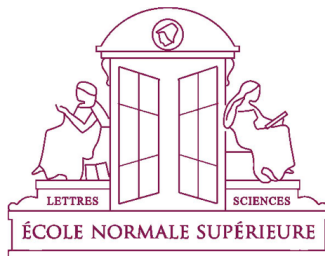


A Case Study of Tipping Points: The Wind-Driven Double-Gyre Problem

Michael Ghil

**Ecole Normale Supérieure, Paris, and
University of California, Los Angeles**



Please visit these sites for more info.

<http://www.atmos.ucla.edu/tcd/>

<http://www.environnement.ens.fr/>

Motivation

- The **North Atlantic Oscillation (NAO)** is a leading mode of **variability** of the Northern Hemisphere and beyond.
- It affects **the atmosphere and oceans** on several **time and space scales**.
- Its **predictive understanding** could help interannual and **decadal-scale climate prediction** over and around the North Atlantic basin.
- The **hierarchical modeling** approach allows one to give proper weight to the **understanding provided by the models vs. their realism**, respectively.
- Back-and-forth between **“toy”** (conceptual) and **detailed** (“realistic”) **models**, and between **models** and **data**.

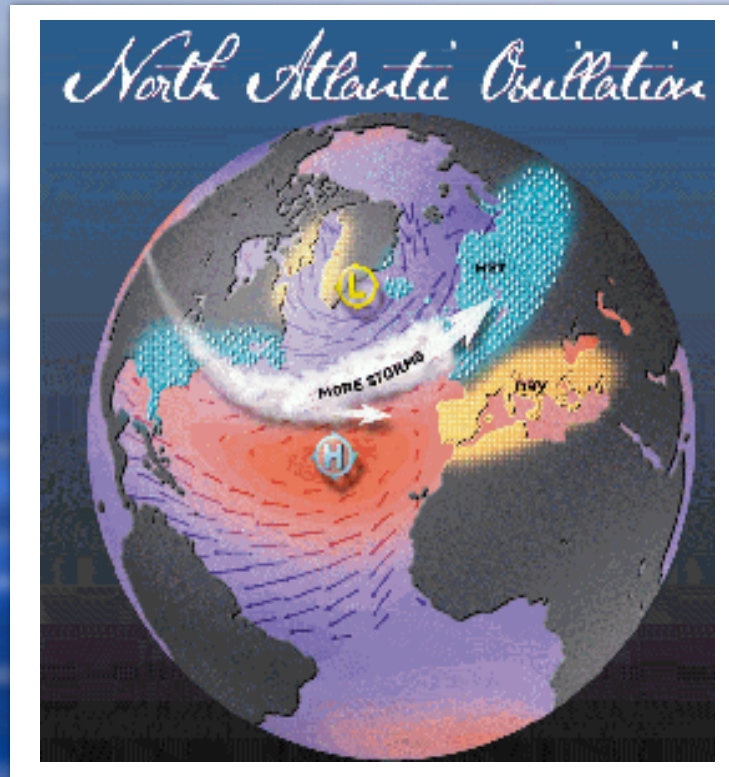
Joint work with *F. Codron, H. A. Dijkstra, Y. Feliks, S. Jiang, F.-F. Jin, H. Le Treut, E. Simonnet, S. Speich, and S. Wang*

Outline, Tipping Points II

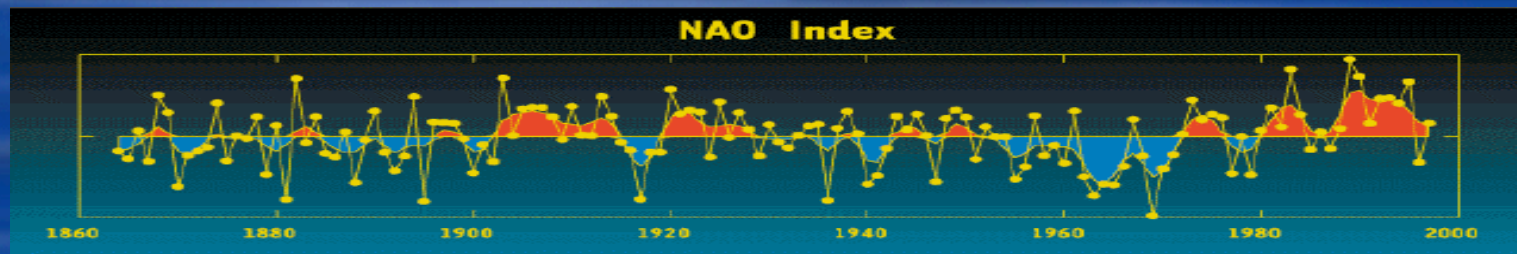
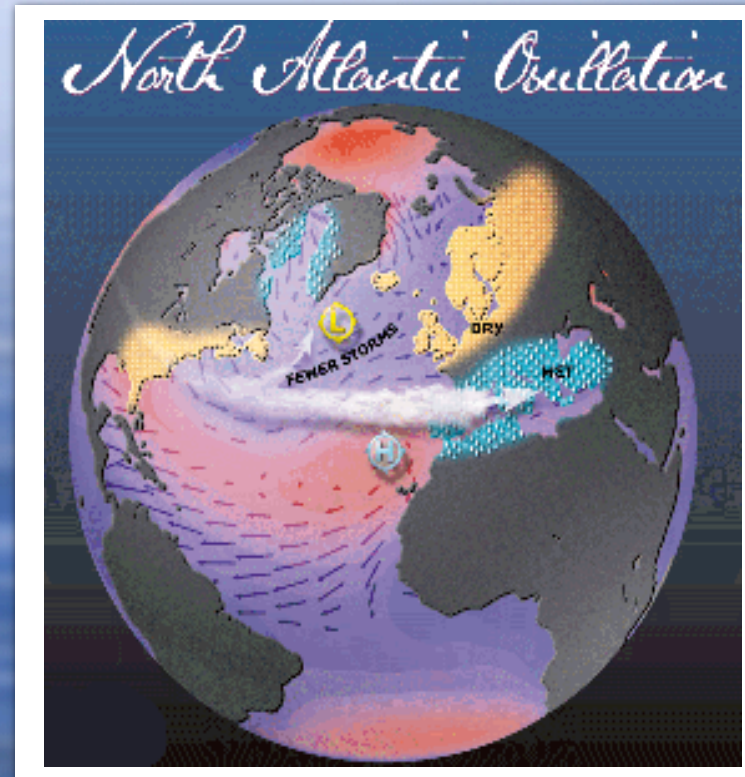
- ♦ The NAO and the oceans' wind-driven circulation
- ♦ The low-frequency variability of the double-gyre circulation
 - bifurcations in a toy model
 - ➔ multiple equilibria, periodic and chaotic solutions
 - some intermediate model results
- ♦ Atmospheric impacts
 - simple and intermediate models + GCMs
- ♦ Some data analysis – atmospheric and oceanic
- ♦ Some very promising NAO results
- ♦ Conclusions and bibliography

The North Atlantic Oscillation (NAO)

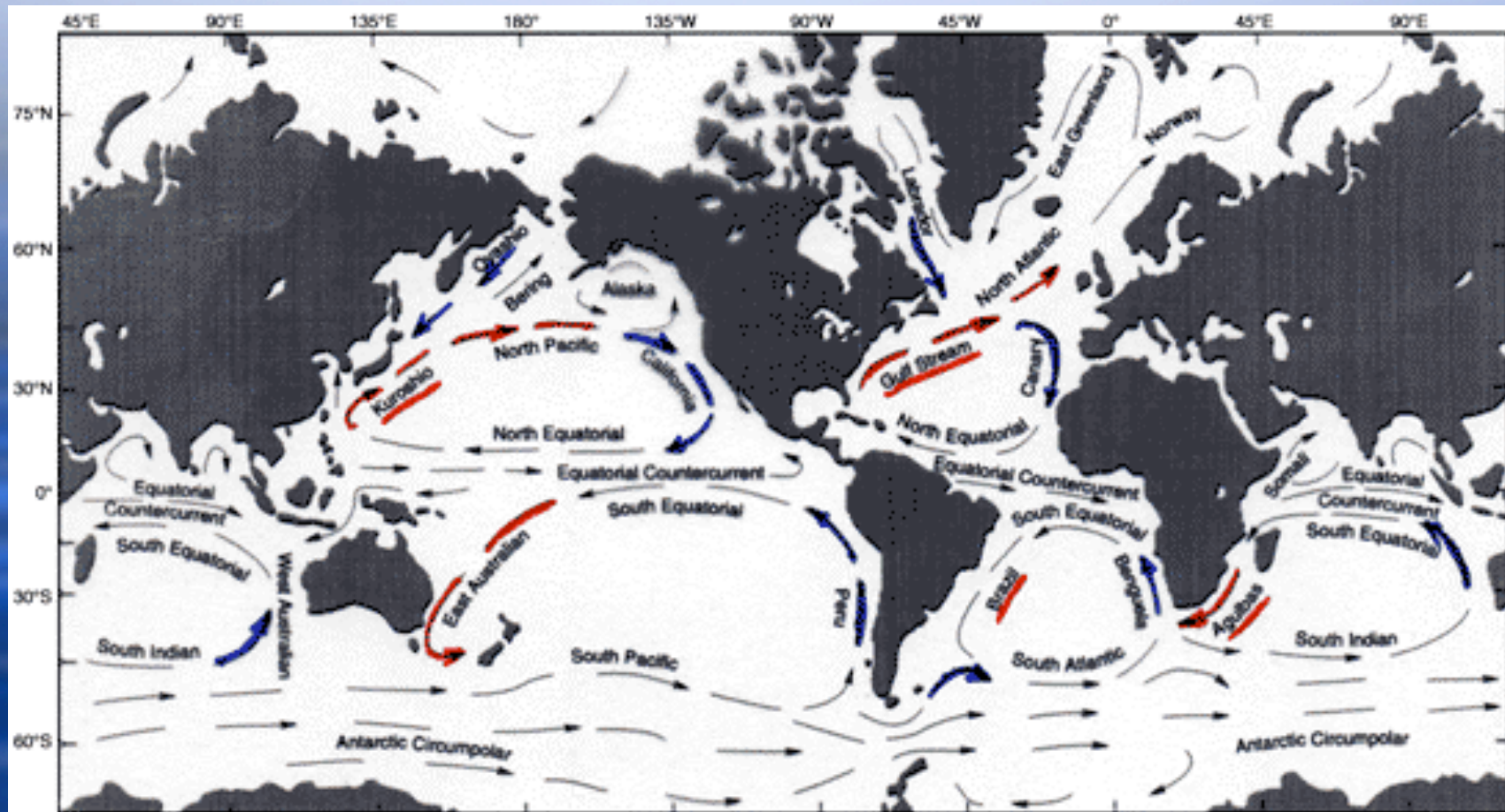
Positive phase



Negative phase



An example of bifurcations and hierarchical modeling: The oceans' wind-driven circulation



