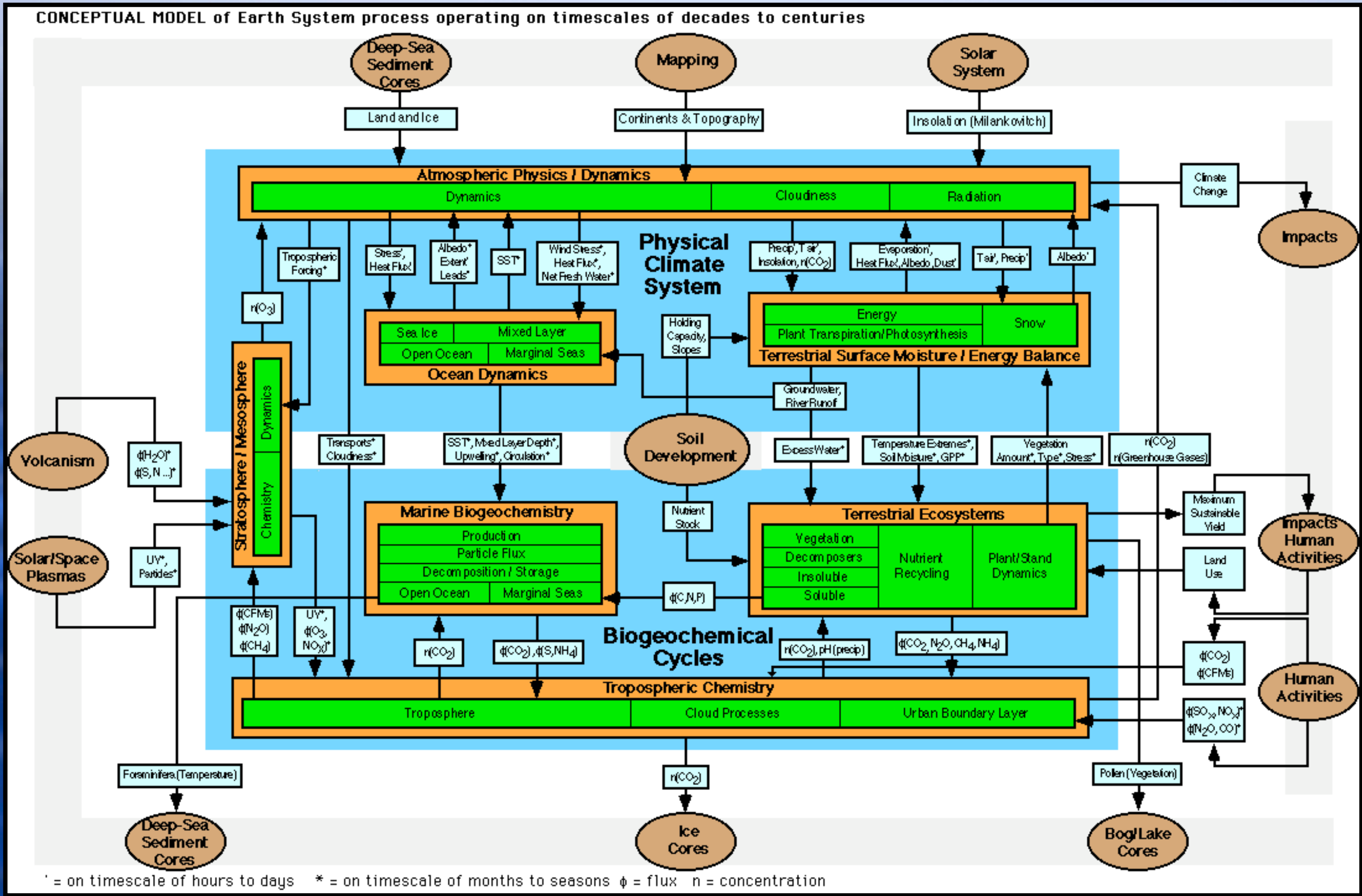
Ghil, M., and E. Simonnet, 2007: *Nonlinear Climate Theory*, Cambridge Univ. Press, Cambridge, UK/London/New York, in preparation (approx. 450 pp.).