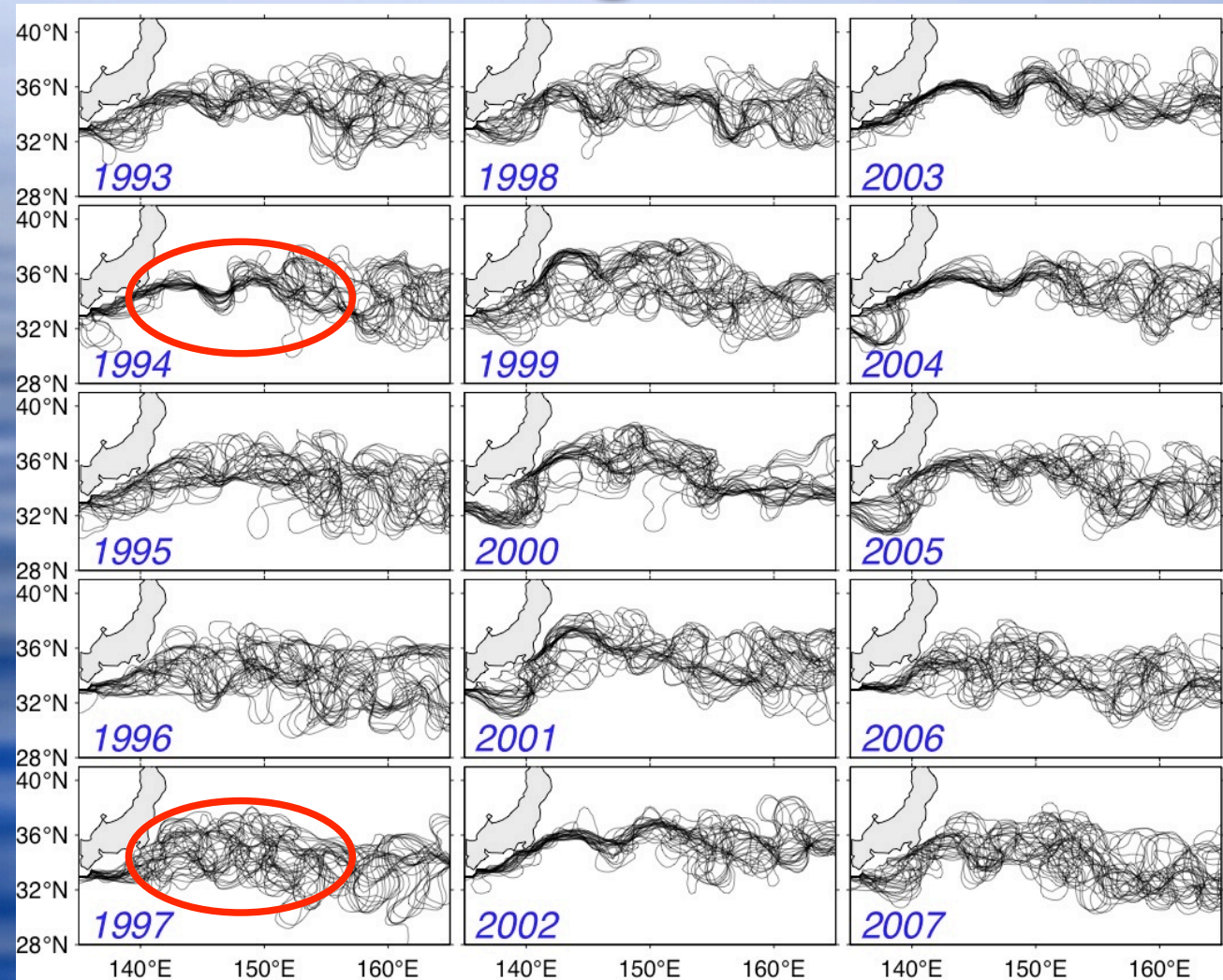
J. Apel (1987), Principles of Ocean Physics

The mean surface currents are (largely) wind-driven

Kuroshio Extension (KE) Path Changes

Monthly
paths from
altimeter:
Stable vs.
unstable
periods

Qiu & Chen
(*Deep-Sea Res.*, 2009)



“Limited-contour” analysis for atmospheric low-frequency variability

*10-day sequences of
subtropical jet paths:
blocked vs. zonal
flow regimes*

Kimoto & Ghil, JAS, 1993a

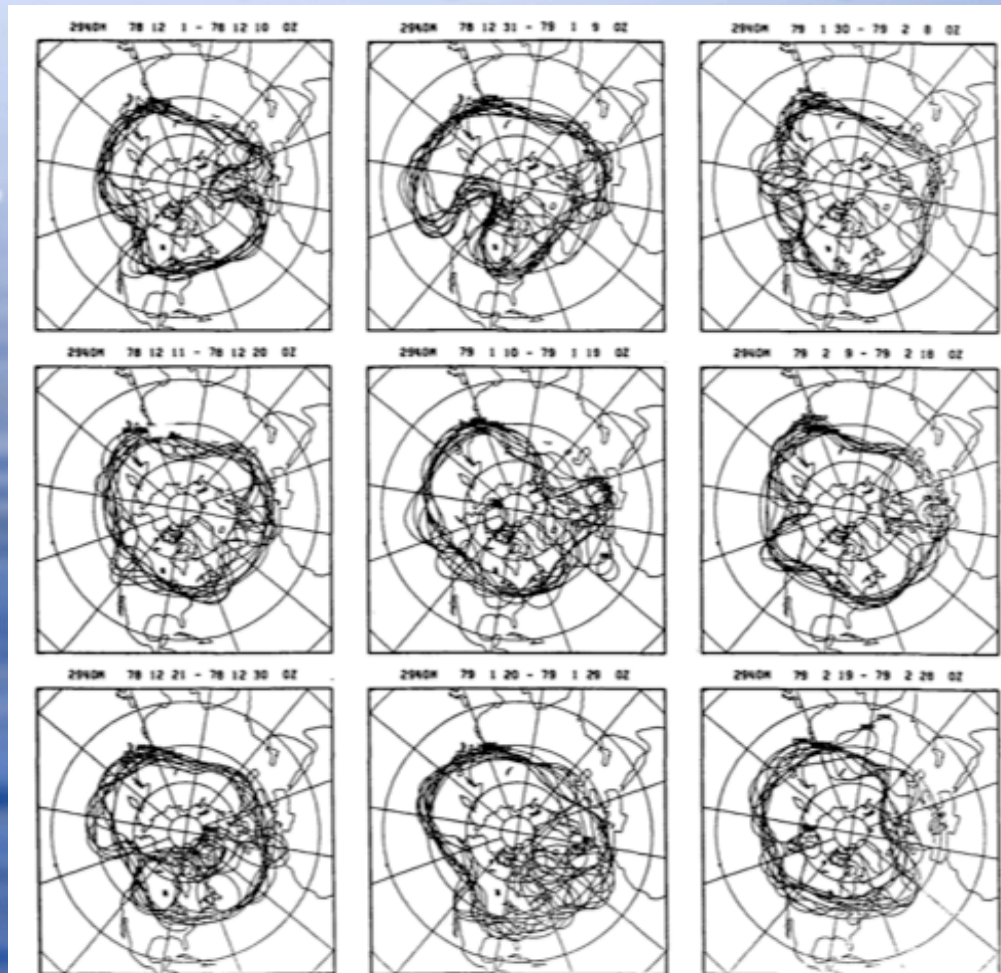


FIG. 1. Limited contour analysis of Northern Hemisphere (NH) flows. Daily contours of a prescribed height (2940 m in this case—roughly corresponding to the jet axis) are superimposed for successive 10-day intervals during NH winter 1978/79. Persistence is illustrated by some of the panels (see text for details).

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- ♦ The low-frequency variability of the double-gyre circulation
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 - => multiple equilibria, periodic and chaotic solutions
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- ♦ Conclusions
 - The coupled climate system: is it the tail or the dog?
 - Natural climate variability: a source of decadal predictability?

The double-gyre circulation and its low-frequency variability

An “intermediate” model of the mid-latitude, wind-driven ocean circulation: 20-km resolution, about 15 000 variables

Shallow-water model

$$\begin{aligned}\frac{\partial U}{\partial t} + \nabla \cdot (\mathbf{u}U) &= -g'h \frac{\partial h}{\partial x} + fV + \underline{\alpha_A} A \nabla^2 U - RU - \underline{\alpha_\tau} \frac{\tau^x}{\rho} \\ \frac{\partial V}{\partial t} + \nabla \cdot (\mathbf{u}V) &= -g'h \frac{\partial h}{\partial y} - fU + \underline{\alpha_A} A \nabla^2 V - RV \\ \frac{\partial h}{\partial t} &= -\left(\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y}\right)\end{aligned}$$

where

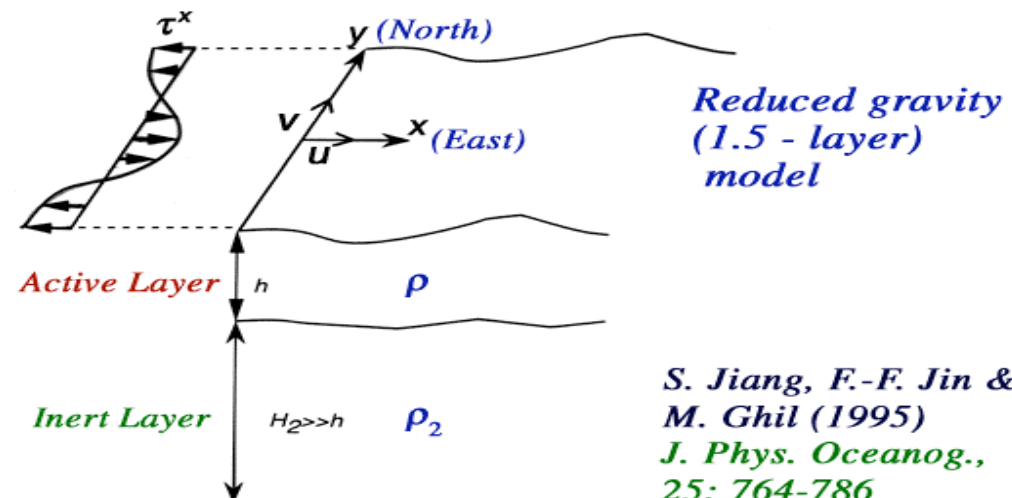
$$U\hat{e}_x + V\hat{e}_y = h\mathbf{u} = h(u\hat{e}_x + v\hat{e}_y)$$

g' : reduced gravity $(= g(\rho_2 - \rho)/\rho)$

A : viscosity coefficient $(= 300 \text{ m}^2\text{s}^{-1})$

R : Rayleigh coefficient $(= 1/200 \text{ day}^{-1})$

τ^x : wind stress $= \tau_0 \cos 2\pi/L$ ($\tau_0 = 1 \text{ dyn cm}^{-2}$ & $L = 2000 \text{ km}$)

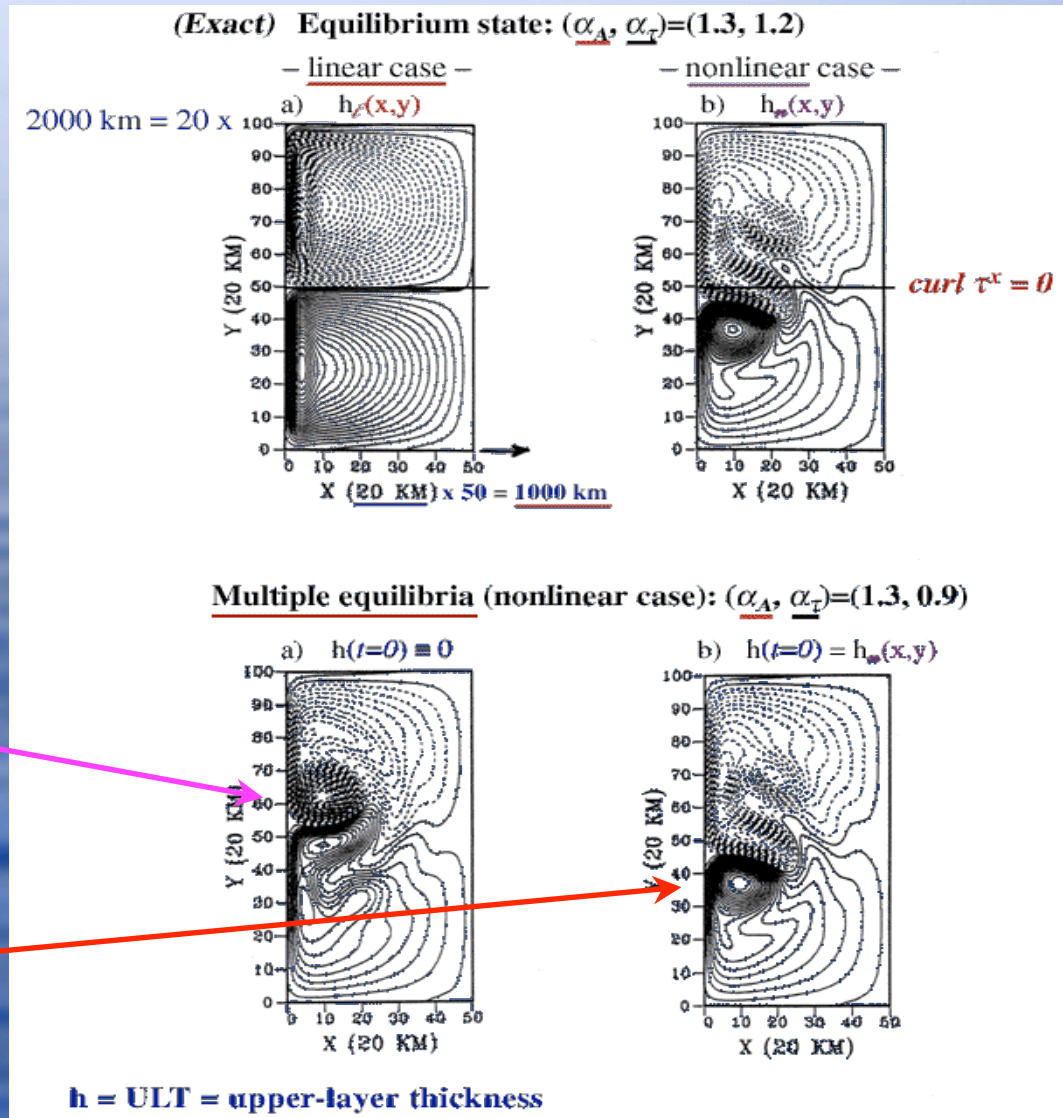


The JJG model's equilibria

Nonlinear (advection) effects break the (near) symmetry: (perturbed) pitchfork bifurcation?

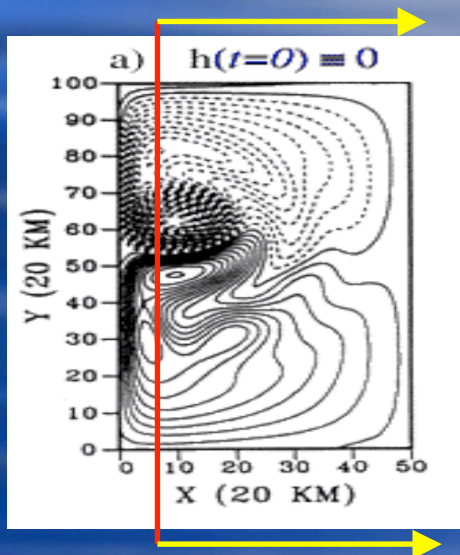
Subpolar gyre dominates

Subtropical gyre dominates



Time-dependent solutions: periodic and chaotic

To capture space-time dependence, meteorologists and oceanographers often use Hovmöller diagrams

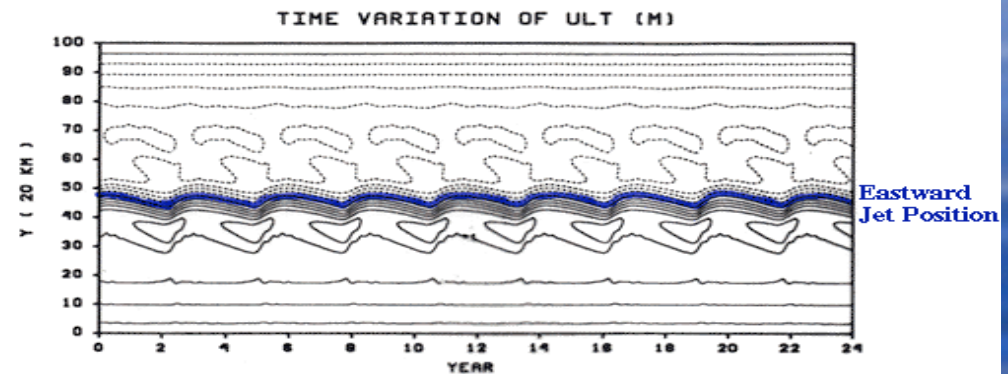


Time-dependent solutions

1. Periodic, w/ interannual period (2.8 years)

$$\alpha_A = 1.0$$

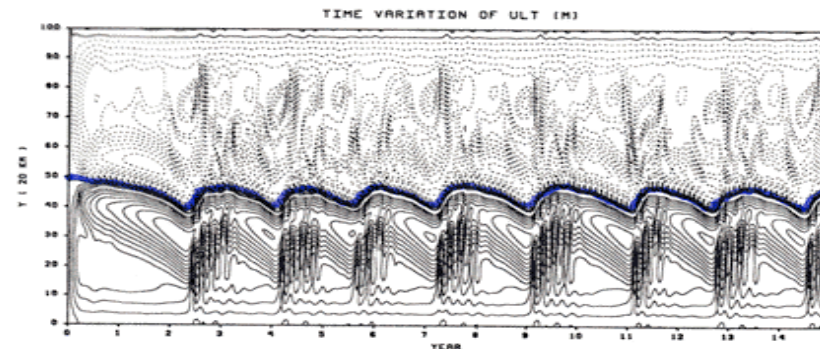
$$\alpha_\tau = 0.8$$



2. Aperiodic (weakly chaotic)

$$\alpha_A = 1.0$$

$$\alpha_\tau = 1.6$$

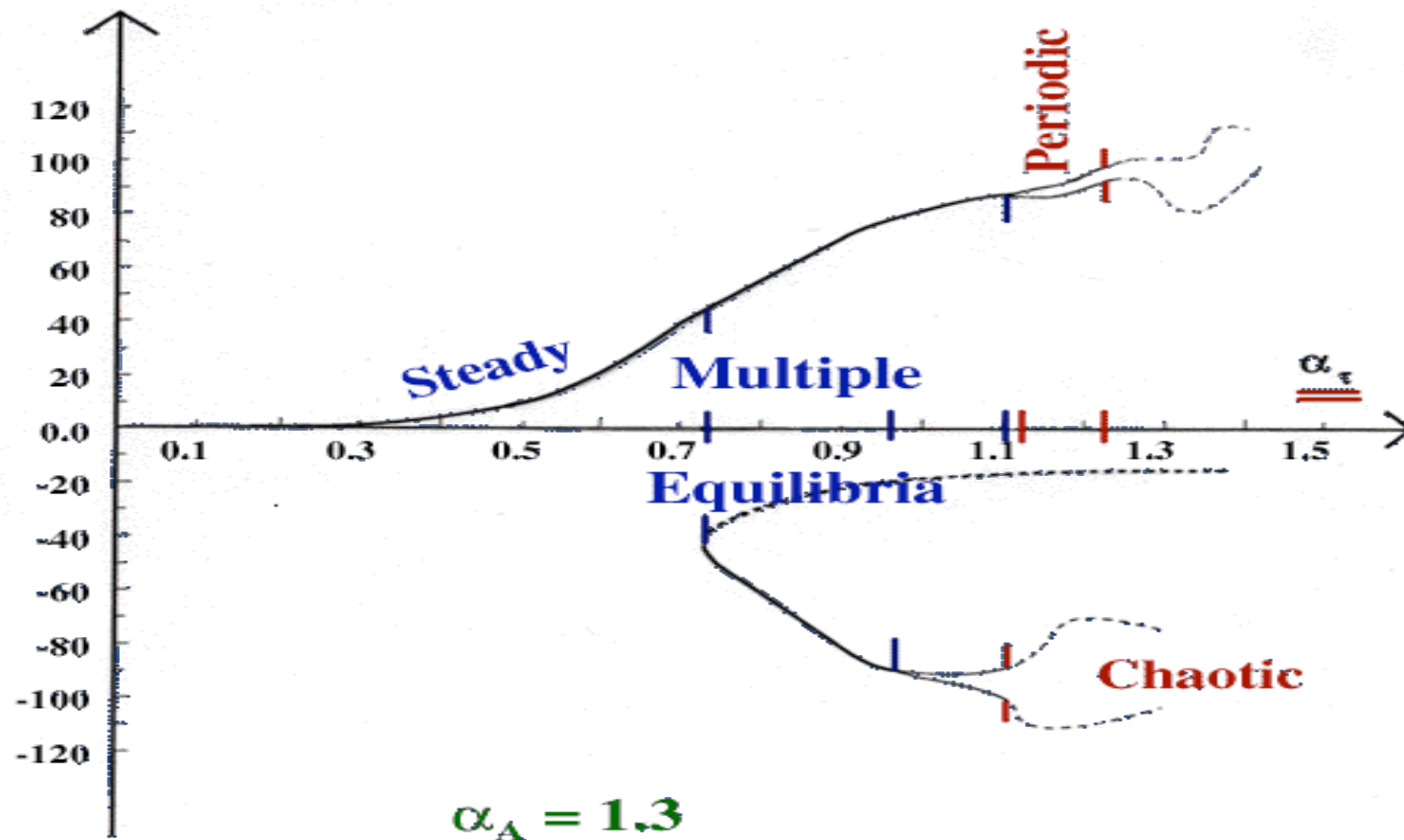


Poor man's continuation method

Bifurcation diagram

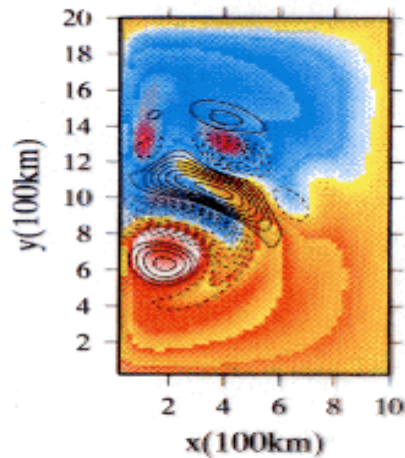
Perturbed pitchfork + Hopf + transition to chaos