Houghton, J. T., G. J. Jenkins, and J. J. Ephraums (Eds.), 1991: *Climate Change, The IPCC Scientific Assessment*, Cambridge Univ. Press, Cambridge, MA, 365 pp.

Solomon, S., *et al.* (Eds.). *Climate Change 2007: The Physical Science Basis. Contribution of Working Group I to the Fourth Assessment Report of the IPCC*, Cambridge Univ. Press, Cambridge, UK and New York, NY, USA, 2007.

# Reserve slides

# F. Bretherton's "horrendogram" of Earth System Science

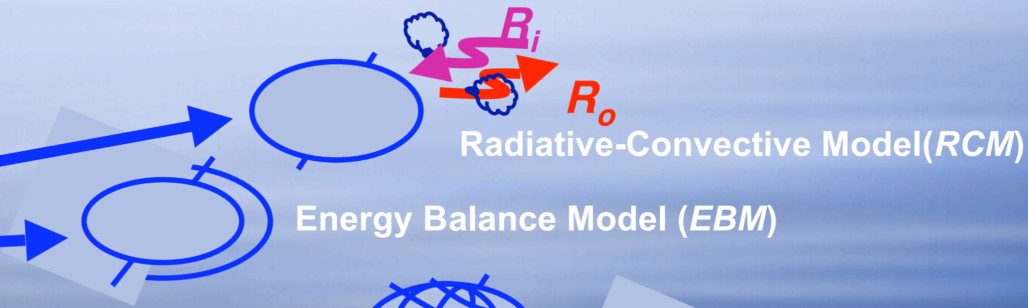


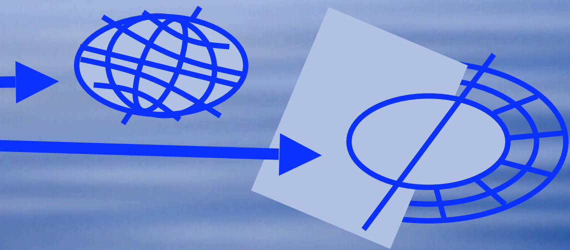

Earth System Science Overview, *NASA Advisory Council*, 1986

# Climate models (atmospheric & coupled) : A classification

- ***Temporal***
  - stationary, (quasi-)equilibrium
  - transient, climate variability

- ***Space***
  - 0-D (dimension 0)
  - 1-D
    - vertical
    - latitudinal
  - 2-D
    - horizontal
    - meridional plane
  - 3-D, GCMs (General Circulation Model)
    - horizontal
    - meridional plane
  - Simple and intermediate 2-D & 3-D models



- ***Coupling***
  - Partial
    - unidirectional
    - asynchronous, hybrid
  - Full