Position of Merging Point (km)

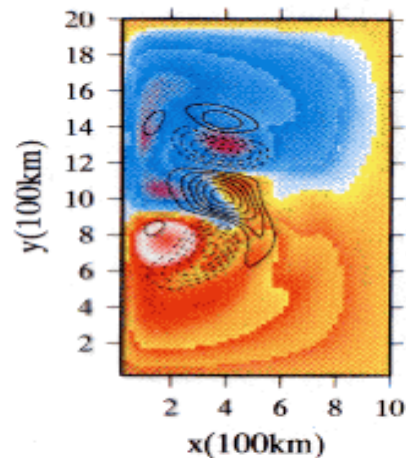


Interannual variability: relaxation oscillation

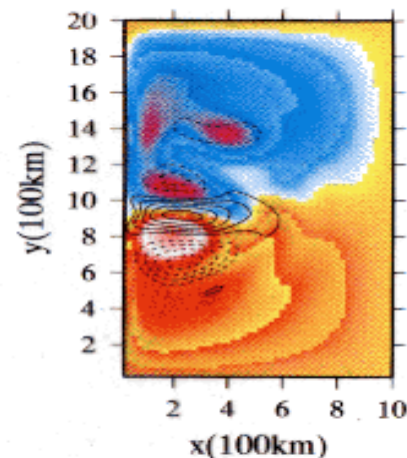
0 years



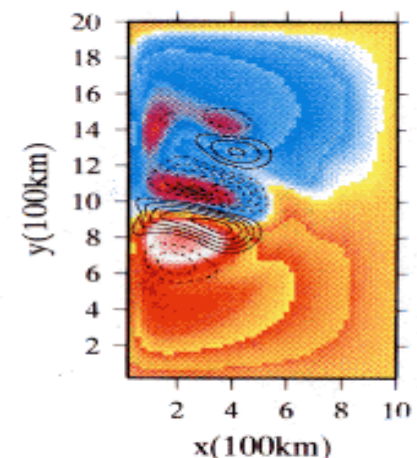
0.4 years



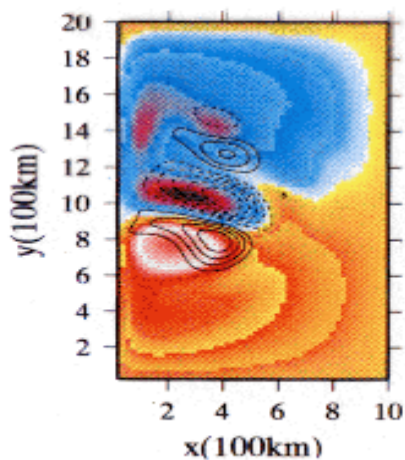
0.8 years



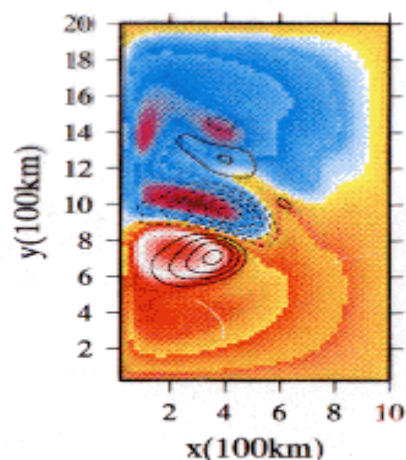
1.2 years



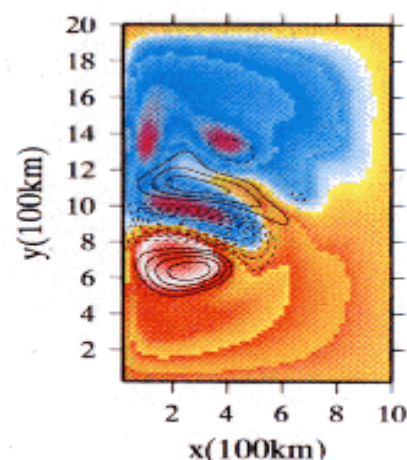
1.6 years



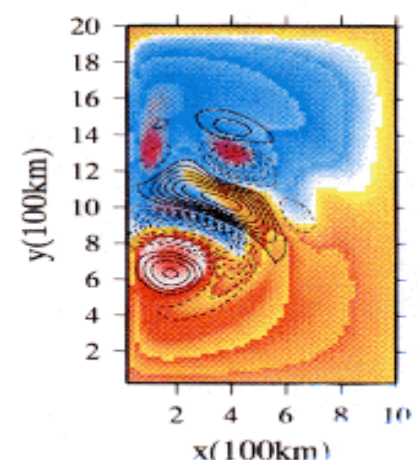
2.0 years



2.4 years



2.8 years



Global bifurcations in “intermediate” models

Bifurcation tree in a QG,
equivalent-barotropic,
high-resolution (10 km)
model: pitchfork,
mode-merging,
Hopf, and homoclinic

2005]

Simonnet et al.: Quasi-geostrophic double-gyre circulation

937

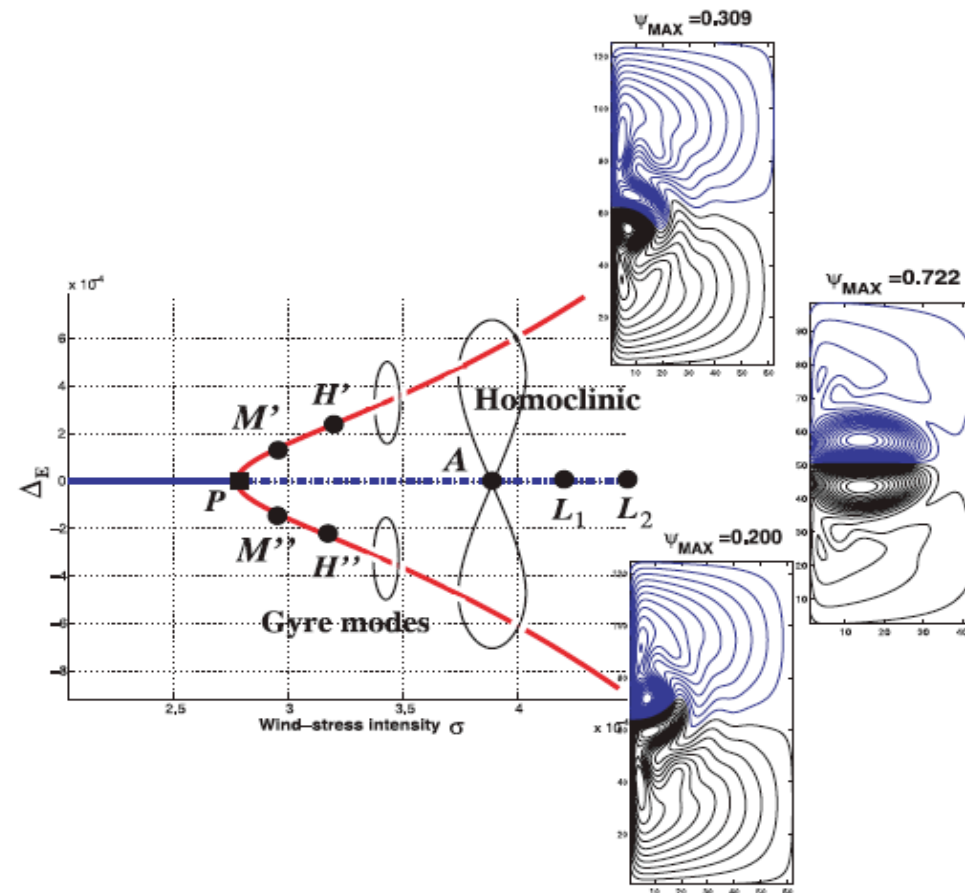


Figure 1. Schematic bifurcation diagram of an equivalent-barotropic QG model, plotted in terms of an asymmetry measure Δ_E (see Section 3a further below) vs. wind-stress intensity. The limit cycles are schematically drawn for illustrative purpose and the streamfunction patterns corresponding to the three steady-state branches—subtropical, antisymmetric, and subpolar (from top to

Homoclinic orbit: numerical and analytical

2005]

Simonnet et al.: Quasi-geostrophic double-gyre circulation

939

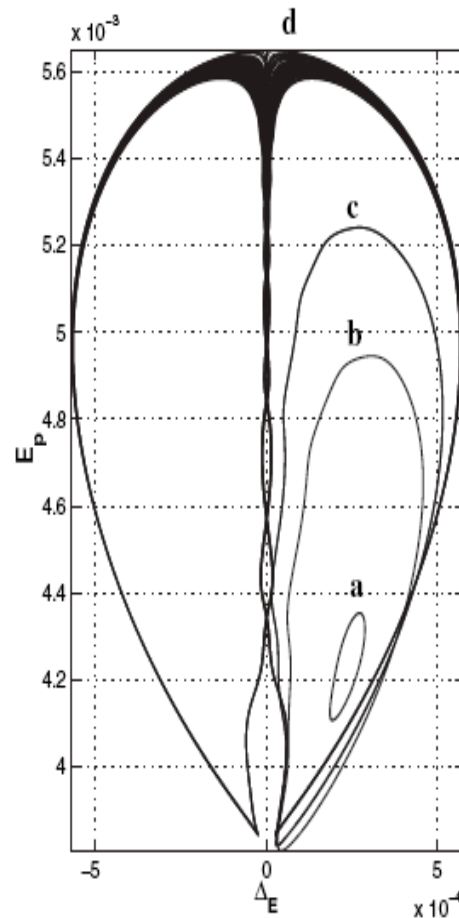


Figure 2. Unfolding of the relaxation oscillations induced by the gyre modes, shown in the plane spanned by the total potential energy of the solution E_p and the difference Δ_E between the subpolar potential energy and the subtropical one (see text for details). The orbits of several limit cycles are

2005]

Simonnet et al.: Quasi-geostrophic double-gyre circulation

941

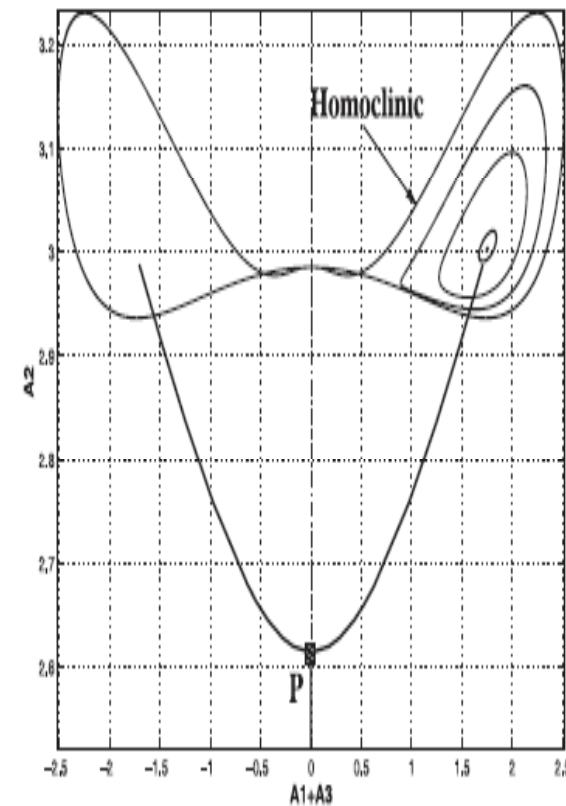


Figure 3. Bifurcation diagram of the highly truncated, four-mode model (5), projected onto the $(A_1 + A_3, A_2)$ plane for $\mu = 1$ and $s = 2$; P stands for pitchfork bifurcation at $\sigma = \sigma_p = 7.61$, while $\sigma = \sigma_{hc} \approx 10.4299$ at the homoclinic bifurcation. The branches of periodic orbits are replaced by several explicitly computed limit cycles.

The double-gyre circulation: A different rung of the hierarchy

Another “intermediate” model of the double-gyre circulation: slightly different physics, higher resolution – down to 10 km in the horizontal and more layers in the vertical, much larger domain, ...

Bo Qiu, U. of Hawaii,
pers. commun., 1997

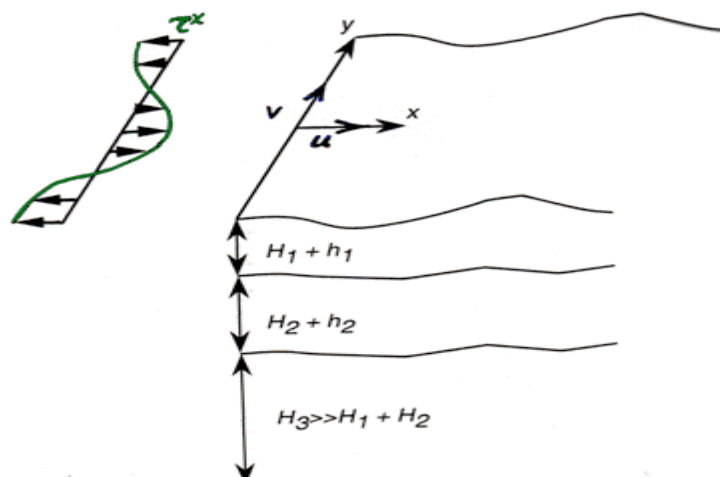
2.5-layer model

$$\begin{aligned} \frac{\partial}{\partial t}(\nabla^2 h_1 - \lambda_1^2(h_1 - h_2)) + \beta \frac{\partial h_1}{\partial x} = & -\frac{g'}{f_0} J[h_1, \nabla^2 h_1 - F_1(h_1 - h_2)] \\ & + A_h \nabla^4 h_1 - C \nabla^2(h_1 - h_2) + \frac{f_0}{\rho_0 g' H_1} \text{curl} \vec{\tau} \\ \frac{\partial}{\partial t}(\nabla^2 h_2 - \lambda_2^2(h_2 - h_1)) + \beta \frac{\partial h_2}{\partial x} = & -\frac{g'}{f_0} J[h_2, \nabla^2 h_2 - F_2(h_2 - h_1)] \\ & + A_h \nabla^4 h_2 - C \nabla^2(h_2 - h_1) - R \nabla^2 h_2 \end{aligned}$$

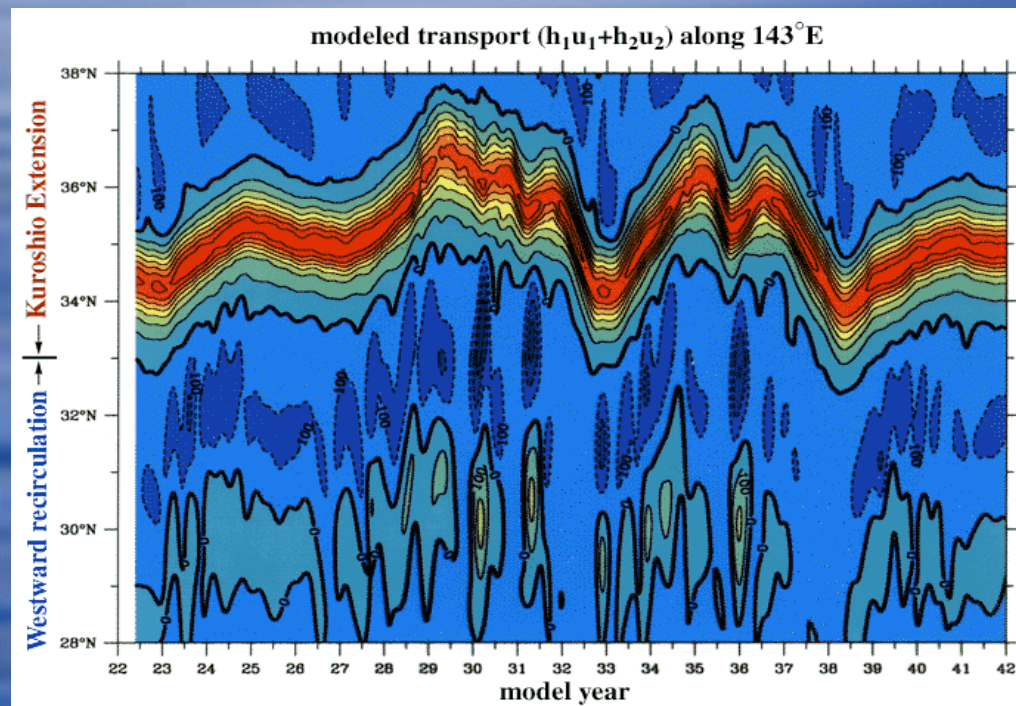
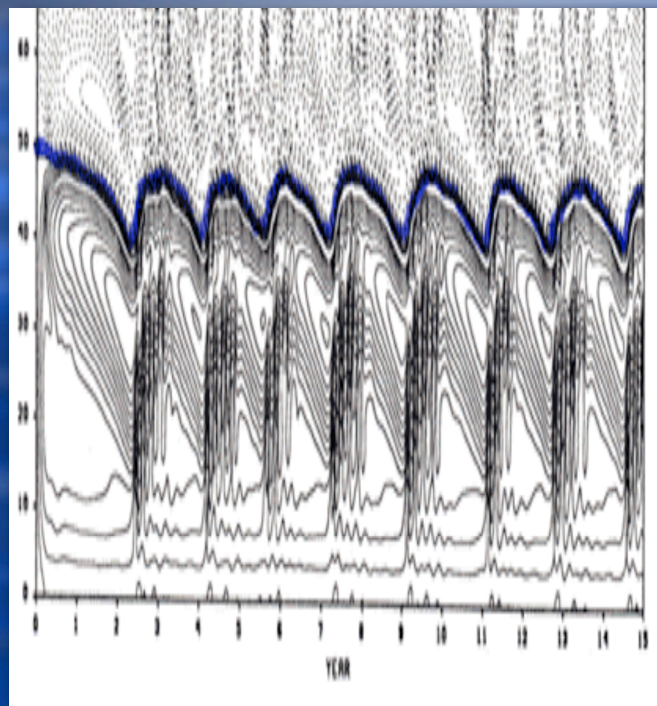
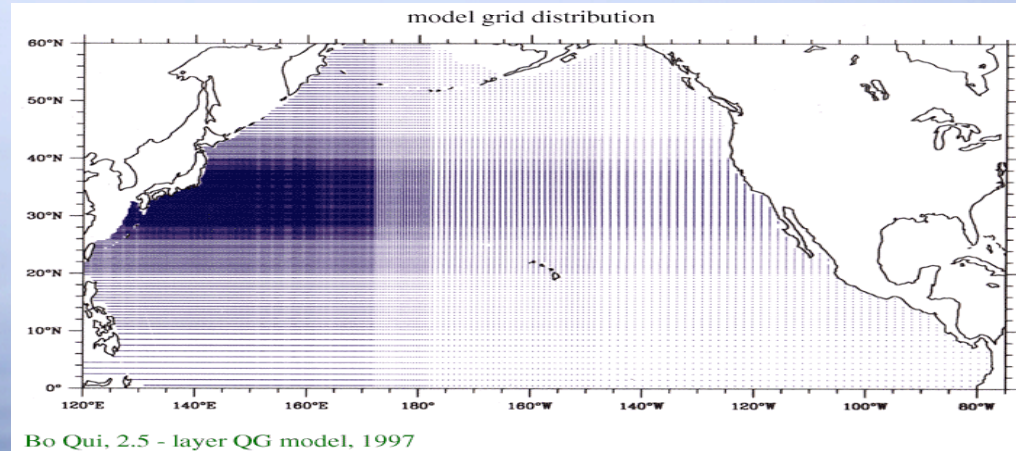
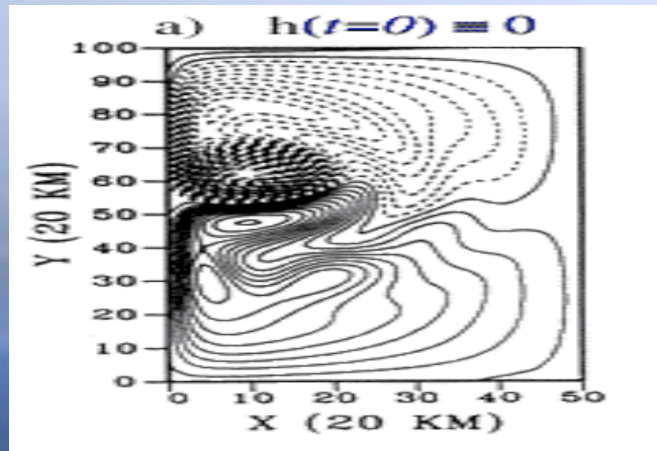
where

- h_1, h_2 : height anomaly for upper and lower layer (stream functions)
- H_1, H_2 : mean height for upper and lower layer
- λ_1, λ_2 : Rossby radius of deformation $\equiv \sqrt{h' H_1 / f_0^2}, \sqrt{h' H_2 / f_0^2}$
- $\vec{\tau}$: wind stress
- A_h : viscosity coefficient
- C, R : Rayleigh coefficient for interface and lower layer
- f_0, β : Coriolis and beta parameters
- ρ_0, g' : mean density and reduced gravity

Quasi - geostrophic model



Model-to-model, qualitative comparison



Model-and-observations, quantitative comparison

Spectra of
(a) kinetic energy of
2.5-layer shallow-water
model in North-Atlantic–
shaped basin; and
(b) Cooperative Ocean-
Atmosphere Data Set
(COADS) Gulf-Stream
axis data

2005]