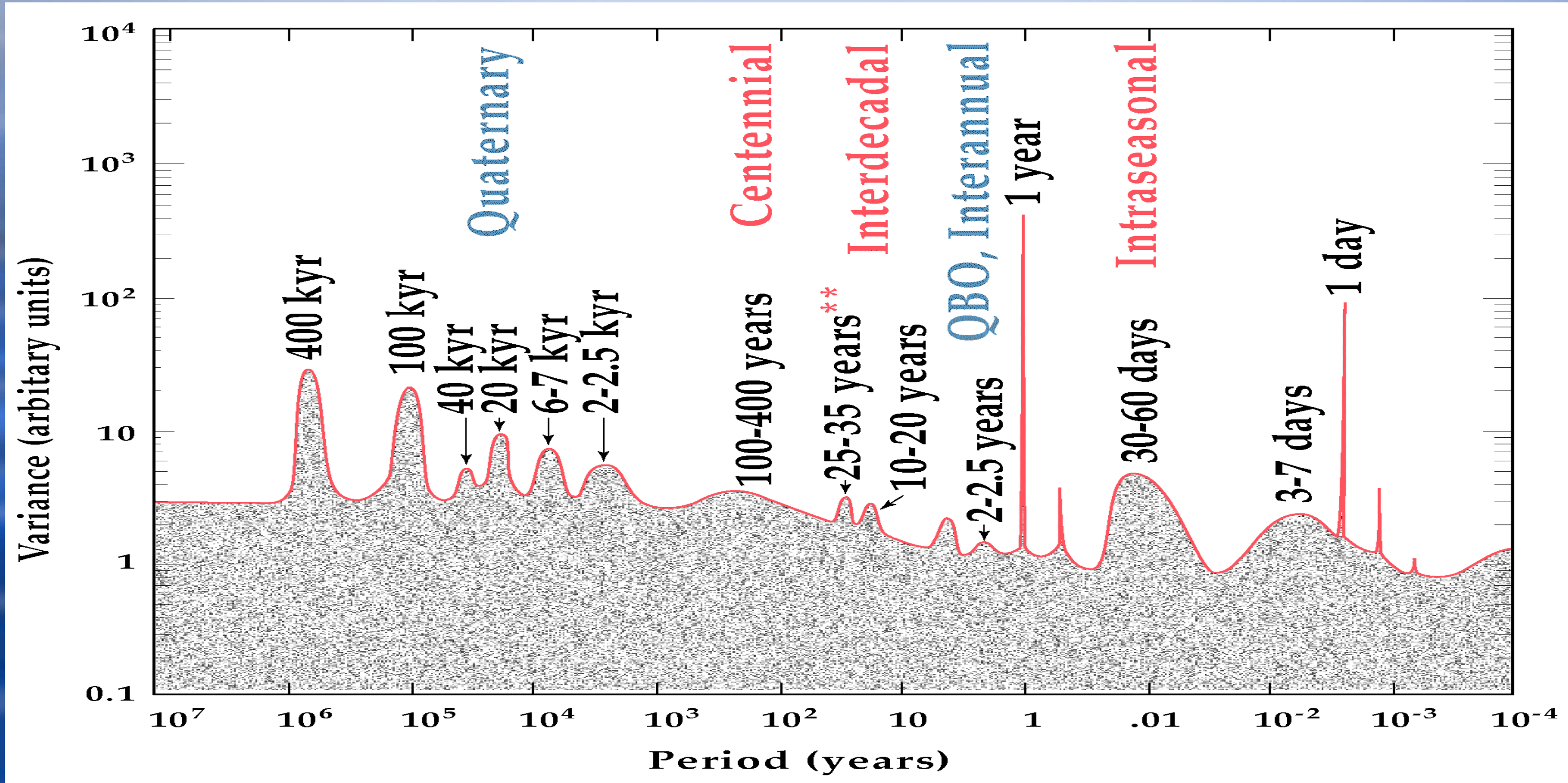
**Hierarchy:** from the simplest to the most elaborate,
iterative comparison with the observational data