Simonnet et al.: Quasi-geostrophic double-gyre circulation

947

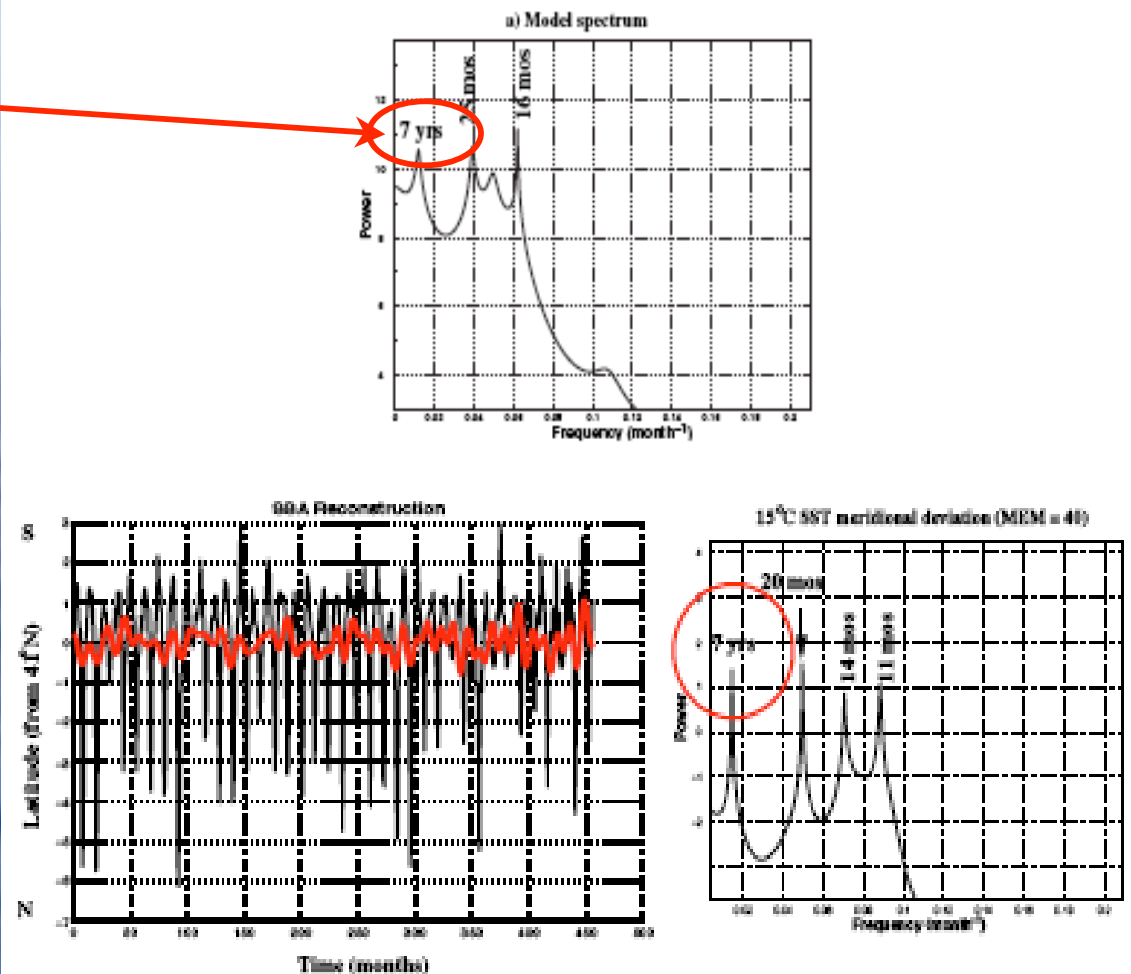


Figure 7. Comparison between low-frequency variability in an idealized double-gyre model and in observations of the Gulf Stream axis. (a) Spectral results for a 2.5-layer SW model for a basin that approximates the North Atlantic in size and shape, using an idealized wind stress. Maximum

More spatio-temporal data

Multi-channel SSA analysis of the UK Met Office monthly mean SSTs for the century-long 1895–1994 interval

Marked similarity with the 7–8-year “gyre mode” of a full hierarchy of ocean models, on the one hand, and with the North Atlantic Oscillation (NAO), on the other: explanation?

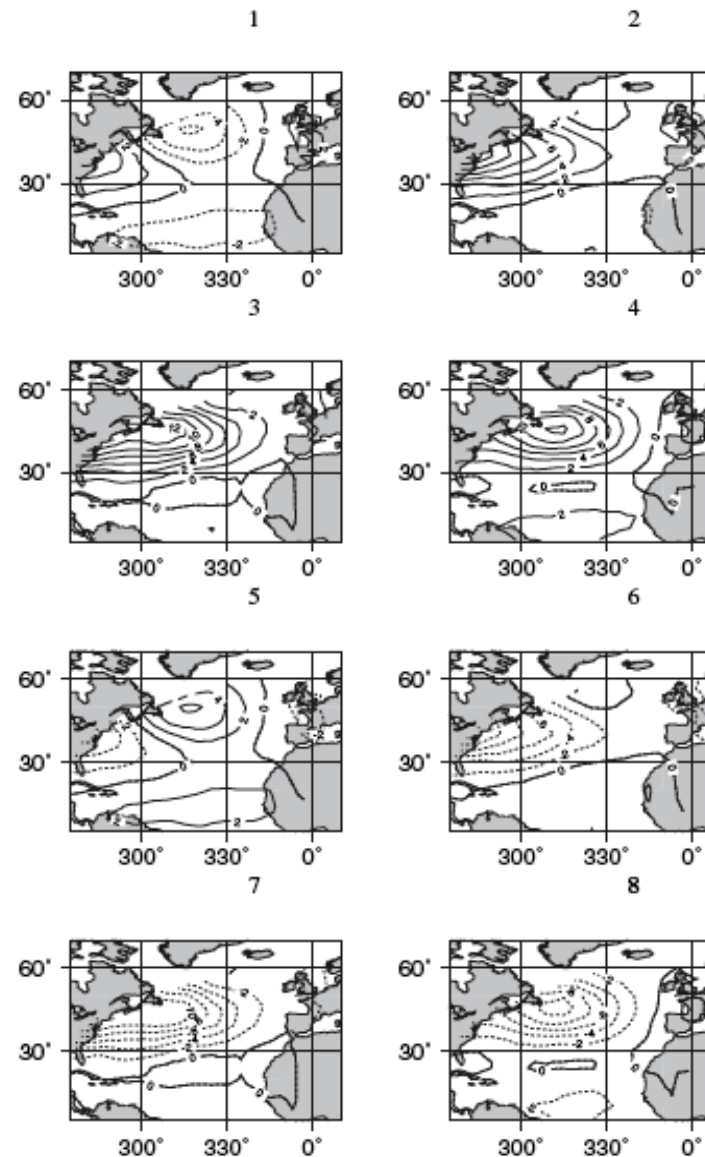
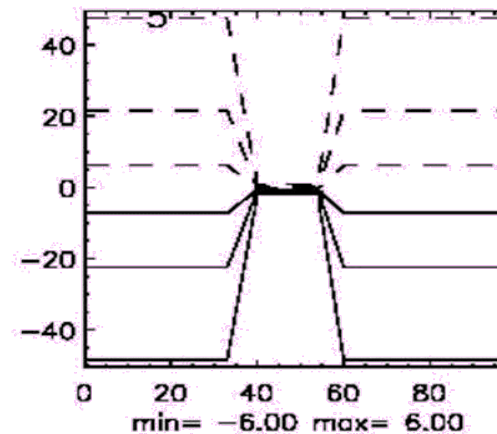


Figure 8. Phase composites of the reconstructed 7–8-year SST oscillation. The MSSA window length is 40 year and the contour interval is 0.02°C.

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Atmospheric impact of mid-latitude SST anomalies: **A highly contentious issue**



- ◆ A quasi-geostrophic (QG) atmospheric model in a periodic β -channel, first barotropic (Feliks *et al.*, *JAS*, 2004; FGS'04), then baroclinic (FGS'07).
- ◆ Marine atmospheric boundary layer (ABL), analytical solution.
- ◆ Forcing by idealized oceanic SST front.

Ocean-atmosphere coupling mechanism (II)

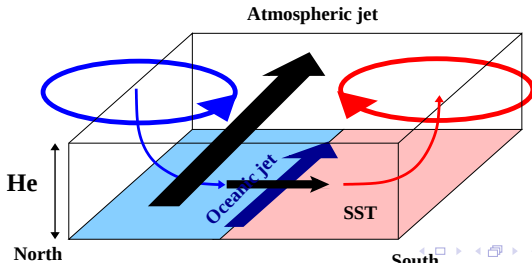
Vertical velocity at the top of the marine ABL

- The nondimensional $w(H_e)$ is given by

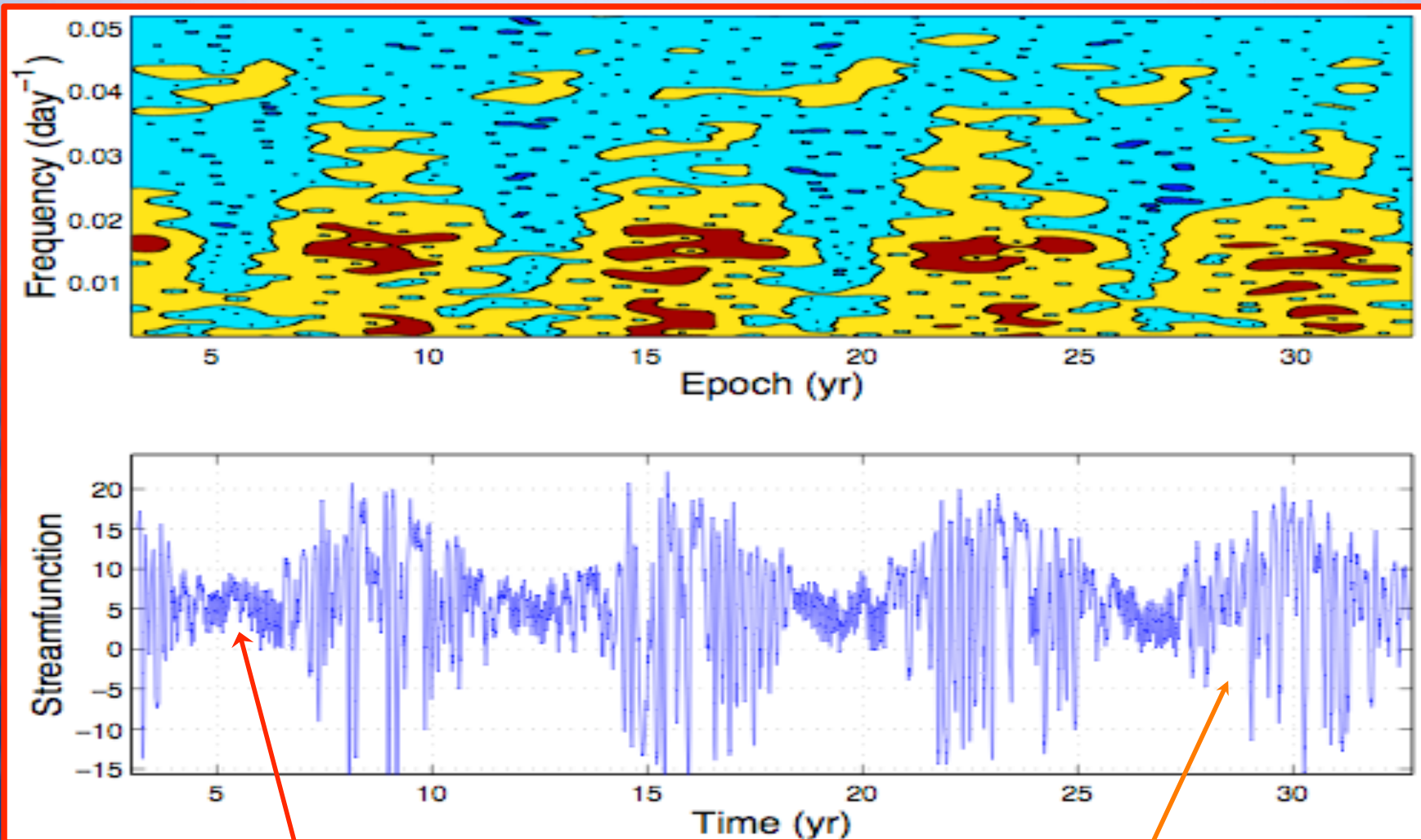
$$w(H_e) = \left[\gamma \zeta_g - \alpha \nabla^2 T \right],$$

with $\gamma = c_1(f_0 L/U)(H_e/H_a)$ and $\alpha = c_2(g/T_0 U^2)(H_e^2/H_a)$, where H_a is the layer depth of the free atmosphere (~ 10 km), and ζ_g the atmospheric geostrophic vorticity.

- Two components: one **mechanical**, due to the geostrophic flow ζ_g above the marine ABL and one **thermal**, induced by the SST front.



Evolutionary spectral analysis



30-day oscillation

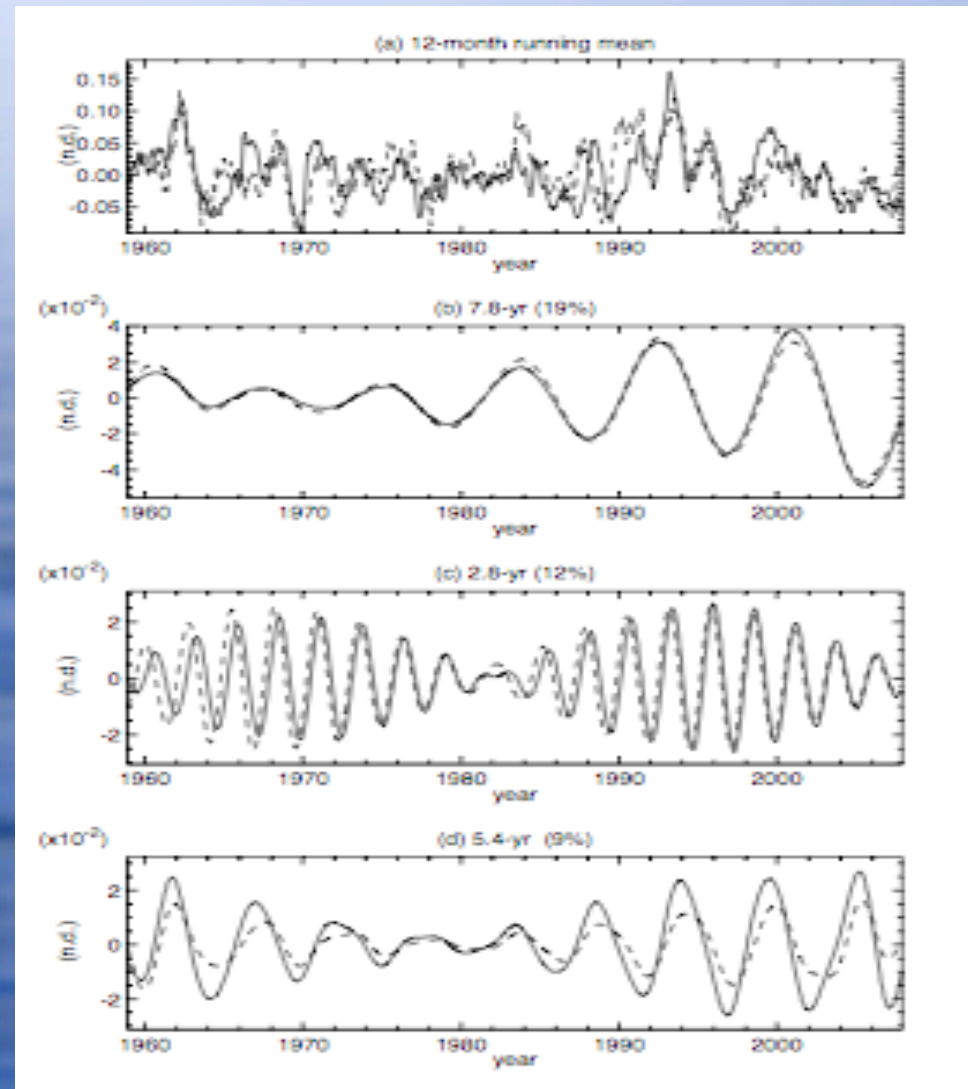
70-day oscillation

The 7–8-yr mode in atmospheric data

Likewise a contentious issue

Simulate atmospheric response to SODA data over the Gulf Stream region

- ◆ Use SST (–5 m) data from the SODA reanalysis (50 years)
- ◆ Use the FGS'07 QG model in periodic β -channel
 - baroclinic + marine ABL
- ◆ Figure shows NAO index:
 - simulated (solid)
 - observed (dashed)



Concluding remarks

- ♦ Tipping points and bifurcations: do they really help?
 - Yes, if properly understood and carefully applied!
- ♦ Can we predict them?
 - Yes, depending on the problem and the data!

Some references

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- Jiang, S., F.-F. Jin, and M. Ghil, 1995: Multiple equilibria, periodic, and aperiodic solutions in a wind-driven, double-gyre, shallow-water model, *J. Phys. Oceanogr.*, **25**, 764–786.
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Reserve slides

Climate models (atmospheric & coupled) : A classification

- **Temporal**

- stationary, (quasi-)equilibrium
- transient, climate variability

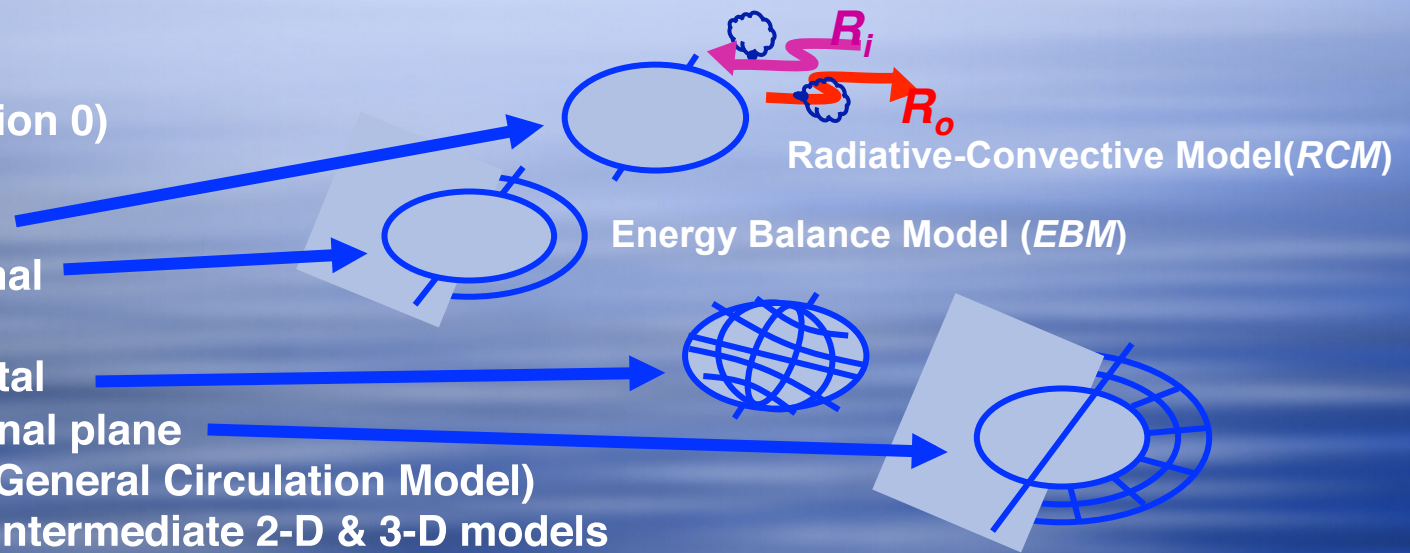
- **Space**

- 0-D (dimension 0)
- 1-D
 - vertical
 - latitudinal
- 2-D
 - horizontal
 - meridional plane
- 3-D, GCMs (General Circulation Model)
- Simple and intermediate 2-D & 3-D models

- **Coupling**

- Partial
 - unidirectional
 - asynchronous, hybrid
- Full

Hierarchy: from the simplest to the most elaborate,
iterative comparison with the observational data



Modeling Hierarchy for the Oceans

Ocean models

- ◆ 0-D: box models – chemistry (BGC), paleo
- ◆ 1-D: vertical (mixed layer, thermocline)
- ◆ 2-D – meridional plane – THC
 - also 1.5-D: a little longitude dependence
 - horizontal – wind-driven
 - also 2.5-D: reduced-gravity models (n.5)
- ◆ 3-D: OGCMs - simplified
 - with bells & whistles (“kitchen sink”)