# Composite spectrum of climate variability

**Standard treatement of frequency bands:**

***1. High frequencies – white (or "colored") noise***

***2. Low frequencies – slow ("adiabatic") evolution of parameters***

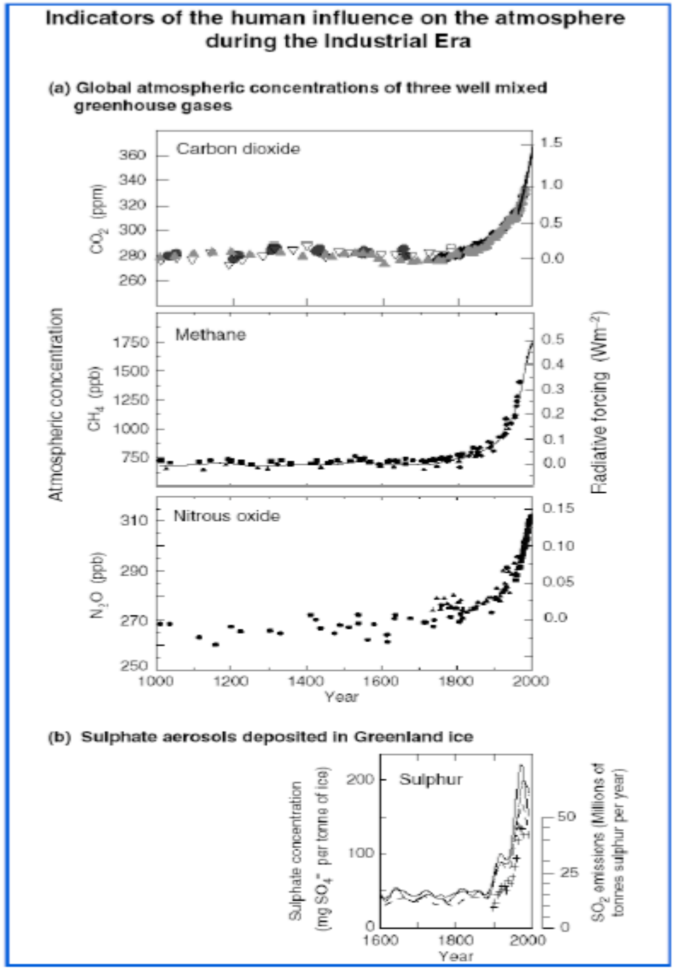



From **Ghil (2001, EGEC)**, after **Mitchell* (1976)**

* "**No known source of deterministic internal variability**"

# GHGs rise

It’s gotta do with us, at least a bit, ain’t it?




*IPCC* (2001)

# The “hockey stick” & beyond

The “hockey stick” of TAR (3rd Assesment Report) is a typically (over)simplified version of much more detailed and reliable knowledge.

National Research Council, 2006: *Surface Temperature Reconstructions For the Last 2000 Years*. National Academies Press, Washington, DC, 144 pp.
http://www.nap.edu/openbook.php?record_id=11676&page=2



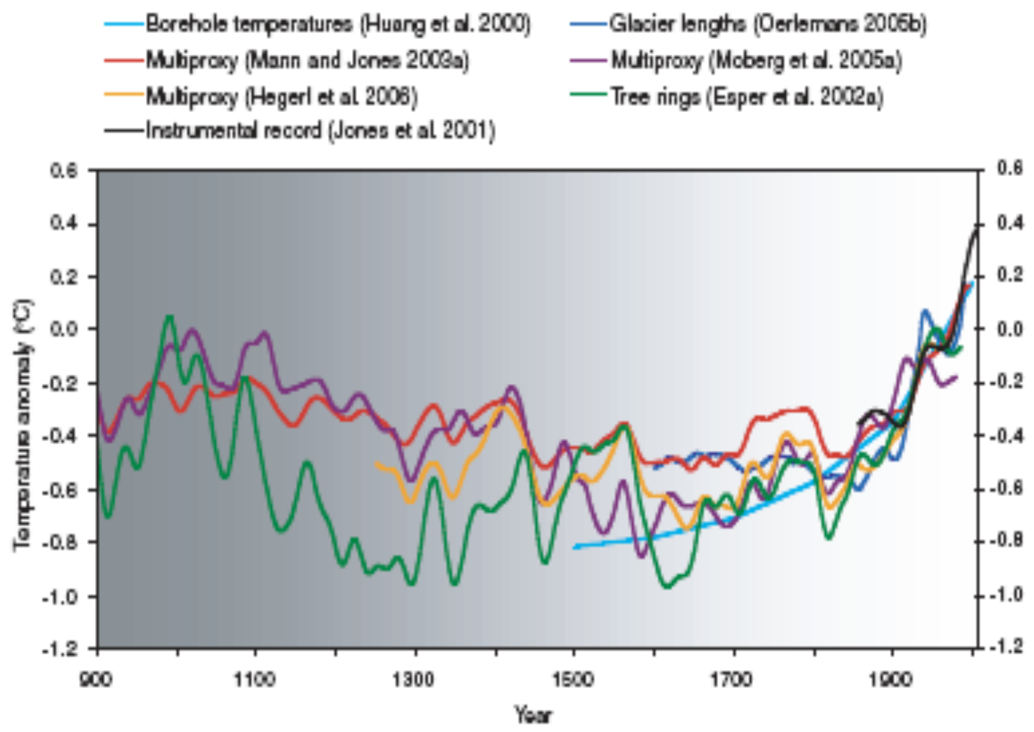
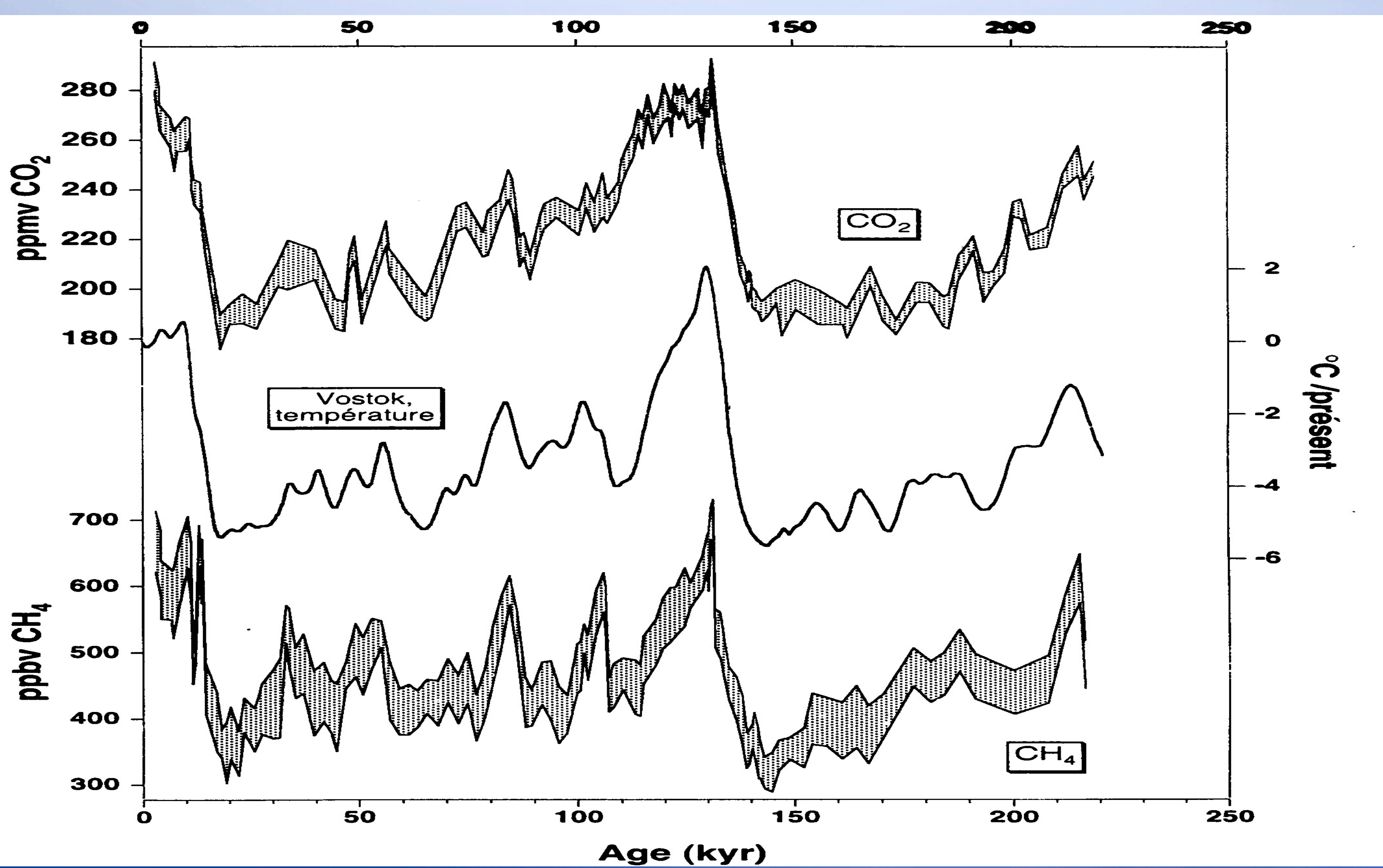