Coupled 0-A models

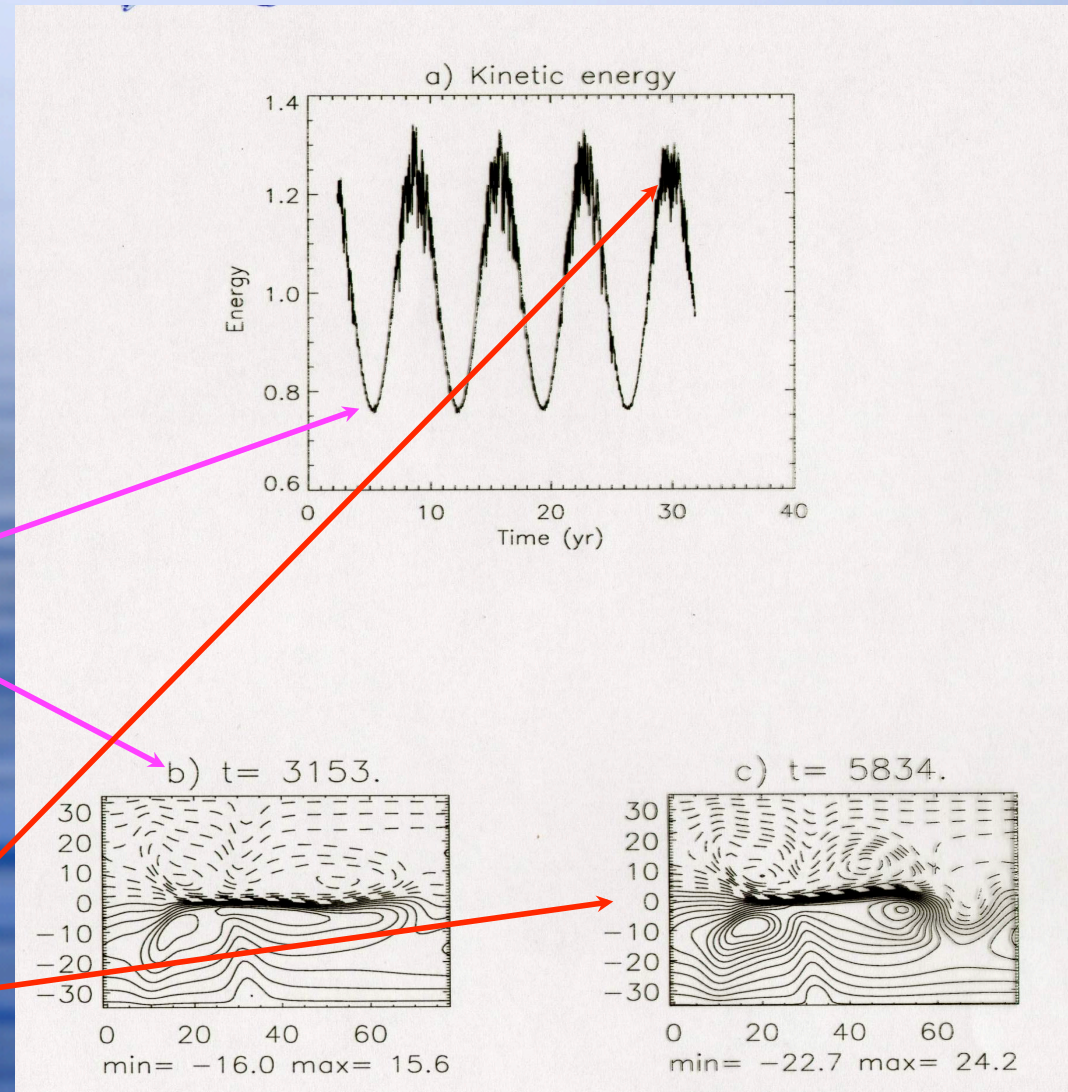
- ◆ Idealized (0-D & 1-D): intermediate couple models (ICM)
- ◆ Hybrid (HCM) - diagnostic/statistical atmosphere
 - highly resolved ocean
- ◆ Coupled GCM (3-D): CGCM

Forced 7-year cycle in the FGS'04 model

Slow amplitude
modulation of 1 °C in
the SST front

Low-energy phase

High-energy phase



Spin-up of atmospheric jet

SST front:

$L_{oc} = 600$ km,

$\Delta T = 3.5$ °C,

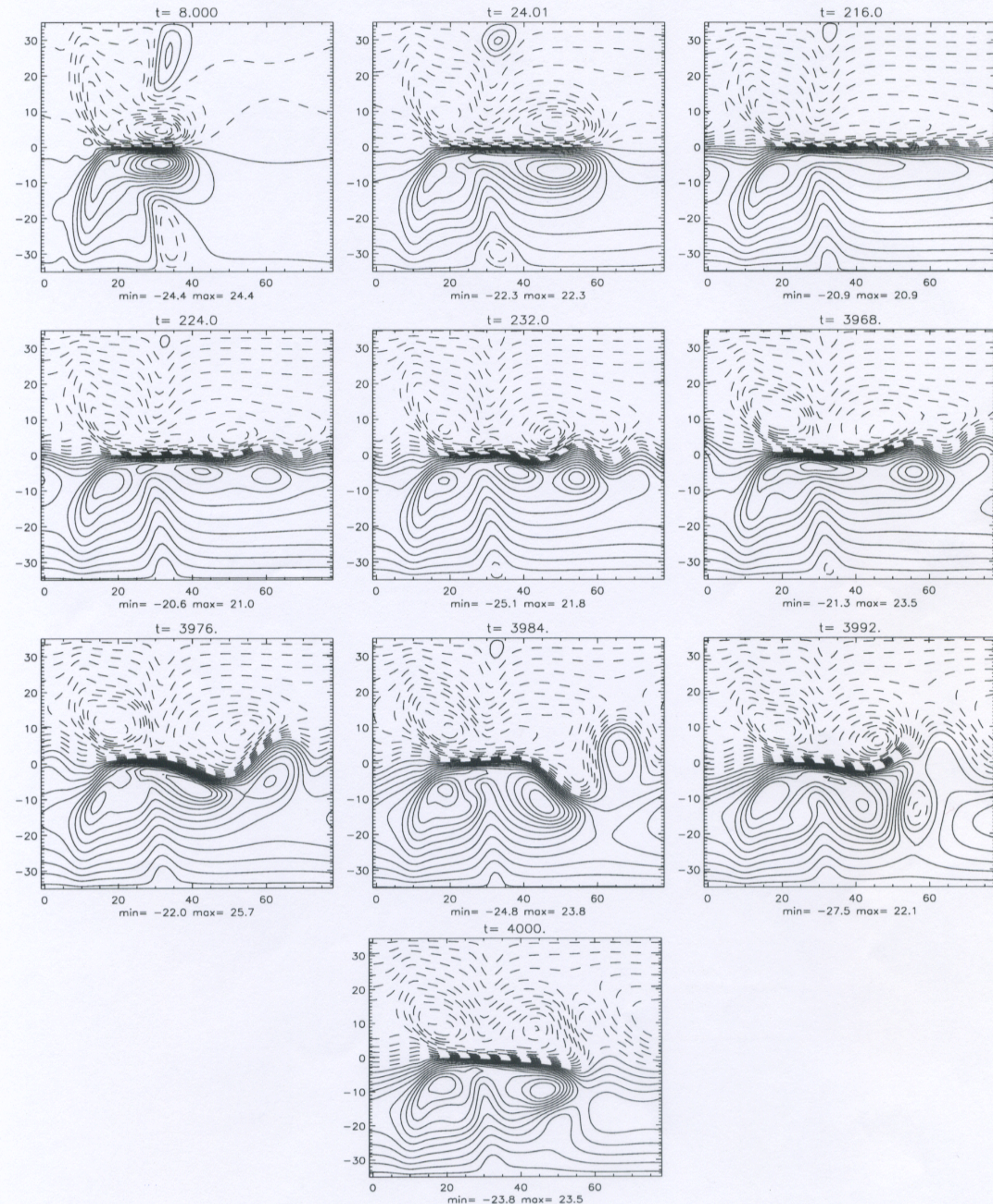
$d = 50$ km

Atmospheric jet

spins up from

$L_a = 2000$ km to

$L_a = 4000$ km, much greater speed and strong recirculation



Can we, nonlinear people, help?

The uncertainties
might be *intrinsic*,
rather than mere
“tuning problems”

If so, maybe
*stochastic structural
stability* could
help!

Might fit in nicely with
recent taste for
“stochastic
parameterizations”

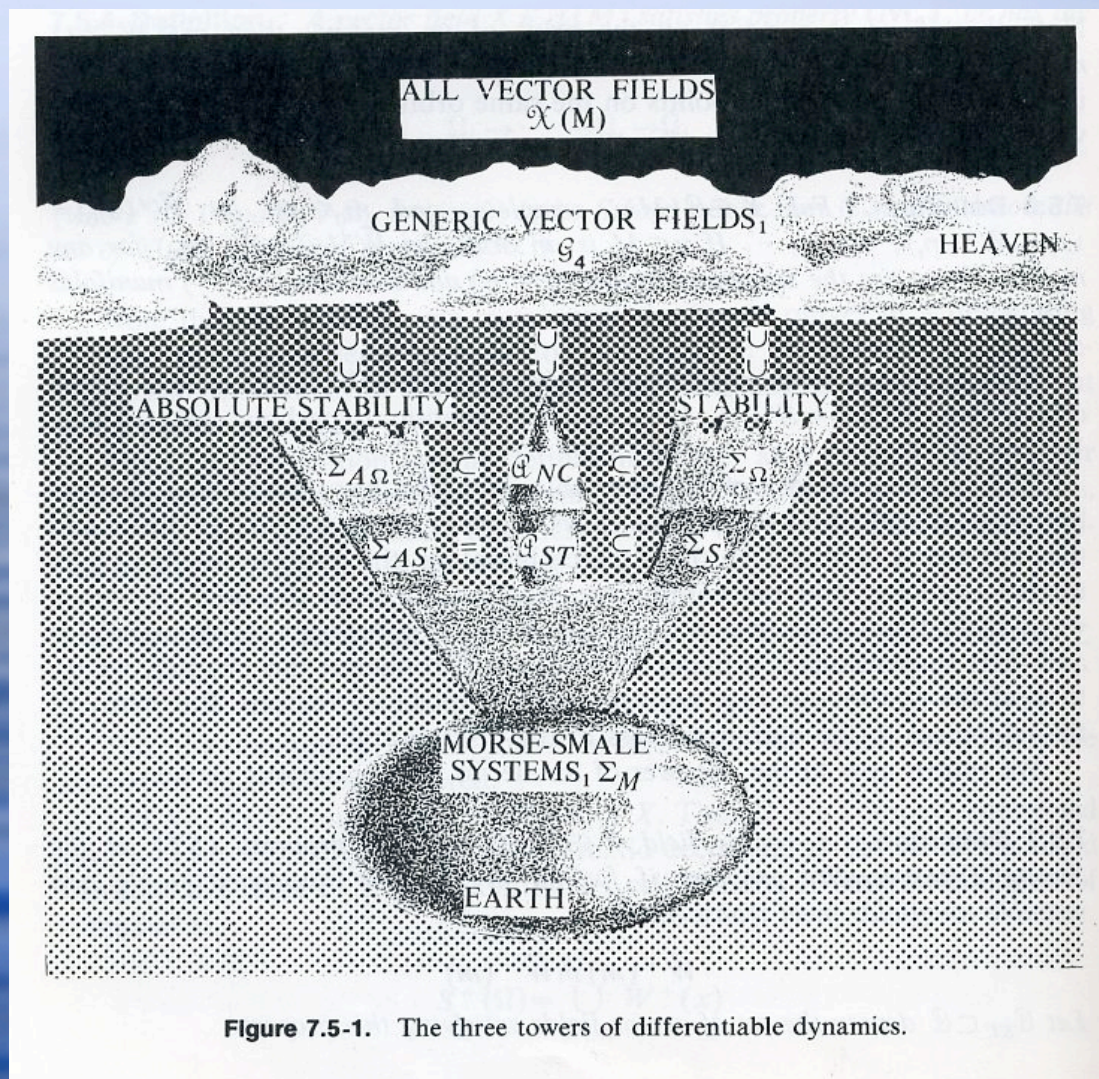


Figure 7.5-1. The three towers of differentiable dynamics.

The DDS dream of structural stability (from Abraham & Marsden, 1978)