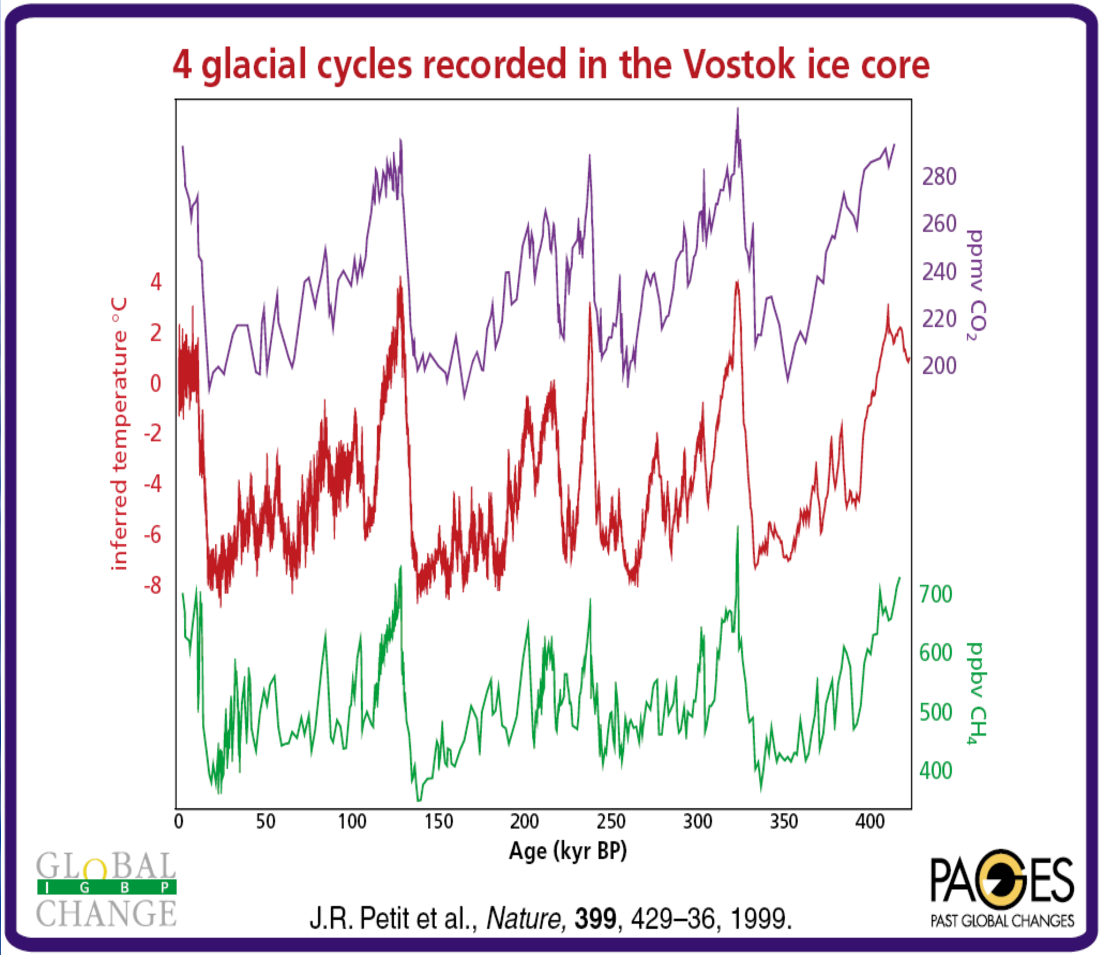



**FIGURE S-1** Smoothed reconstructions of large-scale (Northern Hemisphere mean or global mean) surface temperature variations from six different research teams are shown along with the instrumental record of global mean surface temperature. Each curve portrays a somewhat different history of temperature variations and is subject to a somewhat different set of uncertainties that generally increase going backward in time (as indicated by the gray shading). This set of reconstructions conveys a qualitatively consistent picture of temperature changes over the last 1,100 years and especially over the last 400. See Figure O-5 for details about each curve.




**Isotopic (proxy) temperatures and GHGs at Vostok, over the last glacial cycle**; courtesy of **P. Yiou**

# $T_s$ and GHGs over 400 kyr




*The same lead–lag relations are apparent over these 4 glacial cycles …*

# Extreme Events:
# Causes and Consequences
# (E2-C2)

- EC-funded project bringing together researchers in mathematics, physics, environmental and socio-economic sciences.
- €1.5M over three years (March 2005–Feb. 2008).
- Coordinating institute: Ecole Normale Supérieure.
- 17 'partners' in 9 countries.
- 72 scientists + 17 postdocs/postgrads.
- PEB: M. Ghil (ENS, Paris, P.I.), S. Hallegatte (CIRED), B. Malamud (KCL, London), A. Soloviev (MITPAN,Moscow), P. Yiou (LSCE, Gif s/Yvette, Co-P.I.)

# Sun-Climate Relations

- It ain't new:
  v. ~1000 papers (in 1978!), as well as Marcus *et al.* (1998, *GRL*).
- "Corrélation n'est pas raison."
- Requires serious study of solar physics.

Climatology Supplement

*Nature* 276, 348 - 352 (23 November 1978); doi:10.1038/276348a0

## Solar–terrestrial influences on weather and climate

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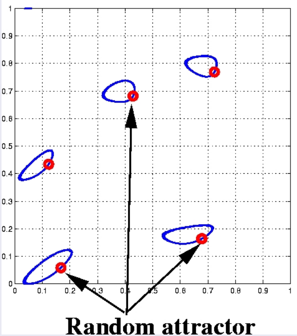
During the past century over 1,000 articles have been published claiming or refuting a correlation between some aspect of solar activity and some feature of terrestrial weather or climate. Nevertheless, the sense of progress that should attend such an outpouring of 'results' has been absent for most of this period. The problem all along has been to separate a suspected Sun–weather signal from the characteristically noisy background of both systems. The present decade may be witnessing the first evidence of progress in this field. Three independent investigations have revealed what seem to be well resolved Sun–weather signals, although it is still too early to have unreserved confidence in all cases. The three correlations are between terrestrial climate and Maunder Minimum-type solar activity variations, a regional drought cycle and the 22-yr solar magnetic cycle, and winter hemisphere atmospheric circulation and passages by the Earth of solar sector boundaries in the solar wind. The apparent emergence of clear Sun–weather signals stimulated numerous searches for underlying physical causal links.

# A short analysis of the noise effect from RDS theory

- The web of resonance is nonlinearly altered. It is linked with stochastic normal form theory.
- This web lives in a sea of "chaos + noise".
- A random attractor computed on a partial resonance region:



# Concluding remarks

## Some insights

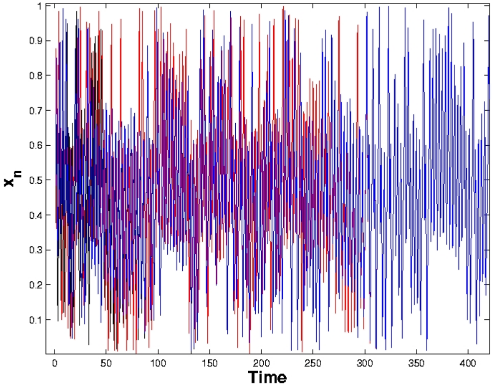
- Reduction of the attractor dimension: $\lim_{\sigma\to 0}\dim\{\mathcal{A}_\sigma(\omega)\} < \dim\mathcal{A}_0$ as the noise intensity $\sigma \to 0$.
- Stochastic parametrization $\Rightarrow$ gain of structural stability for random attractors.
- These results hold for **relevant deterministic models** that are stochastically perturbed.
- RDS theory offers a meaningful framework for performing classification in stochastic modeling.

## Some perspectives

- Effect of colored noises and lag-correlation on stochastic classes.
- Accurate description of noise on non-hyperbolic chaos (Lorenz system, Newhouse phenomena, Hénon map...)

# An example of random attractor for Arnold's family

- Several trajectories for different initial data and one single realization $\omega$:




**Conclusion:** Noise transforms the deterministic 1-D attractor to a random fixed point attractor (0-D!)